

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2020 (am)

Subject CP2 – Modelling Practice Core Practices

Paper One

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. You are given this question paper and Excel file.
2. Mark allocations are shown in brackets.
3. Attempt all questions. Questions are to be answered as per ‘exam requirements’.

If you encounter any issues during the examination please contact the Examination team at
T. 0044 (0) 1865 268 255

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Exam requirements

1 Modelling steps and data checks

Read the background document that describes the scenarios that need to be modelled and documented for this project.

Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed robust reasonableness checks at each stage of your calculations.

- (i) Validate all the data provided on the scores, making any adjustments where necessary. [3]
- (ii) Calculate the average of the 25 metre scores for each competitor, using the data adjusted in part (i). [1]
- (iii) Calculate summary statistics, for senior competitors only, junior competitors only and also overall (i.e. all 100 competitors), as follows:
 - number of competitors
 - average score per card across the competitors
 - highest individual average score
 - lowest individual average score. [2]
- (iv) Calculate the 20 ‘points dropped’ figures for each competitor at 25 metres, i.e. 100 minus the recorded score. [2]
- (v) Calculate the individual mean and sample standard deviation for each competitor of these 25 metre ‘points dropped’ figures. [2]
- (vi) Perform checks on the random numbers that have been provided, including validation that they can be treated as having been generated from a continuous Uniform distribution on $[0,1]$. (It is not necessary to make any changes to the data.) [4]
- (vii) Simulate a series of scores for each competitor, using the results from step (iv), the Uniform random numbers and the formulae provided. A simulated score is required for each of the 10 cards for the 50 metre and the 100 metre events. [4]
- (viii) Calculate the average simulated score across all 10 cards for each competitor for:
 - (a) the 50 metre simulation.
 - (b) the 100 metre simulation. [2]
- (ix) Use your results from part (vii) to calculate the simulated trial score for each competitor. [2]

- (x) Determine the number of competitors to be offered a place in the Senior team and the Junior team, using the Minimum Qualifying Scores provided. [2]
 - (xi) Determine the overall sponsorship budget required in the base scenario. [1]
 - (xii) Using the number in the Senior team calculated from part (x), determine the highest Minimum Qualifying Score for juniors that would ensure the same number of juniors as seniors would receive a scholarship. [2]
 - (xiii) Produce one chart to illustrate how the different distances impact the average score per card achieved by both seniors and juniors. [3]
- [Sub-total 30]**

[**Note:** all scenarios outlined above should be modelled separately in your spreadsheet. The user should not need to change the parameters to see the results.]

2 Modelling technique and practice

- (i) Demonstration of good modelling techniques and practice [7]
 - (ii) Other checks [5]
- [Sub-total 12]**

3 Audit trail

Produce an audit trail for your spreadsheet model that includes the following aspects:

- purpose of the model
- data and assumptions used
- methodology, i.e. description of how each calculation stage in the model has been produced
- explanation of any further checks performed.

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, to continue work on it or to use it again for a similar purpose in the future.

Marks available for audit trail:

Audit approach

- | | | |
|-------|--|-----|
| (i) | Communication skills (the audit trail provides enough detail to be read as a self-standing document) | [4] |
| (ii) | Fellow student can review and check methods used in the model | [7] |
| (iii) | Senior actuary can scrutinise and understand what has been done | [7] |
| (iv) | Written in clear English | [4] |
| (v) | Written in a logical order | [3] |

Audit content

- | | | |
|--------|---------------------------------------|------|
| (vi) | All steps clearly explained | [8] |
| (vii) | Clear signposting included throughout | [4] |
| (viii) | Statement of assumptions made | [5] |
| (ix) | All model steps accurately covered | [16] |

[Sub-total 58]

[Total 100]

Background

A major sporting event is taking place next year. The government has agreed to provide funding for sports scholarships to cover training and equipment for talented individuals.

The director of the National Shooting Organisation has asked your manager, a qualified actuary, to investigate the likely funding needed from the government for scholarships awarded to the National Shooting team.

Scholarship trials are taking place in 3 months' time, after which the teams for the event will be selected. Team selection will be on the basis of each individual's score in the trials.

Your manager has been asked to determine the expected number of eligible individuals for the scholarships and hence the likely budget needed to fund them.

The National Shooting team will consist of a Senior team and a Junior team. The Junior team is for competitors who are aged 21 and younger. There is a Minimum Qualifying Score (MQS), which determines eligibility for inclusion of an individual in each team.

Shooting competitions are scored using 10 rounds of 10 shots known as a 'card'. In one card, a competitor can score up to a maximum of 100 points.

The Team Director has provided details of the last 20 cards shot by 100 individuals who have been actively participating in the sport. He has only provided details of competitors who always shoot a score of 90 or above (out of 100) per card.

The data provided shows competitor scores for 20 cards shot over a distance of 25 metres.

The trials will consist of 20 cards in total, with the first 10 cards being shot over a distance of 50 metres and the next 10 being shot over a distance of 100 metres. For each competitor, the trial score (i.e. the score used to determine team selection) will be the total score achieved over the 20 cards. This gives a maximum possible trial score for each competitor of 2,000 points. Competitors achieving the relevant MQS will be offered a place in the National Team.

The scholarship amounts that will be awarded per team member and the MQS required for team selection are as follows:

	<i>MQS</i> <i>(score out of 2,000)</i>	<i>Scholarship</i> <i>(\$)</i>
Senior team	1,950	20,000
Junior team	1,940	15,000

Competitors achieving a score equal to or higher than the MQS will be offered a place in the team and receive the scholarship amount. There is no restriction on numbers: all members achieving the MQS will be offered a place in the team and will receive the scholarship.

Base scenario

Your manager has asked you to analyse the data provided and then to produce a set of simulations of the trial scores for each of the 100 individuals identified by the Team Director.

The simulation requires adjusted scores that are calculated on a ‘points dropped’ basis, i.e. points dropped is equal to 100 minus the score obtained.

To simulate the scores for the 50 metre and 100 metre events from the 25 metre data, the following formula is to be used:

Distance *Score (out of 100) for card i for competitor n (where $1 \leq i \leq 100$)*

$$50 \text{ metres} \quad \text{Min} \left[100, 100 - rv_{i,n} + \frac{\text{points dropped } s.d.n}{4} \right] \text{ for } 1 \leq i \leq 10$$

$$100 \text{ metres} \quad \text{Min} \left[100, 100 - rv_{i,n} + \frac{\text{points dropped } s.d.n}{2} \right] \text{ for } 11 \leq i \leq 20$$

where:

- points dropped *s.d.n* is the standard deviation of the points dropped over the 20 results from the 25 metre data for competitor *n*.
- the random variable ($rv_{i,n}$) component in the formula above is an exponential random variable that varies by card (*i*) and competitor (*n*).

The random variable ($rv_{i,n}$) can be calculated using the mean and standard deviation of the ‘points dropped’ for each competitor *n*, based on the 25 metre data, as follows:

$$rv_{i,n} = \frac{-\ln(U_{i,n})}{\lambda_n}$$

where:

- λ_n for competitor *n* is simulated from an exponential distribution with parameters

$$50 \text{ metres} \quad \lambda_n = \frac{1}{\text{points dropped } mean_n + \text{points dropped } s.d.n}$$

$$100 \text{ metres} \quad \lambda_n = \frac{1}{\text{points dropped } mean_n + 2 \times \text{points dropped } s.d.n}$$

- $U_{i,n}$ is a random variable from the Uniform (0,1) distribution.

To simulate the 20 cards for each of the 100 competitors, your manager has provided you with 2,000 random numbers that were generated from a continuous Uniform distribution on [0,1].

After completing the simulations of the trial scores, your manager has asked you to use these simulations to determine the number of places to be offered in each of the teams and hence the overall sponsorship budget required.

Equality scenario

The Team Director has heard that the board of the National Shooting Organisation wants to promote the sport to younger people and so wants to ensure equal numbers of seniors and juniors receive a scholarship. Your manager would like you to model this by changing the MQS for juniors so that the number of juniors being awarded a scholarship equals the number of seniors.

Your manager is out of the office and cannot be contacted for the next 3 hours. They would like the above calculations finished and documented in an audit trail ready for their return.

END OF PAPER