

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINERS' REPORT**

April 2020 Examinations

### **Subject SP6 - Financial Derivatives Principles**

#### **Introduction**

The Examiners' Report is written by the Principal Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Mike Hammer  
Chair of the Board of Examiners  
July 2020

### **A. General comments on the aims of this subject and how it is marked**

The aim of Financial Derivatives Principles (SP6) is to develop a student's ability to understand different types of financial derivatives and their uses, the markets in which they are traded, methods of valuation of financial derivatives, and the assessment and management of risks associated with a portfolio of derivatives. It builds on material covered in earlier subjects, particularly Loss Reserving and Financial Engineering (CM2).

Candidates are reminded to ensure that their answers are sufficiently detailed to demonstrate understanding, as there were instances where inadequate explanations led to candidates scoring less well on questions than they might have done. The model solutions are intended to reflect the level of detail that a high scoring candidate might be able to produce. For many questions there are more marks available than the question requires to achieve full marks. This reflects that the examiners will give credit for valid alternative solutions, particularly in questions focussed on higher level skills.

Candidates who give well-reasoned points, not in the marking schedule, are awarded marks for doing so.

### **B. Comments on student performance in this diet of the examination**

Overall the paper was well attempted by most candidates. Suitably prepared candidates were able to score highly across all questions demonstrating the ability to apply their SP6 knowledge and techniques to unfamiliar situations.

This was the first online / open book based SP6 paper and candidates benefitted from the ability to check calculations and create charts in Excel. Within the exam we still expected to be shown intermediate steps of the relevant calculations. Given the time constraints, we felt that the open book nature of the paper was of limited value to candidates. We appreciated that writing out formulas in Word can be time consuming and gave full marks for appropriate formulas written in plain text, as illustrated in the marking schedule.

The questions where most marks were missed were:

- 1 (ii) - some candidates commented on basis risk, without first describing the associated risks of the case study and how quantos could help
- 2 (iv) - this was a difficult question that only the best prepared candidates were able to score full marks on
- 5 (ii) - some candidates missed marks by not working through the pay-offs for each of the cases with respect to possible values of S

### **Pass Mark**

The pass mark for this exam was 60.

37 candidates presented themselves and 22 passed.

## Solutions

### Q1

- (i) A quanto option is an option where the price of the underlying asset is measured in one currency,... [0.5]  
... but the payoff is another currency. [0.5]

An example would be a call option with the underlying being the price of the commodity. [1]

The strike price is given in the currency of country X, say  $K$ . [0.5]

The price of the commodity is in the currency of country Y, say  $S$ . [0.5]

At expiry the variable being measured is in the currency of country X. [0.5]

This is calculated as a constant, say  $C$ , multiplied by the price of the commodity in the currency of Y. [0.5]

At maturity the payoff is then  $\max(CS - K, 0)$ , all paid in the currency of X. [0.5]

[Award up to one mark for further relevant examples]

[Max 4]

- (ii) The risks associated with the future buying of the commodity in the commodity markets include:
- Price risk in Country Y of the commodity [0.5]
  - Foreign exchange rate risk [0.5]
  - Physical availability [0.5]
  - Political risk [0.5]

Quanto options can be used with the aim of hedging these risks:

- Using call options [0.5]
- With payoff in currency of Country X [0.5]
- This should reduce foreign exchange rate risk [0.5]
- And the price risk in Country Y of the commodity [0.5]

Some issues or risks remain:

- The quanto is cash settled so it would not actually deliver the physical commodity [0.5]
- ... therefore physical availability risk remains [0.5]
- Hedging cost / option premium [0.5]
- Basis risks [0.5]
- ... Such as choice of strike price (further OTM less protection) [0.5]
- ... Number of contracts to purchase (might not be known) [0.5]
- ... Modelling required / assumptions made [0.5]
- Political risk will remain [0.5]
- Rolling the hedge might be difficult or expensive [0.5]
- Issues related to collateral, credit risk to counterparty, clearing [0.5]
- OTC versus listed options issues [0.5]
- Regulatory risk [0.5]
- Reporting such as accounting disclosures [0.5]
- Liquidity issues of quanto options [0.5]
- There may already be a natural hedge, e.g. revenue in Country Y [0.5]

[Max 6]  
[Total 10]

*Candidates scored reasonably well on this question, in relation to the second part some candidates commented on basis risk, without first describing the associated risks of the case study and how quantos could help.*

**Q2**

- (i) An exotic option is usually an over-the-counter option, [0.5]  
which have more complex properties than a vanilla option [0.5]  
and are less actively traded. [0.5]  
[Max 1]

- (ii) A cash-or-nothing binary put option pays a **fixed amount of cash** to the option holder if the option is exercised when the underlying asset is **below** the strike price. [1]

An asset-or-nothing binary call option pays the **underlying asset price** at time of exercise to the option holder if the option is exercised when the underlying asset is **above** the strike price. [1]

[Total 2]

- (iii)  $S(T) | F_t = S(t) e^{(r-q-\sigma^2/2)(T-t) + \sigma(W(T)-W(t))}$

Written without using “equation” functionality in Word is also acceptable, e.g.

$$S(T)|F(t) = S(t)\exp((r-q-\sigma^2/2)(T-t)+\sigma(W(T)-W(t)))$$

[2]

[Total 2]

- (iv) Using the Cameron-Martin-Girsanov theorem, as the boundedness condition is trivially satisfied, the  $Q$  Brownian motion becomes: [0.5]

$$W^Q(t) = W(t) + \int_0^t -2\sigma ds = W(t) - 2\sigma t, \text{ or}$$

$$\text{Or } W[Q](t) = W(t) + \int_0^t -2\sigma ds = W(t) - 2\sigma t \quad [0.5]$$

$$dS(t) = S(t)(r - q)dt + S(t)\sigma(dW^Q(t) + 2\sigma dt), \text{ or}$$

$$dS(t) = S(t)(r-q)dt + S(t)\sigma(dW[Q](t) + 2\sigma dt) \quad [0.5]$$

$$dS(t) = S(t)(r - q + 2\sigma^2)dt + S(t)\sigma dW^Q(t), \text{ or}$$

$$dS(t) = S(t)(r-q+2\sigma^2)dt + S(t)\sigma dW[Q](t) \quad [0.5]$$

The quick way of integrating this is to spot that this is lognormally distributed. Instead of a drift term of  $r - q$  under the  $P$  measure it is  $r - q + 2\sigma^2$ . Substituting this into (iii) results in: [0.5]

$$S(T)|F_t = S(t)e^{(r-q+3\sigma^2/2)(T-t)+\sigma(W^Q(T)-W^Q(t))}, \text{ or}$$

$$S(T)|F(t) = S(t) \exp((r-q+3\sigma^2/2)(T-t) + \sigma(W[Q](T)-W[Q](t))) \quad [0.5]$$

[Total 3]

(v) Using the Cameron-Martin-Girsanov theorem: [0.5]

$$\frac{dQ}{dP} = e^{-\int_0^T \gamma dW(t) - 0.5 \int_0^T \gamma^2 dt}, \text{ or}$$

$$dQ/dP = \exp(-\int_0^T \gamma dW(t) - 0.5 \int_0^T \gamma^2 dt) \quad [0.5]$$

$$\frac{dQ}{dP} = e^{-\int_0^T -2\sigma dW(t) - 0.5 \int_0^T 4\sigma^2 dt}, \text{ or}$$

$$dQ/dP = \exp(-\int_0^T [-2\sigma dW(t) - 0.5 \int_0^T 4\sigma^2 dt])$$

$$\frac{dQ}{dP} = e^{2\sigma W(T) - 2\sigma^2 T}, \text{ or}$$

$$dQ/dP = \exp(2\sigma W(T) - 2\sigma^2 T). \quad [0.5]$$

$$\frac{dP}{dQ} = e^{-2\sigma W(T) + 2\sigma^2 T} \text{ (using the information in the question), or}$$

$$dP/dQ = \exp(-2\sigma W(T) + 2\sigma^2 T) \quad [0.5]$$

$$= e^{-2\sigma(W^Q(T)+2\sigma T)+2\sigma^2 T} \text{ (using (iii)), or}$$

$$= \exp(-2\sigma(W[Q](T)+2\sigma T)+2\sigma^2 T) \quad [0.5]$$

$$= e^{-2\sigma W^Q(T)-2\sigma^2 T}, \text{ or}$$

$$= \exp(-2\sigma W[Q](T) - 2\sigma^2 T) \quad [0.5]$$

[Total 3]

[Total 11]

*This was the worst answered question in the paper, with most marks being missed in relation to part (iv). This was a difficult question that only the best prepared candidates were able to score full marks on.*

**Q3**

- (i) An investment asset is held by a significant number of investors for investment rather than consumption purposes. [1]

E-Coin does not appear to be easily consumed... [½]

... as it is a digital currency... [½]

.. and therefore would be considered an investment asset. [½]

[Max 2]

- (ii) *[Half a mark for each point]*

The asset (i.e. E-Coin).

The contract size (i.e. how many E-Coins)

Delivery arrangements (i.e. how the E-Coin will be transferred on delivery)

Delivery months

How the price of the E-Coin will be quoted...

...e.g. currency or tick size.

Daily price movement limit

Position limits (i.e. maximum number of positions an investor may hold)

[Max 3]

- (iii) The exchange may be exposed to ...

credit (or counterparty) risk from the investors defaulting... [1]

... which would depend on the value of the E-Coin positions in the future.[0.5]

... particularly because the investors may not have substantial amounts of assets to post as margin if the markets move against them... [0.5]

.. which could be exacerbated if there is a high concentration of less

experienced investors trading the E-Coin futures... [1]

...and given that E-Coin's price could be very volatile. [0.5]

Reputational risk if the less experienced investors lose significant sums trading the futures... [1]

... particularly if the exchange doesn't make the risks of trading the futures clear. [0.5]

Operational risk if the exchange's systems are not suitable for less experienced investors. [0.5]

Legal risk of contractual non-validity [0.5]

*[Markers: please award credit for suitable suggestions]*

[Max 3]

- (iv) The exchange will mitigate credit risk by ...

- ... requiring initial and variation margin to be posted by the investors (via their clearing house member). [1]
- In particular, the exchange could propose a high initial margin given the price of E-Coin could be very volatile. [0.5]
- ... having a reserve fund to cover any unexpected losses. [0.5]
- ... enforcing position limits. [0.5]
- ... conducting regular monitoring of the exposures of different clearing members and investors to the E-Coin futures. [0.5]

Reputation risk and legal risk can be mitigated by...

- ... ensuring all the documentation and risks of the E-Coin futures are clear to investors... [0.5]
- ... ensuring the E-Coin futures meet the prevailing regulatory requirements [0.5]
- ... marketing the E-Coin futures at more experienced investors. [0.5]
- Operational risks can be mitigated by thoroughly testing any new systems to ensure they are appropriate for less experienced investors. [0.5]
- The limited supply and nature of E-Coins could lead to very high levels of volatility or issues with the supply of the assets... [0.5]
- ...which could also increase the risk of a failure to deliver. [0.5]
- The exchange could limit access to professional investors [0.5]
- ... perform credit checks or other requirements for access [0.5]
- ... limit price moments (e.g. per day) [0.5]
- ... suspend trading [0.5]

[Markers: please award credit for suitable suggestions]

[Max 3]

[Total 11]

*This question was well answered by most candidates.*

#### Q4

(i) Mortality Swaps

Let  $S(t)$  represent a survival index where  $S(t) = p(0) \times p(1) \times \dots \times p(t-1) \dots$  [1]  
 ...and  $p(t)$  represents the probability that an individual that an individual does not lapse from time  $t$  to time  $t+1$ . [1/2]

The bank would pay a fixed amount  $k \hat{S}(t)$  for  $t = 1, 2, \dots, T$  where  $\hat{S}(1), \hat{S}(2)$  etc are fixed at time 0 when the contract is arranged. [1/2]

... in return the bank would receive a floating amount  $kS(t)$  . [1/2]

If more policyholders lapse than expected, then the life insurer will receive the same fixed amounts  $k \hat{S}(t)$ , but pay a lower amount  $kS(t)$  to the bank, which will mitigate the loss the life insurer faces from higher lapses. [1]

#### Principal-at-risk Mortality Bond

The life insurer would issue a bond to the Bank... [½]  
 ...which would pay coupons at a specified rate that are not affected by lapse rates... [½]  
 ... and the principal repayment would be reduced if lapse rates in one or more years were to exceed a certain threshold... [1]  
 ... which would hedge the life insurers losses when lapses rise. [½]

#### Survivor Cap

There is no contract that the bank could offer that is directly analogous to a “survivor cap”. [½]  
 A “lapse cap” would only pay out if there were reduced lapses which would not help hedge the impact of higher lapses. [½]  
 The bank could offer a lapse floorlet... [1]  
 ... with payoff  $Max(\hat{S}(t) - S(t), 0)$  ... [1]  
 ... which would payout when lapses increase and  $S(t)$  reduces. [½]  
 A series of lapse floorlets could be combined into a lapse floor. [½]

[Max 6]

(ii) Moral hazard is the action of a party who behaves differently (typically inappropriately or less carefully) from the way they would behave if they were fully exposed to the consequences of that action. [1]

[Total 1]

(iii) Lapses can be caused by factors specific to a life insurer (e.g. poor customer service) and industry wide factors (e.g. change in regulation or tax). [0.5]  
 If the life insurer is hedged against their specific lapse experience rather than a national index, then the life insurer may not act as carefully to limit the instances of lapses... [1]

... by, for example, cutting back on product offerings, reducing underwriting standards, or providing poorer customer service. [1]

If the payouts could be based on publically available statistics, then lapses specific to the behaviour of the life insurer would not lead to a payment on the securities, reducing the moral hazard. [0.5]

[Max 2]

(iv) The bank could limit the notional of the contracts offered to mitigate lapse risk (e.g. to 90% of exposed to risk)... [1]

... to ensure that the life insurer is always partially exposed to lapses. [0.5]

The bank could limit the maximum payoff such that the life insurer is exposed to lapses above a certain point. [1]

The bank could require that certain Key Performance Indicators need to be met in order for the contracts to pay off in part/full. [1]

[Markers: please award credit for alternative sensible suggestions]

[Max 2]

- (v) The life insurer could...
- ... adopt a policy of natural hedging... [1]
  - ... for example, by offering products with guarantees to customers which they lose should they lapse. **[Please award credit for any other example]** [0.5]
  - ... enter into a reinsurance deal to share some or all of the lapse risk. [1]
  - ... securitise a block of business to transfer all risks into the capital markets. [1]
  - ... increase the prices of products that are exposed to lapse risk and reserve the additional cost to act as a buffer. [1]
  - ... engage proactively with customers and their representatives (e.g. IFAs) to ensure the products meet their needs and are being appropriately serviced. [1]
  - ... set a penalty on lapse [0.5]
  - ... provide loyalty awards [0.5]

[Max 3]

[Total 14]

*This question was well answered by most candidates.*

### Q5

- (i) The graph of  $X(S(T))$  can be determined by looking at different ranges of  $S(T)$ :

$$0 \leq S(T) \leq K$$

For these values of  $S(T)$ :

$$\begin{aligned} X(S(T)) &= \max(K - (K - 0.5S(T)) - 2(K - S(T)), 0), \\ &= \max(2.5S(T) - 2K, 0). \end{aligned} \quad [0.5]$$

This is greater than zero only when  $S(T) > 0.8K$ . [0.5]

For  $0.8K < S(T) \leq K$ ,  $X(T)$  is a straight line with  $X(0.8K) = 0$  and  $X(K) = 0.5K$ . [0.5]

$$2K \geq S(T) > K$$

For these values of  $S(T)$ :

$$\begin{aligned} X(S(T)) &= \max(K - (K - 0.5S(T)) - 2(S(T) - K), 0), \\ &= \max(2K - 1.5S(T), 0). \end{aligned} \quad [0.5]$$

This is greater than zero only when  $S(T) < 4/3 K$ . [0.5]

For  $K < S(T) \leq 4/3 K$ ,  $X(T)$  is a straight line with  $X(K) = 0.5 K$  and  $X(4/3 K) = 0$ . [0.5]

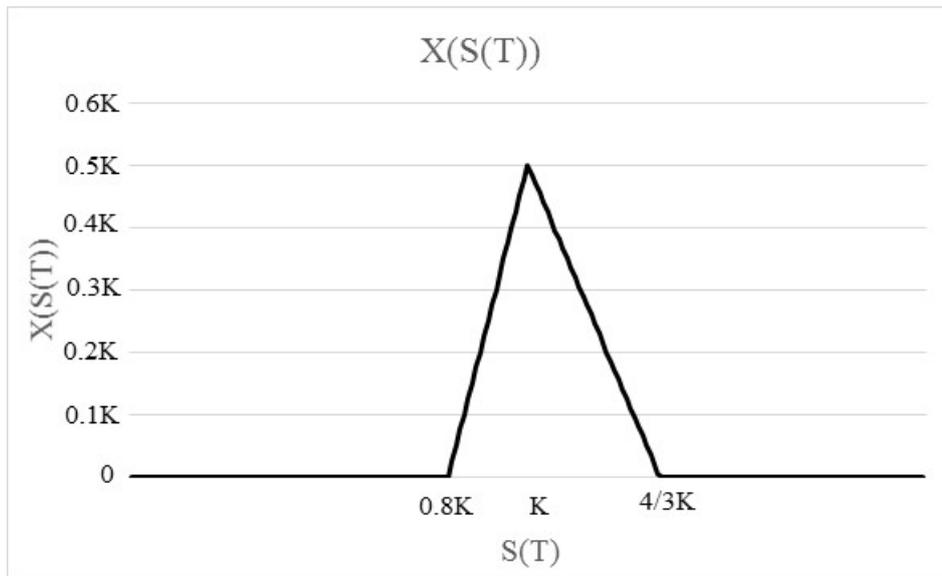
$S(T) > 2K$

For these values of  $S(T)$ :

$$X(S(T)) = \max(K - (0.5S(T) - K) - 2(S(T) - K), 0),$$

$$= \max(4K - 2.5S(T), 0).$$
 [0.5]

$$= 0.$$
 [0.5]



Note to markers: when creating the chart in Excel, numerical examples for  $K$  are acceptable.

Full marks are available for drawing the correct chart:

Correct left zero payoff:  $X = 0$  for  $S < 0.8 K$  [1]

Correct peak:  $X = 0.5 K$  for  $K$  [1]

Correct right zero payoff:  $X = 0$  for  $S > 4/3K$  [1]

Label the y-axis [1]

Label the x-axis [1]

[Max 5]

(ii) From part (i)  $X(S(T))$  can be written as:

$$X(S(T)) = \begin{cases} 0 & S(T) \leq 0.8K, \\ 2.5S(T) - 2K & 0.8K < S(T) < K, \\ 2K - 1.5S(T) & K < S(T) < 4/3K, \\ 0 & S(T) \geq 4/3K. \end{cases}$$

[1]

In order to show the result in the question the portfolio of options should be set-up and the pay-off should be checked that it equals the above expression for  $X(S(T))$ . Let the pay-off from the derivative portfolio in  $T$  years be  $V$ . [0.5]

The pay-offs from the derivatives are as follows:

Buying 2.5 European call options:  $2.5 \max(S(T) - 0.8K, 0)$ . [0.5]

Selling 4 European call options:  $-4 \max(S(T) - K, 0)$ . [0.5]

Buying 1.5 European call options:  $1.5 \max(S(T) - 4/3 K, 0)$ . [0.5]

$$V = 2.5 \max(S(T) - 0.8K, 0) - 4 \max(S(T) - K, 0) + 1.5 \max(S(T) - 4/3 K, 0).$$

[0.5]

Cases:

$$0 \leq S(T) \leq 0.8K$$

$$V = 0 - 0 + 0 = 0.$$

[0.5]

$$0.8K < S(T) \leq K$$

$$V = 2.5(S(T) - 0.8K) = 2.5S(T) - 2K.$$

[0.5]

$$K < S(T) < 4/3 K$$

$$V = 2.5(S(T) - 0.8K) - 4(S(T) - K) = 2K - 1.5S(T).$$

[0.5]

$$S(T) \geq 4/3 K$$

$$V = 2.5(S(T) - 0.8K) - 4(S(T) - K) + 1.5(S(T) - 4/3 K) = 0.$$

[0.5]

Therefore  $V = X(S(T))$ .

It is assumed that derivatives can be bought in non-integer amounts... [0.5]

... that the premiums can be ignored... [0.5]

... there are no costs associated with clearing and collateralising .... [0.5]

... and no is made allowance for tax. [0.5]

[Max 6]

(iii) There is significant basis risk to the investment bank. [0.5]

This is as a result of the proposed underlying asset having different characteristics to the derived manufacturing company's shares. [0.5]

Primarily this is as a result of the derived manufacturing company's shares being privately held whereas the underlying shares are actively traded. [0.5]

This reflects the differences between private and public equity. [0.5]

This could be due to:

- Private companies may not pay a dividend; and [0.5]
- Private companies may be in the earlier stages of growth resulting in more volatile derived stock prices. [0.5]

*(Note to markers: any other suitable examples can be awarded 0.5 marks up to a maximum of 1 mark.)*

The calculation of the derived share value of the manufacturing company may introduce additional basis risk. [0.5]

Basis risk may also be introduced if there are no options with the required time to maturity required by the bank. [0.5]

Similarly if the strike prices required are not available. [0.5]

[Max 3]

**[Total 14]**

*This question was well answered by most candidates. Some candidates missed marks by not working through the pay-offs for each of the cases with respect to possible values of  $S$  for the second part of this question.*

**Q6**

- (i) Credit ratings represent the credit worthiness of a corporate bond or issuer (i.e. the ability and propensity of an issuer to pay interest and principal on a bond)

[1]

When a bond is downgraded to a lower credit rating it will imply a higher probability of default or higher loss given default.

[1]

This would lead to a lower demand for the bond in the market...

[0.5]

... or pricing models used by investors to value the bond determining a lower value...

[0.5]

... which would reduce the price of the bond.

[0.5]

The regulatory regime will also require insurers to sell the Sub-Investment grade bond which will increase the supply of Sub-Investment Grade bonds, further depressing their prices.

[1]

[CR Unit 16, pp9-11]

[Max 2]

- (ii) The regulatory regime will ensure that insurers' portfolios are only comprised of bonds rated Investment Grade...

[0.5]

... which will limit the expected probability of default ...

[1]

... or loss given default on the whole portfolio...

[0.5]

... especially because most defaults occur once a bond has downgraded to Sub-Investment grade, rather than "jumping" to default.

[0.5]

For example, BBB rated bonds are described as having "adequate protection" on S&P's rating scale, whereas BB bonds "face uncertainties".

[0.5]

The regulatory regime will also foster a culture of regularly monitoring the credit worthiness of the portfolios, which could reduce credit risk.

[1]

[Max 2]

- (iii) The regulatory regime will only mitigate credit risk to the extent that the credit rating incorporates the credit risk of the bonds...

[0.5]

... but the credit rating represents one simple comparator statistic that may not fully reflect the credit risk of each bond.

[1]

For example, ratings may not fully allow for the recovery amount on default / the liquidity of the bonds

[0.5]

[0.5 maximum for any relevant example]

Credit ratings have also failed in certain historical events to forecast losses (e.g during the financial crisis).

[0.5]

Rating does not take correlation into account

[0.5]

Binary nature of Investment Grade versus non-Investment Grade has several issues, such as gaming and forced selling

[0.5]

Responsiveness - the credit ratings may not be refreshed frequently ...

[1]

... so the creditworthiness of the bonds could deteriorate materially or even default, before the bond has downgraded.

[0.5]

Scope - it is possible that the corporate bonds may cease to be rated by rating agencies. [0.5]

Consistency - credit ratings may be issued from a group of Credit Rating Agencies whose individual views could differ. [1]

The regulatory regime would need to set out clearly what rating should be referenced when determining whether a bond is Sub-Investment Grade. [0.5]

Transparency of fee structure - the rating agencies may have too familiar a relationship with the bond issuers, possibly opening themselves to undue influence or being misled. [0.5]

Competition - the rating agencies may have a monopoly position in the market which may reduce the incentive to continue to develop and adjust to changing market conditions. [0.5]

[Other valid points may be made]

[CR Unit 16, pp13-14]

[Max 4]

(iv) The table below shows the key calculation steps:

Year	Probability of Survival	Probability of Default in year
1	0.9600	0.0400
2	=0.9600*(1-4%) = 0.9216	=0.96*4%=0.0384
3	=0.9216*(1-4%) = 0.8847	=0.9216*4%=0.0369

[1.5]

The value of the payment leg is therefore:

$$= (0.9600 + 0.9216 + 0.8847) \times \$100m \times 2\% = \$5.53m \quad [1]$$

The value of the receive leg from the expected default payments is:

$$= (0.0400 + 0.0384 + 0.0369) \times \$100m \times (1 - 40\%) = \$6.92m \quad [1]$$

Therefore the total value is

$$= \$6.92m - \$5.53m = \$1.39m \sim \$1.4m \quad [0.5]$$

[Hull pp551-555]

[Sub-Total 4]

(v) For a \$100m notional we know the increase on value post downgrade is

$$= \$8.2m - \$1.4m = \$6.8m \quad [1]$$

Hence the total notional required is:

$$= \$30m / \$6.8m \times \$100m = \$441m \sim \$440m \quad [1]$$

[Sub-Total 2]

(vi) (a) There is a greater chance of the bond defaulting over a longer time horizon... [1]

- ... so the increase in value of the \$100m notional CDS will be higher... [0.5]  
... so the required notional will decrease. [0.5]
- (b) On default the payoff of the CDS will reduce... [1]  
... so the increase in value of the \$100m notional CDS will be lower... [0.5]  
... so the required notional will increase. [0.5]  
[Max 3]
- (vii) The insurer is required to sell the Sub-Investment Grade immediately, but on downgrade of the bond to BB the CDS will not settle given the outstanding term will be around 3 years and the bond has not defaulted. [1]  
In order to fund the \$30m cost, the insurer must either sell the CDS... [1]  
... or use the collateral / margin provided. [0.5]
- Selling the CDS may be unattractive due to transaction costs... [0.5]  
...or being selected against by market participants, who will likely be aware of the regulatory regime the insurer is operating in... [1]  
... or not be completed in a timely fashion. [0.5]
- Poor liquidity in single name CDS [0.5]  
Basis risk between derivative and physical [0.5]  
Notional amount is very different [0.5]  
Roll risk of CDS [0.5]  
Expertise and infrastructure required in using CDS [0.5]
- The margin provided by the counterparty may not be in the required form (e.g. it may be in physical assets rather than cash.) [0.5]
- The increase in value of the CDS may not also be as expected, if the assumptions used by the insurer are not borne out in practice or reflected in market pricing. [1]
- The margin will hedge the original balance sheet move but unless they sell the CDS they are then running an unhedged position against the credit of Big Bond. [0.5]
- [Max 3]  
[Total 20]

*This question was well answered by most candidates.*

**Q7**

(i)

Pension funds typically use derivatives to implement investment strategies that enable closer hedging between assets and liabilities. [1]

The pension fund might also use derivatives to access investment premia more capital effectively... [0.5]

...or to hedge against capital market moves. [0.5]

**Interest rate hedging**

Adverse movements in interest rates are a significant risk for the pension fund. [0.5]

Interest rate swaps and government bond repos are used to capital efficiently increase the interest-rate sensitivity of the pension fund's investment portfolio to be more closely matched to the interest-rate sensitivity of its liabilities. [1]

Swaptions may also be used [0.5]

**Inflation hedging**

Increases in inflation is also a significant risk for the pension fund. [0.5]

Inflation swaps are a capital efficient and alternative way to better match inflation linked liabilities. [0.5]

Inflation linked government bonds can also be used to better match inflation linked liabilities, including in repo. [0.5]

A further example might also include LPI Swaps or LPI bonds in the UK. [0.5]

Longevity swaps can also be written in derivative form to hedge the risk that pension fund members live longer than budgeted for. [0.5]

**Equity risk**

Falls in equity markets can pose a risk [0.5]

Protective put options can be used to hedge [0.5]

Foreign exchange risk can be hedged using FX derivatives [0.5]

Property valuation risk can be hedged using property derivatives [0.5]

[Max 4]

(ii)

Reducing equity exposure appears to be an obvious solution to reducing the risk of extreme falls in the equity market... [0.5]

... but the reduction in this exposure could lead to the following risks and problems:

- The pension fund is underfunded and likely needs investment returns to meet its objectives

- Equities provide excess expected returns over bonds or cash
- Equities probably play a key part in improving funding and meeting objectives
- Selling equities might therefore jeopardize meeting funding objectives
  
- selling the equity portfolio will incur selling costs;
- the equity market may be low resulting in reduced realised cash;
- further costs will be incurred in buying new assets (as it is unlikely that the assets will remain as cash);
- also costs will be incurred for investment advice;
- the assets that replace the equities will have their own risk characteristics...
- which could increase concentration risk in other asset classes;
- there may be conflicts with a sponsoring employer over reducing equity exposure.

[0.5 marks for each bullet]

*(Note to markers: this list is not exhaustive and other sensible reasons should be given credit.)*

[Max 2]

- (iii) Put options give the holder the right to sell at a pre-agreed price at a pre-agreed moment in time [0.5]

Put options allow holders to protect downside risk, while maintaining upside potential [0.5]

The pension fund would buy out of the money put options... [0.5]

... on an equity index (or specific equities) depending on the equities in the pension fund's portfolio. [0.5]

The number of puts bought depends on the amount of equity portfolio it wants to protect. [0.5]

The strike price will depend on the level of protection required. For example if the current equity underlying is 100 and if the pension fund wants downside protection to -10% then the puts will have strike prices of 90. [1]

In this example if the underlying falls below 90 then the put will rise in value in line with the loss on the equity underlying. This will therefore mitigate the effect of the fall in the equity underlying up to 10% (subject to the number of puts bought). [0.5]

The time to expiry will depend on the type of strategy adopted. For example the pension fund may buy short dated puts and sell them before expiry and buy new ones (i.e. roll the options). The alternative is purchasing sufficiently long dated puts if they are available. [1]

[Max 3]

- (iv) 0.5% of the equity portfolio is  $0.5\% \times 20,000,000 = 100,000$ , therefore there is a budget of 100,000 for hedging. [0.5]

30% of the current equity index level is 1,400. [0.5]

The put option price for a strike price of 1,400 is 0.11. [0.5]

As a result the number of options purchased is  $100,000 / 0.11 = 909,091$ . [0.5]

[Total 2]

- (v) The number of options to be purchased is a large proportion of the number of put options traded at 1,400 the previous day. [0.5]

This suggests liquidity in the market could be a problem at this strike price. [0.5]

Further, purchasing large number of options at this strike price could significantly distort the market and increase the option price during trading. [0.5]

This would increase the option price and reduce the number of options to be purchased, which may affect the hedge itself. [0.5]

The options close in strike price to 1,400 have greater liquidity and therefore could be a suitable substitute... [0.5]

... either instead of purchasing any at 1,400 or for completing the hedge should liquidity reduce significantly during trading. [0.5]

Moreover these other options appear to be better value, as a result of greater liquidity. [0.5]

An alternative approach would be purchase options over the period of several days to avoid distorting the market. [0.5]

A more drastic approach would be to consider this form of hedging given the high volume of options required given the size of the markets. [0.5]

*(Note to markers: alternative answers should be given credit.)*

[Max 3]

- (vi) The equity index:  $0.8 \times 2,000 = 1,600$ . [0.5]

Using the Garman-Kohlhagen formula for a put option with standard notation:

Time to maturity = 1/12 years.

Volatility = 50%.  
Strike price = 1,400.

The risk free rate and dividend yield are provided in the question.

$$d_1 = \frac{\ln(1,600/1,400) + (0.02 - 0.04 + 0.5 * 0.5^2)(1/12)}{0.5\sqrt{1/12}} = 0.98575. \quad [0.5]$$

$$\Phi(-d_1) = 0.16213. \quad [0.5]$$

$$d_2 = d_1 - 0.5\sqrt{1/12} = 0.84142. \quad [0.5]$$

$$\Phi(-d_2) = 0.20006. \quad [0.5]$$

Value of a put option:

$$1,400 \times e^{-0.02 \times 1/12} \times 0.20006 - 1,600 \times e^{-0.04 \times 1/12} \times 0.16213 = 21.07 \quad [0.5]$$

$$\text{Total value of put options: } 909,091 \times 21.07 = 19,158,000 \text{ (5 s.f.)} \quad [1]$$

[Total 4]

- (vii) The value of the equity portfolio has fallen by 20% as well. This results in the equity portfolio having a value of  $20,000,000 \times 0.8 = 16,000,000$ . [0.5]

In this situation the hedging strategy has performed well as the value of the equity portfolio has declined by 4,000,000 but the puts have a value of 19,158,000 (this does not include the ~100,000 cost of purchasing the options). [0.5]

The hedge has protected against the entire loss on the equity portfolio... [0.5]

... and in fact has over hedged so has profited materially from the drop in equity values... [0.5]

... which suggests the pension fund may be purchasing too many options... [0.5]

... although part of the rise in the hedge is the increase in implied volatility which has occurred at the same time as the market fall. [0.5]

[Max 2]

[Total 20]

**[Paper Total 100]**

*This question was well answered by most candidates. Some marks were missed in relation to (v) and (vii).*

**END OF EXAMINERS' REPORT**