Exploring Mortality Experience in Pension Schemes: Traditional vs. Data-Driven Methods

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Abstract

Mortality rates are influenced by numerous factors that fluctuate over time. Traditional analyses often use a limited set of pre-determined factors, such as age and gender, for simplicity. Improving this process could benefit stakeholders by providing a deeper understanding of mortality factors, leading to more informed financial planning and risk management.

Although traditional methods provide a robust framework, they may overlook underlying indicators and patterns; alternatively, data science potentially captures more complex relationships between mortality factors within a specific population.

SAPS S3 tables typically provided a good fit to the LGPS EW mortality experience data as a baseline measure. Subdividing by IMD leads to significant underestimation of exit amounts at lower IMDs and overestimations at higher values in comparison to their IMD-specific models. Applying the SAPS S4 series IMD tables generates similar inconsistencies with extreme groups. Region-specific models did not typically perform significantly better than the baseline. Graduated mortality models, fitted directly to the data, usually significantly outperformed fits from SAPS tables. Although ML regression and classification models produced reasonable fits, they typically were not superior to the baseline model, due to a lack of smoothness, and would benefit from further refinement.

Contents

List of Abbreviations

Akaike Information Criterion (AIC)

Analysis of Experience (AoE)

Analysis of Surplus (AoS)

Area Under the Curve (AUC)

Bayes Information Criterion (BIC)

Channel Islands (CI)

Continuous Mortality Investigation (CMI)

Corrected Akaike Information Criterion (AICc)

Crude Mortality Rate (CMR)

Coronavirus Disease 2019 (COVID-19)

Degrees of Freedom (DoF)

Directly Standardised Mortality Rates (DSMR)

East of England (EE)

East Midlands (EM)

Exposure to Risk (EtR)

False Positive Rate (FPR)

Female (F)

Gompertz (G)

Gompertz-Makeham (GM)

Government Actuary's Department (GAD)

Indirectly Standardised Mortality Rate (ISMR)

Institute and Faculty of Actuaries (IFoA)

Ill Health (IH)

Isle of Man (IM)

Index of Multiple Deprivation (IMD)

Local Government Pension Scheme England and Wales (LGPS EW)

London (LN)

Lower-layer Super Output Area (LSOA)

Machine Learning (ML)

Machine Learning Model With Extra Variables (MLXL)

Male (M)

Nation-Specific (NS)

Normal Health (NH)

North East (NE)

North West (NW)

Northern Ireland (NI)

Nomenclature of Territorial Units for Statistics (NUTS)

Office for National Statistics (ONS)

Quasi Bayesian Information Criterion (QBIC)

Root Mean Squared Error (RMSE)

Receiver Operating Characteristic (ROC)

Residual Sum of Squares (RSS)

Scotland (SC)

Self-Administered Pension Scheme (SAPS)

South East (SE)

South West (SW)

Spouse (SP)

Statutory Funding Objective (SFO)

Standardised Mortality Ratio (SMR)

State Pension Age (SPA)

Technical Actuarial Standards (TAS)

True Positive Rate (TPR)

United Kingdom (UK)

Ultraviolet (UV)

Variance Inflation Factor (VIF)

Wales (WL)

West Midlands (WM)

Yorkshire and the Humber (YH)

Chapter 1

Introduction

1.1 Motivation

Various methods can be utilised to analyse mortality experience in pension schemes, especially with recent advancements in the capabilities and accessibility of data science techniques. Whilst traditional methods of fitting mortality tables have established a robust analytical framework, exploring data-driven processes may uncover underlying indicators and patterns related to mortality; this benefits users by deepening their understanding of their pension scheme data, further informing financial planning decisions.

Although mortality rates are influenced by a multitude of factors, the full scope of their relationships is yet to be fully understood [1] as behaviours, technology, the environment and other factors evolve continually evolve over time [2]. Traditional methods for analysing mortality experience typically use a limited number of pre-defined factors; these create more simplistic calculations, often due to limited data availability [3]. Data science techniques have become increasingly more popular, due to their capacity to analyse more variables, potentially identifying additional metrics related to mortality. Furthermore, external data sources and indicators could supplement pension scheme membership data, enhancing model performance and provide more meaningful conclusions.

Improvements to mortality experience analysis offer several benefits. Firstly, they reveal deeper insight into the reasons behind data fluctuations; stakeholders can anticipate potential cash flow requirements, making more informed financial adjustments accordingly. Secondly, deviations from expected mortality trends can be detected earlier; these early warning indicators alert users to atypical movements. Additionally, enhancing mortality analysis refines decision-making behind insurance arrangements; trustees and sponsors can better assess the suitability of insuring death benefits and catastrophe policies. Overall, this approach minimises cash flow uncertainty, ensuring more effective risk management.

Limited or low-quality data hinders the credibility of statistical analysis for some pension schemes, giving way to schematic inaccuracies. Conversely, data science techniques potentially offer a more adaptive analytical approach; these are particularly effective for pension schemes struggling to obtain high-quality data.

1.2 Outline

This research explores traditional and non-traditional mortality analyses, determining which methods produce the most favourable outcomes in addition to highlighting factors influencing the calculation of mortality rates.

Chapter 2 examines common factors affecting mortality rates and their typical availability within pension scheme membership data.

Chapter 3 addresses the regulation of pension scheme valuations, discussing the general objectives for different types of stakeholders and their unique priorities and uncertainties. The importance of an AoS and AoE is explored in relation to different pension scheme sizes.

Chapter 4 focuses on the necessary data items of an AoE and the implication of low-quality data. A general overview of different types of mortality models is outlined, illustrating common limitations and applications. Additionally, this chapter provides insight into the calculations and terminology involved when conducting an AoE to determine mortality rates.

Chapter 5 compares a variety of mortality analyses to a specific pension scheme data set, which is firstly summarised to deepen understanding of its profile. The EtR methodology is explained and thusly applied to an AoE using the SAPS S3 series mortality tables.

Following the examination of the relative prudence of assumptions, as recommended in recent scheme valuations, best estimate assumptions are derived to serve as a baseline for other AoE techniques.

Further subdivision of the data by IMD and region, using postcode data, is assessed along with the impact caused by IMD-based SAPS S4 tables. Next, mortality rates are graduated from the data using a Gompertz-Makeham formula. Lastly, alternative approaches to analysing mortality experience using ML regression and classification methods are investigated; initial attempts are made to improve the ML performance by supplementing membership data with external variables at the LSOA granularity level.

Chapter 2

Factors affecting mortality rates

Mortality rates are calculated by determining the probability of dying within a given time-frame, such as a year, by using the ratio of deaths to individuals; longevity is estimated using a specific set of mortality rates to determine remaining life expectancy, which is the average number of years until death [4]. It can be calculated at any given age using the sum of mortality rates for that age and older. Two common variants are period life expectancy and cohort life expectancy. The former assumes a fixed mortality rate for each age from a specific date or time period; consequently, it inaccurately estimates an individual's expected remaining lifetime as it disregards the inevitable fluctuations of mortality rates over time. Conversely, the latter has a more refined framework that incorporates potential future changes to mortality rates, resulting in more accurate life expectancies [5]. However, it is more subjective due to its predictions regarding future mortality trends, whereas period life expectancy typically reflects empirical data [6].

Many actuarial calculations require assumptions about future mortality rates to estimate the amount of benefits needed to be paid, including pension, annuity and life assurance payments, as well as pension contributions or premium payments received. Setting a mortality assumption for a category of people takes several factors into account, such as the overall objective of the calculation and the data availability. The mortality experience for a singular group is unlikely to be reflective of the national population; therefore, general population statistics cannot usually be used directly. For instance, mortality analyses of general population data include unemployed individuals with medical issues that prohibit them from working; in contrast, pensioners of final salary pension schemes that were healthy and employed until retirement should theoretically be healthier than the general population. Moreover, these individuals have probably secured stable jobs, which have potentially contributed to their health, as opposed to pensioners without such benefits.

Given sufficient data, the past mortality experience of a group over a period of time can identify mortality patterns; factors such as the suitability and quantity of data, in terms of the number of lives and duration of data availability, are essential. A significant shift in the group's composition may diverge the mortality experience for historic and current members. If available, past mortality experience can be compared to rates from mortality tables published by the CMI or ONS [7, 8]. The most appropriate published table is determined by the amount of available data or similarity of the category of individuals. Once selected, the rates contained in a published table can be adjusted to reflect actual experience; for instance, by applying a proportion of the rates or adjusting the age bracket [5].

Pension schemes generally do not have access to as much information as insurance companies, limiting the extent of specific mortality factors that can be considered; an individual pension scheme or life office is unlikely to have sufficient data in order to derive meaningful conclusions about historic rates of change in mortality rates. Assumptions about future changes should be consistent with historical data; for example, there should be continuity between past rates of change and future mortality assumptions. These assumptions typically involve two key components: an estimation of past mortality rates, residing in the mortality base table, and an assumption about future changes found within the mortality improvements table. Estimated changes in mortality rates between the effective date of the base table and the present may differ from future assumptions [5]. Despite the wealth of information and available data on past mortality experiences, predicting future mortality changes can be complicated by a multifaceted and constantly evolving mortality landscape. Although a definitive model has not been identified as superior in forecasting future rates, it is crucial that assumptions regarding these rates align with historical patterns. Maintaining consistency increases the reliability of projected future outcomes.

CMI mortality tables for pensioners and annuitants often have variants for both number of lives and pension amounts. The former is based on number of deaths, which can be adjusted for those with multiple pensions; the latter provides mortality rates per unit of pension, which are applied to pension amounts by age, estimating the amounts ceasing to be paid during a year. Generally, these rates are lower than those calculated on a lives basis, which is indicative of lower mortality rates for those with larger pensions. A correlation is suggested between wealth, using higher pension as a proxy, and higher life expectancy, suggesting that social class influences the mortality gradient [7, 9, 10, 11].

Base mortality rate assumptions are frequently set with specific reference to the group of lives under consideration. Separating mortality assumptions into base mortality rates and future changes allows base mortality rates to customise specificity, whilst future changes utilise more general assumptions as specificity is often infeasible [5]. While the baseline assumption may be specific to a scheme, individual schemes do not usually have sufficient evidence to make a scheme-specific allowance for future improvements; as a result, these schemes often have to broaden their choice by using external data [4].

Over the past century, mortality rates have experienced a significant decline on a global scale due to advances in medicine, public health and living conditions. Infectious diseases, once major causes of widespread death, are largely controlled through vaccinations, sanitation and antibiotics; on the other hand, non-communicable diseases, such as heart disease, cancer and diabetes, currently dominate mortality patterns, particularly in developed countries. The COVID-19 pandemic, caused by an infectious respiratory disease, was a recent and unexpected instance leading to a large number of excess deaths. Certain groups of individuals, especially those in an older age bracket and individuals with pre-existing medical conditions, were considered vulnerable to higher mortality risks from the disease. Currently, life expectancy is projected to continually increase, despite a declined rate of improvement; the long-term impact of events such as COVID-19 has yet to be discovered, highlighting a lag between mortality rates and observable future outcomes. Survivors of the pandemic may experience unaccounted health issues in the future, potentially affecting their mortality; conversely, some may even owe their survival to previously undiscovered superior health traits, leading to lower than expected mortality rates as they age. Although, the full impact on mortality can only be observed in the distant future, other factors may confound these observations [12].

Whilst several factors correlate with mortality rates, the extent of their influence has yet to be fully understood. Historically, mortality rates have significantly changed and will continually do so. As it stands, there is currently no consensus on the overall long-term trend; the accuracy of assumptions is limited and unlikely to be borne out in practice. Sensitivity and scenario testing of a mortality model mitigates this issue by presenting a wide range of possibilities; unfortunately, this can be disadvantageous as many results are subjective and arbitrary, with little indication of the probability of such events occurring [13].

Common factors that are influential to mortality include age, gender, pension status, occupation, location, health, socio-economic status, climate, lifestyle and education. As some of these serve as proxies for other underlying variables directly impacting mortality, cross-correlations must be considered when analysing mortality trends. Differences in mortality are rarely attributed to a single factor and interactions between several factors are difficult to thoroughly assess due to confounding variables. Multiple risk factors affecting mortality often interact with one another; for example, there are proportionately more smokers in lower socio-economic groups than in higher groups and occupation, postcode and income are often closely related. In some cases, one risk factor can serve as an effective proxy for others due to a strong correlation; for instance, a postcode can be used instead of occupation and income. If weakly correlated, joint influences of all risk factors should be used to model their interactions. Care should be taken to avoid duplication and make allowances for the impact of multiple factors when performing an analysis [4]. Popular statistical modelling software packages are often useful in providing evidence-based guidance on which risk factors are significant in mortality data and should be retained; insignificant variables, or those with well-established proxies, may be dropped and linked factors requiring modification can be flagged.

Interestingly, some factors with a significant direct impact on mortality rates are infrequently used by actuaries in life assurance or pensions; conversely, others with minimal direct influence are extremely useful in practice. One example illustrating this scenario is postcode: although a geographic change does not directly affect life expectancy, postcode and mortality rates are strongly correlated as their relationship is further influenced by other factors such as lifestyle and wealth. Therefore, postcode can be considered a useful proxy for these other variables, which often have considerable overlap in terms of impact on mortality. The depth of information about a factor regarding a particular group and the availability of quantitative information about its effect on mortality rates will determine its usefulness [5].

Unfortunately, pension schemes cannot comprehensively collect data on every factor influencing mortality; actuaries must therefore adopt mortality techniques best suited to the limited information available. Potentially, schemes could collect additional data if sufficient evidence justifies it. Although gathering more variables related to direct or indirect factors influencing mortality in an AoE analysis would be beneficial in providing a fuller picture, it would be difficult to collect from pension scheme members due to the sensitive and personal nature of this data. A plausible workaround would be to collect data at a wider group level after determining the optimal degree of granularity necessary. The following sections outline various factors impacting mortality rates and discusses their suitability in deriving actuarial assumptions.

2.1 Age

Age is the most prominent factor affecting mortality, significantly and directly influencing mortality rates, which explains its common usage in pension scheme analysis. Advancements in age intuitively increases the likelihood of death, particularly from natural causes; however, a more nuanced relationship should be considered between age and mortality at different stages of life.

During the first year of life, infants face high risks to mortality; specifically, newborns experience the highest probability of death on their day of birth due to a sudden change in environment and various medical complications. This risk sharply declines over subsequent days and weeks as more vulnerable infants tend to pass away earlier. An infant's chances of survival increase over time as their vital organs, such as the immune system and lung capacity, continue to develop [14]. After the initial year, death rates dramatically decline throughout childhood; nevertheless, there is a sudden increase during adolescence, primarily due to external causes, including accidents, falls, overdoses, poison, violence and suicides [15]. Whilst deaths from external causes remain relatively stable, adult mortality rates rise sharply due to a higher prevalence of diseases [16]. Throughout the ageing process, our bodies incur damage from injuries, stress and DNA mutations at a cellular level; consequently, the deterioration of our organs increases our susceptibility to further harm and our body's ability to repair itself becomes increasingly impaired [17].

2.2 Gender

Due to its significant influence on mortality rates, gender is commonly collected by pension schemes for analytical purposes. Male mortality rates are typically higher than female rates at any given age; this is attributed to biological differences in hormonal activity, greater male participation in high-risk activities and an elevated reluctance among males in seeking medical care for health issues [18]. Despite a longer life expectancy, women disproportionately experience higher rates of ill health, due to factors such as lower wealth, financial instability and increased stress from demanding caregiving responsibilities. The disparity between male and female mortality rates has narrowed in recent years, partly due to reduced smoking rates amongst men, increased awareness around men's health issues and advancements in medical treatments [19].

2.3 Medical history

Although medical history has a significant direct impact on mortality rates, this data is infrequently collected due to its sensitive nature. Higher mortality rates are closely related to recent health issues, progressively diverging over time. The mortality rate of individuals that have retired due to ill health is considerably higher than those retiring normally; this difference gradually diminishes with age [20]. The impact of specific diseases on mortality could inform mortality assumptions if the relevant information is available. A difference in regional quality of medical care facilities affects mortality rates such as staffing levels, staff competence, work experience, qualifications and availability of equipment [5].

2.4 Genetics and family medical history

Although genetics have a significant direct impact on mortality rates, this data is infrequently collected due to its sensitive nature. Mortality rates can be derived from the intricate relationship between genetic and environmental factors. Beneficial genes enhance longevity whereas others affect susceptibility to diseases [21]. Certain genetic mutations are inherited and influence lifespan; spontaneous mutations also occur after birth due to environmental factors, such as exposure to toxins, free radicals or radiation. While genetics may affect disease susceptibility, there is insufficient data to utilise this information for predictive purposes in mortality analyses.

Toxins are poisonous substances that harm cells and interfere with normal biological processes. Free radicals are unstable molecules in the body that damage DNA at a cellular level and are naturally produced during normal metabolic processes; exposure to toxic substances, such as tobacco smoke and UV radiation, increases their presence and the risk of contracting diseases, including cancer. Radiation is energy that travels in the form of waves or particles and most commonly encountered as sunlight exposure. Radiation damages cells to a varying degree; with limited exposure, cells may repair themselves or die without causing long-term harm; however, high doses of radiation can lead to widespread cellular death, organ failure and, ultimately, death [22].

Family medical history refers to a record of diagnosed health conditions of wider family members. Family medical history has both a minimal direct influence on overall mortality rates and usefulness in pensions due to its rare availability and sensitive nature. However, it plays a significant role in determining an individual's risk of certain health conditions and mortality rates. A family member with a health condition can influence an individual's mortality risk; families often share genetic traits and environmental habits, affecting health outcomes. An individual related by blood to someone with a specific medical diagnosis also might be at a higher risk of developing it. Inherited conditions result from genetic mutations passed down from parents. Genetic history and shared lifestyle habits both increase mortality risk [23].

2.5 Marital status

Although marital status has a moderate direct influence on mortality rates and use in pensions, due to availability of data, it may change and is not always accurately updated. Evidence suggests that spouses experience lower mortality rates compared in comparison to unmarried individuals; this can be attributed to greater social and financial support, shared healthier behaviours and a reduction in stress. Differences in mortality rates increase with age and the married population generally shows greater improvements to mortality over time. Healthier individuals may be more inclined to marry, reducing mortality rates amongst married people [24].

2.6 Socio-economic status

Socio-economic status has a significant direct impact on mortality rates and moderate usefulness in pensions; it refers to an individual's position within a social hierarchy, encompassing various factors such as education, occupation, income, nutrition and current place of residence.

Research indicates individuals in higher socio-economic groups tend to have lower mortality rates, experiencing more rapid improvements in mortality in comparison to those in lower socio-economic groups. The disparity in mortality rates between socio-economic groups could persist in the future, as individuals in higher socio-economic groups have access to more advanced medical treatments, which is likely to continue in the future. Conversely, the gap might converge over time if those in lower socio-economic groups increasingly adopt healthier lifestyle changes, such as improved nutrition and smoking reduction, since there is less room for improvement in higher socio-economic groups [25].

In the absence of direct assignment of a socio-economic group, several proxies can be employed, including industry, postcode and pension amount; however, differences in mortality by socio-economic group does not solely explain the correlations of these factors to mortality rates. Actuaries increasingly determine an individual's socio-economic status through their postcode, using various proprietary databases, such as consumer segmentation tools. The use of postcode information for mortality modelling has become widespread in the insurance industry providing annuity business; similar techniques are now also being implemented by some actuaries when advising trustees and sponsors of pension schemes [5].

2.6.1 Employment status and occupation

Occupation has a significant direct impact on mortality rates and is considered as moderately useful in pensions. Unemployment substantially increases mortality risk due to various reasons; people with residence in locations of high employment rates generally tend to live longer. A positive correlation is present between an area's employment rates and healthy life expectancy for men and women; in contrast, economic inactivity adversely affects healthy life expectancy. Employment is often associated with better health and longevity, whereas unemployment may impact well-being negatively, increasing the likelihood of reporting health conditions. Those in the latter category have an increased risk of death from various causes, such as cancer, heart disease and diabetes.

Whereas unemployment may produce the worst outcome, manual labour employees routinely experience higher mortality rates than individuals with alternative occupations. Taxing job demands within a highly controlled structured environment with low individual freedom of creative expression can negatively impact mental health, leading to increased risk of depression. Jobs with a focus on repetitive tasks adversely affects long-term health, accelerating the risk of disability and mortality; these occupations often follow monotonous and predictable patterns, leading to physical and mental health repercussions [25].

2.6.2 Education

Education has a moderate direct impact on mortality rates but is infrequently used as a proxy in pensions due to an absence of data. Education corresponds to awareness around a healthy lifestyle and thus, mortality risk factors; higher educational attainment generally prompts better personal health decisions, improved access to healthcare and increased longevity. Education is a powerful determinant of health, whereby better health outcomes naturally reduce mortality rates [26].

2.6.3 Income, wealth and pension amount

Although wealth has a moderate direct impact on mortality rates, an absence of data restricts its usage in pensions. Individuals in wealthier areas tend to enjoy a longer lifespan than those in more deprived regions. Whilst income influences health outcomes, the extent of their relationship varies with age, weakening during adolescence. Persistent wealth inequality affects mortality in older age brackets [27].

Pension amount has little direct impact upon mortality rates, yet is a highly useful proxy in pensions due to its correlation with other factors such as income level and socio-economic group. Income level is a significant indicator of mortality as a higher income may provide better access to private medical care; however, pension amount may be unreliable in measuring income depending on the duration of service with a single employer, annual salary and overall amount of earnings. The CMI SAPS investigation has reported differences in mortality amongst individuals in different pension bands [11].

2.6.4 Diet and nutrition

Whilst diet has a strong direct impact on mortality rates, it has limited usage in pensions since such information is rarely available. Poor nutrition increases the risk of developing diseases as a lack of sufficient food or variety of vitamins impedes immunity to pathogens and diseases; it is linked to elevated morbidity and mortality rates [28]. Malnourished individuals are more susceptible to infections and delayed impeded recovery, leading to an increased risk of death. Although obesity inflates the risk of certain diseases, it may paradoxically provide protection in specific situations, such as where people have previously experienced heart failure [29]. An insufficiently healthy diet and an excess of unhealthy foods are key contributors to global levels of mortality. Excessive sodium consumption is associated with high blood pressure and heart conditions, shaping a significant proportion of diet-related deaths.

2.6.5 Location and postcode

Geographic location has a minimal direct impact on mortality rates and usefulness in pensions. Different locations have distinct climates and susceptibilities to natural disasters and diseases, affecting local mortality rates. Whilst extreme climates admittedly increase mortality risk [30], a much larger proportion of mortality can be attributed to temperatures with slight deviations to their normal range [31]. Mortality experience tends to follow a seasonal pattern, with cold winters typically leading to higher rates than other months. However, a relatively high mortality rate in any season often rebounds, resulting in a more resilient starting population and a relatively lowered mortality rate in the subsequent season [32]. Life expectancy may differ between urban and rural regions, with rural residents generally experiencing higher mortality rates than those in cities; access to medical facilities, lifestyle and economic conditions are factors contributing to this differentiation. Migration between locations adds a layer of complexity, as individuals' overall health may be influenced by multiple areas, obscuring cause and effect.

Postcode has a low direct impact on mortality rates but is highly prevalent in pensions since this data is often available. The highest mortality rates are typically found in the north of England and are typically greater in more deprived areas with a tendency towards young adults and males for smoking related causes. Similarly, mortality levels are consistent between the least deprived areas, regardless of region. The Interim Life Tables published by the ONS can be used to identify changes in mortality rates for England, Scotland, Wales and Northern Ireland [8]; age-standardised mortality rates by each local authority are also published. The annual publications by the ONS regarding regional mortality reveal significant discrepancies in mortality rates between regions and local authority areas. Life expectancy for smaller populations can be heavily influenced by the availability of local factors, including the presence of nursing homes, and care establishments and other medical facilities within districts [5].

2.7 Smoking status

Smoking status has a significant direct impact on mortality rates but is of very low usefulness in pensions because this data is rarely available. Although smokers have higher mortality rates than non-smokers [33], interestingly, individuals who have quit smoking by the age of 30 exhibit a life expectancy similar to lifelong non-smokers. Those who quit by the age of 40, 50 and 60, approximately gain nine, six and three years of life, respectively, in comparison to those who continue smoking [34].

A notable correlation can be examined between socio-economic class and smoking. Individuals on lower incomes are more likely to smoke than those in a wealthier bracket; those in routine and manual occupations tend to have higher smoking rates than those in managerial and professional roles [35]. Moreover, a cohort effect can be observed between individuals born within a certain time period, whereas the mortality rate differs between those born in different time periods; this can be attributed to factors such as smoking, diet, medical access and general birth rates which also influence the rates of smoking cessation.

Many people believe a reduction in smoking has significantly contributed to notable improvements in mortality observed over the past few decades. One argument suggests that since individuals can only permanently quit smoking once, mortality improvements for non-smokers may exhibit slower progress than current smokers who have the capacity to quit in the future. However, caution must be exercised when defining smoking and non-smoking categories, especially the classification of individuals who have stopped smoking, since significant effects can be observed to the improvement of mortality rates [5].

2.8 Obesity

Obesity has a significant direct impact on mortality rates but little influence in pensions, due to a lack of data. Obesity is associated with an increased risk of chronic conditions, including type 2 diabetes, heart disease and certain types of cancers. Excessive fat deposits naturally impair health and may lead to several health complications. Although obesity heightens the risk of certain diseases, it may incongruously provide some protection in specific situations, such as where people have previously experienced heart failure [29].

Moderate obesity shortens life expectancy by approximately three years; severe obesity can cause a reduction of up to ten years, which is comparable to the consequences of a lifetime of smoking. Regular physical activity, a balanced diet and maintenance of a healthy weight and lifestyle are essential preventive measures [36].

2.9 Alcohol consumption

Alcohol consumption has a moderate direct impact on mortality rates but offer little assistance in pensions, as this information is unlikely to be available. Alcohol contributes to various diseases and injuries, notoriously liver disease; an excessive intake increases the risk of premature death, while light to moderate alcohol consumption potentially offers some degree of protection in respect to cardiovascular health and cognitive function [37, 38].

2.10 Exercise

Exercise has a moderate direct impact on mortality rates but holds very low relevance in pensions, due to the unavailability of such data. Regular exercise lowers the risk of major non-communicable diseases and premature death; adequate levels of moderate and vigorous physical activity yield the best results; consistent exercise reduces overall mortality risk, improving quality of life with minimal risk of physical injury and death [39].

2.11 Stress

Exposure to stress has a moderate direct impact on mortality rates but is not often considered in pensions, as this data is frequently unavailable. Mortality rates increase proportionally to the severity of stress; chronic stress exacerbates health problems by affecting various physiological and psychological systems. Stress management is crucial for overall health maintenance and longevity; even moderate stress has a significant long-term impact, triggering health issues and potentially resulting in an earlier death [40].

Chapter 3

Pension scheme valuations and experience analysis

3.1 Valuation objectives

The purpose of a defined benefit pension scheme valuation is to facilitate the decision-making process of involved parties; various stakeholders hold interest in the scheme's operation, including:

- Individuals responsible for the scheme, typically trustees;
- Employers financially backing the scheme that wish to understand the cost of benefit provision;
- Scheme members keen to generate the adequacy and security of their benefits;
- Shareholders seeking to be kept abreast about the sponsor's profitability;
- Regulators requiring a scheme's compliance.

Despite a shared interest in the scheme, these stakeholders often have conflicting objectives. Trustees must strike a balance between member welfare and financial sustainability; employers aim to control costs in addition to providing attractive benefits; members prioritise the stability and value of their pensions. The partnership of these interests shapes pension scheme decisions and outcomes [41].

Whether they are individual, corporate, member-nominated, employer-nominated or professional, the trustees responsible for running the pension scheme have a primary duty to act in the interests of all involved members. They must adhere to the scheme's guidelines and rules, refrain from profiting from their duties, execute all necessary administration, appropriately invest scheme assets and ensure the security of member benefits by frequently monitoring funding levels and the sponsor covenant. A sponsor covenant refers to the sponsor's ability and willingness to pay, or the trustees' requirements of payment from their sponsor; it guarantees that scheme benefits are sufficiently paid when due. Trustees should always consider the sponsor covenant, except in a few exceptional circumstances.

If a scheme is exceptionally well-funded, whereby assets comfortably cover liabilities, the sponsor's direct involvement may become less critical; in such cases, their financial support might not be immediately required to supply benefit payments. When the sponsor covenant has proven robust and certain, trustees can rely on the sponsor's commitment to meet obligations; a strong covenant inspires confidence that contributions will be provided when necessary. Conversely, if considered weak or non-existent, trustees must take additional precautions due to mounting uncertainty regarding the sponsor's ability to fulfil their obligations; a weak covenant significantly impacts a scheme's risk profile [42].

Trustees need to maintain a strong and reliable relationship with the employer to retain their support and confidence for the scheme; they can exercise discretion to ensure members' continued appreciation of benefits, as illustrated by allowing pension enhancements under certain circumstances [41].

Employers will want to control the pension scheme's costs, establishing their stability, flexibility and predictability in order to keep pension scheme benefits competitive; supporting employees by meeting needs, providing attractive benefits that are appreciated will increase levels of staff procurement and retainment. Tailoring benefits may be beneficial in targeting specific groups. Employers will want to optimise value for money they have contributed to the pension scheme, balancing the opportunity cost of funds that could be used to expand and improve the business [41].

Members of the pension scheme desire stability, predictability and security from benefits, providing good value for money from affordable contributions paid towards the scheme. Members will want portable, flexible and understandable benefits offering generous contingencies upon death and ill health; furthermore, benefits need to sufficiently maintain an individual's desired standard of living throughout retirement [41].

A typical outcome of valuations determines the necessary contributions paid towards the scheme for ongoing funding over a short-term period, allowing for current surplus or deficit. Such valuations aim to safeguard the rights of beneficiaries, avoiding the risk of excessive or insufficient funds and improperly preparing for future benefit provision. Consequently, assumptions adopted for funding valuations are generally prudent, where contribution rates are likely overstated, increasing the probability of meeting liabilities; however, as the party responsible for the provision of the scheme's contributions, the sponsor may prefer more realistic assumptions to avoid increasing unnecessary immediate costs. Nevertheless, they should keep in mind the percentage of contribution paid doesn't affect the scheme's total cost, only the timing of payments to meet the expected cost. Paying a lower rate of contributions in the short term would require higher contribution levels in the future; these may unexpectedly arise at an inconvenient time, leading to financial difficulties or an increased opportunity cost where the sponsor could have maximised the return on funds by investing them into the business. Therefore, the sponsor may prefer contributions to remain stable over time to account for unforeseen changes in cash flow. Trustees are likely to sympathise with the sponsor's requirements to a degree, subject to maintaining a sufficient standard of benefit security, since the scheme relies upon the latter's continual support; thus it is in the interests of scheme members that the sponsor remains profitable and prosperous. If an employer is unable to adopt the acceptable degree of prudence regarding their funding requirements, the scheme may appear unaffordable; they could counteract this by reducing member benefits, lowering future costs and contribution requirements [43].

3.2 Analysis of surplus

The AoS serves as a verification tool for contribution rates determined during the valuation process, providing a detailed breakdown of changes between previous and current valuation results. Profits and losses are calculated for each major component between the two valuation dates. These arise from deviations within the experience when comparing previous valuation's expectations and changing the current valuation assumptions. Experience-related profits and losses are identified by conducting an AoE, juxtaposing the actual experience observed during the intervaluation period with the expected experience, which is derived from assumptions set at the previous valuation. Profits and losses due to assumption changes can be calculated using the difference between values of member benefits from historic and new assumptions. An unattributed balancing item represents the amount of unallocated profit or loss within the analysis. The unattributed AoS items should be minimised to a level that provides sufficient reassurance regarding the accuracy of valuation results. Elimination of all unattributed amounts is not always essential if users remain confident any remaining discrepancies are products of immaterial factors [44].

A typical AoS may account for, but is not limited to, the following items:

- Interest on surplus at the previous valuation;
- Difference between accrual cost and contributions paid;
- Deficit repayment contributions made;
- Difference between actual and expected employer contributions;
- Difference between actual and expected member contributions;
- Difference between actual and expected financial or demographic experience;
- Changes to financial or demographic assumptions;

- Changes in actuarial methods.

When setting the assumptions for a pension scheme valuation, the sponsor covenant, investment strategy, membership profile, scheme size, scheme funding position, nature of business and benefits should be taken into consideration [45].

The sponsor covenant refers to the financial health and commitment of the employer sponsoring the pension scheme. As stated previously, a strong sponsor covenant is indicative of an employer's financial stability, influencing assumptions about future contributions and the pension scheme's capability of meeting its obligations. The relative importance of an assumption will impact the necessary precision and level of detail warranted. Post-retirement mortality is a particularly significant assumption; overestimation of the current state of mortality or underestimation of the expected improvements in future mortality would undervalue liabilities.

The investment strategy outlines how a scheme's assets should be allocated across different classes, such as equities and bonds; it crucially dictates the expected return on investment, volatility and level of risk.

A scheme's membership profile also significantly determines the assumptions in valuations. A scheme with younger members generally has a longer time horizon before benefits are paid out; this impacts the discount rate assumptions since the scheme could potentially take on more investment risk for higher returns over time. The gender composition can lead to assumption adjustments with women typically having longer life expectancies than men; a scheme with a higher proportion of females should assume longer periods of benefit payments, increasing overall liability. Schemes with numerous high earners should account for higher final salary projections, possibly increasing liabilities. A scheme with a high retention rate of employees staying working until retirement age will need different assumptions in comparison to one with a high turnover of staff; the former leads to more stable longevity and salary increase assumptions. Members with physically demanding jobs may have different mortality and morbidity rates. The proportion of members with dependants receiving benefits, conditional upon their death, affects assumptions regarding survivor benefits; greater numbers of married members normally result in higher liabilities due to survivor pensions.

A scheme's size, in terms of the number of members and total assets, impacts the assumptions used in valuation; larger schemes may benefit from economies of scale in administration and investment management whereas smaller ones might face higher per-member costs.

The funding position reflects the scheme's current financial status and is ascertained by comparing assets to liabilities. A well-funded scheme could allocate a percentage of existing surplus towards a riskier investment strategy in pursuit of higher returns or adopt a more conservative strategy to prevent future levels of insufficient funding. Although an underfunded scheme may consider taking an aggressive approach to improve its funding level, this in fact exacerbates the risk of financial degradation, necessitating more conservative investment assumptions.

The nature of the sponsor's business influences the stability and predictability of future contributions. For instance, companies in unpredictable industries with significant fluctuations in performance due to volatile market demand, regulatory changes, economic conditions or technological advancements typically face greater financial uncertainty; this impacts assumptions regarding the sponsor's ability to support the scheme over a long period of time.

Pension scheme assumptions can be financial or demographic. The former includes investment returns, interest on surplus, inflation and salary increase rates; these typically affect the amount of each member benefit payment. On the other hand, the latter typically affect the nature and timing of benefit payments, regulating the number of payments distributed. The funding of a defined benefit pension scheme guarantees that benefit payments are met as long as the assumed proportion of members experiencing each benefit option is accurate. Each demographic factor's impact upon the surplus or deficit found within a scheme relies upon the discrepancy between actual and expected member numbers experiencing each benefit event as well as the difference between actual cost of provision and value of the reserve held. Examples of demographic assumptions include rates of mortality before and after retirement, retirement rates (normal, ill, early, late) and withdrawal rates.

The rate of post-retirement mortality is usually a highly significant demographic assumption necessary for defined benefit pension scheme valuations. Whether a member's death leads to a surplus depends on the comparison between the relative value of such benefits, which may include a lump sum or dependant pension, and the survival member benefits. The insurance arrangements held by the scheme determines whether deaths cause a gain or a strain on its finances [4].

The overall strategy of the assumptions is important, dictating the required level of accuracy and prudence. A funding valuation typically implements prudent assumptions; maintaining consistency with the previous valuations and other schematic approaches is desirable. Compliance with relevant legislation and guidance standards should be noted [43].

The surplus is represented mathematically as:

$$(\text{Actual decrements} - \text{Expected decrements}) \times (\text{Reserve held} - \text{Actual benefit cost}) \tag{3.1}$$

3.3 Regulations

TAS 100 and 300 set clear expectations for actuaries in the UK regarding pension scheme valuations; the former mandates the provision of comparisons between current and previous valuation results; the latter requires a thorough justification of changes to a scheme's funding level, emphasising the quantification of significant elements [46, 47].

The SFO stipulates that UK defined benefit pension schemes must maintain a minimum level of assets relative to their liabilities; this legislation also demands a recovery plan addressing any deficits. Reclamation of surplus funds is often restricted and can incur potential financial penalties; for instance, surplus beyond full buyout cost is only recoverable by a UK employer following trustee discretion, where the scheme rules allow it, and are taxed at 35%; there may be an expectation from members to enhance member benefits with them. Consequently, there are risks to the sponsor when any trapped surplus funds in the scheme are unable to fulfil opportunity costs for other purposes.

Pension schemes have legal requirements to adopt prudent mortality assumptions, incorporating a suitable margin below best estimate rates where members are assumed to live and receive benefits for an extended period. Whilst scheme-specific experience can be used when statistically justifiable, this is typically feasible only for the largest schemes. For smaller schemes, where individual experience is statistically unreliable, adjustments depend upon the aggregated analysis of known characteristics influential to mortality. Regarding future mortality improvement rates, it is prudent to assume higher rates than those based on the latest evidence and accepted projection methodologies.

Under the Pensions Act 2004, schemes are required to generate conservative estimates of accrued liabilities, known as technical provisions [48]. Trustees must seek actuarial advice when selecting assumptions; actuaries must adhere to TAS 300, which sets standards on scheme funding and financing, ensuring the delivery of high-quality advice [47].

Each assumption could be selected conservatively, helping to achieve an appropriate overall level of caution regarding the technical provisions; the degree of prudence may vary across assumptions to meet the overall target. For less critical assumptions, best estimates can be used provided the overall technical provisions remain predominantly and sufficiently prudent.

3.4 Errors and uncertainty

Errors when calculating contribution requirements pose risks to the security of the pension scheme; these can result from using unsuitable models, inappropriate assumptions or incorrect data. Ensuring the accuracy of contributions is essential in maintaining the scheme's stability and protecting member benefits.

Due to the delay between the promise of benefits and their provision, a degree of uncertainty is always present for the sponsor and member; this is related to the level or timing of the actual benefits or the contributions required to meet them. Members chance receiving benefits of a lower value than expected or at a different time than desired; sponsors risk facing costs that are higher than anticipated or paying these contributions at inconvenient times.

A defined benefit pension scheme has various uncertainties impacting both members and sponsors, include changes to membership numbers, transitions in member status such as retirement or withdrawal, the proportion of members with eligible dependants, inflation affecting benefit values, investment performance, legislation amendments and the benefit options exercised by scheme members. Trustees must navigate these differentials to establish financial stability and fulfil long-term commitments, whilst balancing the interests of all stakeholders [49].

Potential limitations of using past data to forecast the future include abnormal or random fluctuations, over-reliance on historical trends, changes to data recording methods or protocols, data errors, balance shifts of homogeneous groups within the underlying data and heterogeneity within the observed group [50].

Abnormal fluctuations refer to irregular changes in data that can unexpectedly distort predictions; these can be caused by rare events, such as natural disasters or economic crises. Reliance on such data leads to inaccurate forecasts due to the low possibility of recurrence.

Trends are long-term movements in data that help shape predictions. Since they can subtly change over time due to various factors, such as medical advancements, policy changes or shifts in behaviour, if the underlying causes are misunderstood or assumed to continue indefinitely, predictions may become flawed.

Random fluctuations are the natural variations in data occurring without a specific pattern or reason; these obscure the data's true signals, making it challenging to identify underlying patterns. Over-dependence on data with high random variability induces unreliable predictions [51].

Changes to the collection of data may introduce inconsistencies. For instance, a shift from manual to automated data mining can result in different levels of accuracy or completion; such changes create discontinuities between historical and current data.

Data errors, such as incorrect entries, missing values or measurement inaccuracies, significantly impact the quality of predictions; leaving these errors unidentified and uncorrected could result in misleading conclusions and poor decision-making.

The composition of the member data changes over time; for instance, when a population diversifies, the group's characteristics shift. Predictions based on outdated data can lead to inaccurate forecasts.

Heterogeneous data consists of diverse subgroups. When the data set includes individuals or entities exhibiting different behaviours or characteristics, making accurate predictions may prove challenging. Disregarding this diversity when aggregating data masks fundamental differences resulting in incorrect conclusions.

The contributions necessary for payment are based on future financial and demographic assumptions; whilst historic data does not comprehensively reflect future patterns, it provides a useful starting point for adjustments, particularly when considering demographic assumptions. Past experience is determined by performing an AoE; mortality rates are highly significant demographic assumptions for pension scheme valuations.

Possible reasons attributed to inappropriate mortality assumptions include basis risk, where the selected base table inaccurately reflects the mortality rates of the underlying population; idiosyncratic risk, where populations experience random fluctuations, deviating from base table mortality rates; trend risk is where future projections of mortality improvements may become inaccurate.

Individuals may not necessarily share the same mortality experience as their wider community; for example, life expectancies of wealthy members in a final salary pension scheme could surpass the average of those living in the same geographical region; thus, when setting mortality assumptions, executing an AoE is critical in reflecting specific characteristics of the relevant population, without explicitly quantifying the impact of the underlying causes of mortality. Additionally, postcode analysis can be used to set mortality assumptions based on location.

Trustees face significant uncertainty when making assumptions about future mortality; various approaches should be explored to circumvent this; firstly, CMI investigations based on SAPS experience analysis of broader population data can be used to inform mortality projections; secondly, trustees must carefully evaluate the characteristics of different projection methodologies whereby deciding a stable value for long-term mortality improvement is crucial; thirdly, allowances should be made for cohort effects and generational differences; lastly, recent mortality trends should be contemplated and their relative weight applied appropriately to the assumptions. Actuaries provide valuable advice to trustees concerning mortality rates and life expectancies at sample ages, sensitivity tests demonstrating the impact of changing assumptions and analysis of both period and cohort life expectations in order to illustrate the significance of improvement rates; moreover, they analyse annuity factors, technical provisions, how to make discount rate shifts equivalent to mortality improvements, stochastic modelling simulations and scenario examples demonstrating the impact of extreme conditions [4].

3.5 Analysis of experience

An AoE quantifies the difference between actual events over a period of time and the predicted assumptions; it reconciles the scheme's initial and final positions over an intervaluation period, validating the feasibility and credibility of the valuation results. Such analysis aids all parties involved in the scheme by prioritising assumptions with the highest financial significance; deeper understanding of the scheme's progression is beneficial in determining appropriate assumptions for the current valuation [41]. Furthermore, an AoE identifies sources of surplus and deficit within the pension scheme; this informs decisions to revise assumptions for future periods to expressly fit the mortality experience, providing a more appropriate prudence margin or accounting for identified trends.

Although analysing past data is typically very useful when setting future assumptions, other information needs to be considered; the credibility and relevance of historic analysis should be evaluated against judgements and expectations concerning future conditions.

Deviations from expected outcomes should be analysed in case adjustments need to be made, given an event's significant impact on the scheme. Importantly, allowances should be made for identified trends and additional prudence may be considered, enhancing the likelihood of future surplus.

Assumptions in a pension scheme may remain static, despite differences between actual and expected events, if they are intentionally conservative; actual events may turn out more favourably, resulting in an intentional surplus. Other differences may arise from isolated events that are unlikely to recur regularly. A minimal discrepancy might be considered immaterial and not warrant updates to the assumptions. Additionally, assumptions are often reflective of long-established trends with shorter-term variations inducing less significant fluctuations randomly around these expected values.

Key assumptions typically include inflation, investment return and the expected lifespan of scheme beneficiaries. Actuaries are essential in helping trustees to understand the sensitivity of technical provisions to changes in assumptions; demographic assumptions, such as mortality and early leaver rates, are examples where large deviations often occur within populations or between employers.

A mortality AoE uses pension scheme data on deaths and amount of scheme members to determine the historical death rates experienced at each age; this is subsequently combined with other information, updating assumptions that are defined by allotted mortality improvements and a base table. These AoE rates should not be directly applied to future mortality assumptions if there is insufficient data, hindering statistical credibility across all categories; instead, the analysis may be used to adjust an existing national mortality table, such as those produced by the CMI or ONS [7, 8].

Particular care is needed when setting future mortality assumptions. Mortality naturally exhibits wide variability amongst individuals, annual variability across the entire population, long-term trends present in certain age categories and historically underestimated reduction rates. Over time, mortality has incrementally decreased, consistently increasing life expectancy with different generations experiencing varying rates of improvement through the cohort effect.

3.6 Scheme size

Small schemes are often affected when significant liabilities are concentrated amongst relatively few members; individual outcomes are inherently more varied than in larger schemes. Trustees need to examine if the concentration of liability poses a significant mortality risk for the scheme, accounting for its membership size. Random fluctuations could be significant in a smaller scheme and the prudence margin may need to be relatively greater than for a larger scheme [41].

Analysis of a scheme's mortality experience usually yields relevant evidence; however, random fluctuations in a small scheme decreases its reliability from which inferences can be extrapolated about the future. Therefore, it may not be worthwhile to undertake this analysis in these cases and utilising more general factors, such as industry, occupation or pension size to adjust a standard table is more beneficial [4].

The extent of an AoE's significance depends on the scheme's membership size; disregarding schemes with unusual characteristics, assumptions are commonly determined by adjusting standard tables in light of recent experience. With a scheme of a statistically significant membership size, suitability of the proposed table should be noted as it may be advantageous to create a bespoke table from the scheme's own experience, incorporating customised and appropriate prudence margins in the base rates.

Actuaries routinely recommend adjusting a standard table to better reflect a scheme's current or expected future experience; these amendments can be made by various methods. An age rating treats members as younger or older than they actually are, shifting mortality rates by the desired number of ages; this is applicable when scheme experience or general factors, such as industry or type of work, displays lighter or heavier mortality rates than a standard table; a lower age could be applied to add a margin for prudence. Alternatively, mortality rates can be made heavier (e.g. 120%) or lighter (e.g. 80%) by altering percentage adjustments relative to those found in a standard table; this method can become more sophisticated by applying different percentage rates to different ages. Additionally, blending multiple standard tables could shape mortality rates more closely to scheme experience [4]. Age ratings were formerly regularly applied when adjusting standard mortality tables due to their ease of application; however, applying a scaling factor is preferred since it preserves the shape of the mortality rates across different ages, allowing more precise refinement.

Significant levels of disruption can be found in both crude and fitted mortality rates due to small-population noise present in death counts, especially at higher and lower age ranges where expected numbers of deaths are low; therefore reviewing the scheme's mortality experience against larger national datasets with greater exposures and a broader range of years is useful to assess the data's credibility [52].

Chapter 4

Analysing mortality experience

4.1 Data quality

Data quality and quantity are crucial to the performance of models; several potential issues can arise. Firstly, insufficient data volumes impede model performance validation, due to inadequate examples necessary for assessing the model's generalisation capabilities; secondly, low quality data and fluctuations in data quality can both obscure the complex relationships between variables, inhibiting the identification of meaningful patterns.

Historical data is often highly influential in determining future trends to demographic assumptions. Recent data usually better represents current rates of mortality and should be given greater emphasis. More distant historical data remains useful for identifying longer term trends; it can increase analysis reliability where recent experience contains abnormal events. Mortality assumptions are routinely determined by selecting a base table that contains mortality rates for a population at a specific exposure date; the table is updated to the current valuation date, utilising mortality improvement expectations to project future rates [51].

To conduct a mortality AoE for pensioners and dependants by age, specific information must be available for every scheme member as of the valuation date, including their assigned category, date of birth and the pension's commencement date. Additionally, further details are required for individuals whose pensions have ceased between the previous and current valuation dates, such as the date of dissolution and the reason behind it. Information such as pension amounts can be used to weight the analysis instead of performing it solely using a lives basis.

An individual's category provides essential insight into shared characteristics of a homogeneous group; typically, this is represented by multiple variables affecting mortality, as discussed in Chapter 2. Unfortunately, only a member's basic personal information is usually collected, other than presently identified factors of significance.

Mortality data is subdivided into homogeneous groups to determine a reasonably accurate single set of assumptions, representative of all individuals within that segment; excessive reduction of the category's size, to disproportionate subdivision, must be prevented as it hampers the data's statistical credibility.

Selection involves defining the homogeneous groups in the mortality analysis that should have different mortality assumptions [3]. Adverse selection relates to interpreting member's behaviour, whereby the most beneficial options within a scheme will be chosen, based on what they currently understand and can utilise effectively; it occurs when an individual's choice actively influences their assigned group. For example, people in poor health may exchange a larger proportion of their pension for a lump sum upon retirement via commutation, given its greater value to those with shorter lifespans. Class selection acknowledges that people with distinct permanent attributes experience variable mortality and, therefore, should be analysed separately; this can be observed in the differences between male and female mortality patterns. Temporary initial selection occurs when the level of risk changes after a selection event; over time, heterogeneity wears off naturally. For instance, mortality rates tend to be lower for new members of a pension scheme, since they have typically demonstrated evidence of reasonable health when hired; as time elapses, the reliability of good health diminishes, which is reflected in incremental mortality rates. Time selection involves subdividing a population based on data related to each calendar year to account for gradual changes to mortality rates; mortality rates have generally declined with exceptions, such as COVID-19 [12]. Spurious selection leads to members being allocated incorrectly due to a confounding factor being present in a supposedly homogeneous group; for example, changes in data recording standards result in older records becoming more prone to invalidity. Heterogeneity will be present within any group, due to superfluous factors independent of a population's subdivision; spurious selection occurs when differences in mortality rates are ascribed to factors that are not the true cause of differentiation.

Date of birth, date of pension commencement and date of pension cessation are used in tandem to track age increments between the pension's start and end dates; this is a prerequisite for allocating EtR to different ages over the intervaluation period and determining the ages at which deaths occurred [51].

Understanding the reasons behind termination helps to accurately distinguish death records and other reasons, such as trivial commutation, excluded from the death count; irrespective of death, the EtR calculation can be impacted if the initial exposure measure is used instead of the central method (see Section 4.3).

Weighting the mortality analysis by pension amounts can provide valuable insights since they fluctuate significantly based on other mortality factors, consolidating the financial implications of mortality risk.

4.2 Mortality analysis models

All forecasts utilise models to create simplified representations of reality; differences occur due to inaccurate representations of the real world, data errors and insufficiencies as well as the random variation to actual observations. The level of uncertainty is likely to be much larger for smaller groups. Model accuracy typically diverges over time; those useful for short-term approximations may not reliably produce valid long-term outputs; therefore, many different types of models are used to forecast future mortality rates [5].

It is desirable for models to produce smooth transitions of mortality rates between neighbouring ages and time periods, such as observed historic rates and future forecasts, avoiding sharp discontinuities that are inherently unlikely. The model should also produce sensible scenarios in long-term projections, although rationality is rather subjective within this context. There is considerable disagreement over whether the current maximum lifespan, approximately 120 years, will remain stable due to diminishing lifestyle gains and medical advancements [53], or continue to increase due to the discovery and development of methods that extend longevity as proven historically. Models could benefit from indicating the level of uncertainty surrounding forecasts, allowing more educated decisions; for example, multiple projections and sensitivities could be displayed, enabling users to reconsider the range in which the results may reasonably lie [2].

Empowering users with the ability to fully comprehend the model's outputs is crucial. Sole reliance on an expert's judgement always carries the risk of subjectivity since they err on the side of caution, potentially creating a bias for longer-term trend continuations and ultimate life expectancy limits. In practice, actuaries collaborate closely with trustees to choose appropriate models and assumptions to increase transparency and robustness of mortality analyses; these decisions are shaped by data availability, scheme characteristics and the desired level of granularity [13].

Many different types of models are useful when analysing mortality in pension schemes and forecasting future rates, including extrapolative, process-based and stochastic.

4.2.1 Extrapolative models

Extrapolative models continue to project historical patterns, typically age, period and cohort into the future although they may be influenced by expert judgement regarding persistent trends [13]. Mortality often unexpectedly changes over time, proving challenging for extrapolative models due to their forecasts relying heavily on historic trends. These models often mistakenly assume past influential factors continue to be significant; radical advancements in medical treatments or disease outbreaks can lead to inconsistencies. Extrapolative models also tend to overestimate the value of past sources of mortality improvement; this is exemplified by the reduction in mortality rates due to declining smoking prevalence where the margin for further additional gains is diminishing.

Extrapolative models assume that the future will reflect the past to a degree; they inherently pass the plausibility test by smoothly transitioning between historical and forecast rates. Nevertheless, the assumption that historical patterns will follow along the same trajectory indefinitely is clearly improbable; there is a high chance that these models will neglect changes to trends and structural shifts. Furthermore, purely extrapolative models dismiss potential biological limits to longevity, future medical advancements, environmental concerns and other risk factors. Actuaries select the most appropriate structural form for the historical data used, fine-tune parameters to optimise performance, define the time period of extrapolated data and decide whether to adopt a period or cohort perspective [5].

Failure in recognising and separating heterogeneous groups hinders the effectiveness of extrapolation, resulting in unreliable forecasting performance; a model's performance, particularly for long-term forecasting, cannot be decisively based on historical success. Some extrapolative models are highly sensitive to data; omitting or including a single year's worth of data can significantly alter the results [13]. This is commonly exemplified by skewed data that immediately follows a random event that has produced an abnormal mortality trend.

4.2.2 Process-based models

Process-based models aim to deduce causes and effects, often extrapolating the impacts of individual causes at a granular level. They fundamentally use assumptions about disease prevalence, treatment availability and effectiveness, lifestyle changes and environmental developments to model mortality by causes of death.

These models serve various purposes, such as identifying excessively optimistic or pessimistic projections, generating scenarios for stress testing and complementing other models to determine best estimates and variability. When successful, these models comprehend how causes of death have historically functioned and predict their evolution. Analysing mortality by cause of death presents challenges related to data recording and isolating causes of death. Nonetheless, if these shortcomings could be addressed, the process-based approach would optimise demographic forecasting and should be further researched [54]. To develop models to a satisfactory level for forecasting overall mortality, improvements are necessary regarding the consistency of death data recording, a more nuanced understanding of underlying drivers behind each cause of death and comprehension of complex interactions and interdependencies between various factors and causes of death [13, 55].

An area of development likely to significantly improve mortality modelling is the growing scientific discoveries behind ageing as well as age-related frailty, disability and disease. There is an increasing consensus amongst biomedical scientists that a comprehensive understanding of ageing may be achieved in the coming decades. Current scientific knowledge is suggestive of an intrinsic malleability to the ageing process, which could be exploited to reduce age-related morbidity and mortality. Additionally, evidence indicates that rapid changes in natural selection traits over the last two centuries might have triggered positive genetic changes across populations within a few generations, potentially extending life expectancy to a significant degree; this is especially relevant when examining desirable genetic compositions balancing high fertility, innate resistance to infectious disease and mortality [5].

Process-based models usually produce objectively clearer results than extrapolative models. Although explanatory models are often regarded as the optimal method for forecasting, their accuracy regarding demographic forecasting produces similar results to other methods. Primarily, data related to causes of death can be deemed unreliable due to inconsistent or arbitrary classifications made by those recording the data. There is also limited knowledge of how interactions between different risk factors affect mortality, though this should theoretically improve when data of a higher quality and quantity is collected [56].

4.2.3 Stochastic models

Stochastic modelling involves generating numerous simulations of the future using random inputs drawn from a distribution of plausible values; this approach illustrates a wide variability of outcomes and their relative likelihood. By selecting assumptions with successful outcomes from a desired proportion of simulations, trustees choose appropriately prudent assumptions to generate technical provisions meeting the promised benefits with a sufficient confidence level [4].

The primary advantage of stochastic methods is their production of a forecast probability distribution, instead of a single deterministic value estimate, allowing users to identify a prediction interval with the desired probability [13]; examples of stochastic mortality models include the Lee-Carter model [57] and Cairns-Blake-Dowd model [58].

Unlike extrapolative and process-based models, stochastic models explicitly allow for randomness and uncertainty; they potentially allow a more realistic and flexible representation of real-world processes. However, these advantages of stochastic models can come at the expense increased model complexity, increased data requirements to define distributions for variables rather than using single figures and less easily interpretable results.

4.2.4 SAPS tables

SAPS tables serve as standardised references for mortality analysis, providing valuable insights into mortality rates; they use comprehensive population data, covering a broad age range, typically from national census data, vital statistics and other demographic sources. Their consistent format increases their reliability and popularity. They track the mortality experience of specific birth cohorts over time; examining these trends can reveal valuable information for assumptions and projections. Published by the CMI, an organisation owned by the IFoA, SAPS tables categorise mortality rates using a multitude of characteristics including gender, lives versus amounts, health status and member versus dependant [7, 9, 10, 11].

The selection of an appropriate baseline mortality table is critical for the assessment of current mortality rates; whilst scheme-specific adjustments can be made, broader data is often a more useful starting point. Transparency and evidence-based assumptions are crucial to this process.

Determining the allowance for future mortality improvements has considerable significance; some individual schemes may lack specific validation for improvements. Prudent assumptions avoid unreasonably low rates based on current evidence; suitably balancing conservatism and realism is essential.

Although they offer valuable baseline data, SAPS tables have limitations; schemes do not always share the same membership characteristics, such as occupational groups or socio-economic status, as the general population. Additionally, these tables historically focus primarily on age and gender breakdowns; further analysis, such as occupation-specific mortality, additional data sources or adjustments, has typically been necessary to compensate for their simplicity; fortunately, the latest S4 SAPS series has introduced mortality tables based on the IMD, which helps address this issue.

4.2.5 Postcode analysis

Postcode analysis investigates mortality patterns rooted in geographic location, often utilising postcodes, attempting to distinguish regional variations in rates using local factors. Actuaries analyse mortality within specific postcodes, scrutinising factors such as socio-economic status, healthcare access, environmental conditions and lifestyle choices; this yields valuable insights into mortality disparities across regions, identifying high-risk areas and informing targeted interventions. Inherently, postcode analysis relies on the availability and quality of postcode-level mortality data, which can be sparse or unreliable; it assumes homogeneity within each area, disregarding individual differences within a single postcode. In the absence of other information, an individual's IMD decile, based on their residence, is a significant predictor of mortality [52].

4.2.6 Graduated mortality tables

Graduating a life table for a population is typically feasible only for huge schemes with sufficient and credible data; these tables ideally create smoother, more stable representations by adjusting source mortality data. Graduation techniques account for noise reduction by smoothing out random fluctuations in the raw data; consequently, these tables display a clearer trend, improving the stability of mortality assumptions. Consistent assumptions are vital when projecting future mortality rates; graduated tables strike a balance between volatile raw data and overly smoothed rates. Furthermore, graduation enables interpolation between available data points, producing estimates for imperceptible ages [51].

Subjectivity is present when applying the appropriate graduation method; actuaries carefully select an approach in accordance with a pension scheme's specific characteristics. Different methods yield diverse results, whereby expert judgement should be exercised. Moreover, there is a risk associated with over-smoothing or under-smoothing; aggressive smoothing may obscure important data features and insufficient smoothing may introduce noise and instability.

4.2.7 ML models

ML offers a data-driven approach to mortality analysis; these models use algorithms to derive patterns from historical data. In contrast to traditional methods, ML models have the capacity to grasp complex relationships and nonlinear patterns or interactions; they are customised to specific scheme characteristics beyond age and gender, such as occupation, lifestyle choices and health conditions. ML models possess vast predictive power, improving mortality projections [59].

ML adapts to various data structures, handling complex mortality dynamics; its flexibility results in a tailored analysis, personalised to scheme-specific factors. These models capture intricate mortality trends overlooked by traditional methods, thereby enhancing predictive accuracy [60].

ML methods are reliant upon substantial historical data for training; large, high-quality datasets are essential in building accurate models. Complex ML models may also lack transparency, obfuscating their decisions. There is a risk of overfitting if the dataset is too small or improperly validated [61].

4.3 Calculating mortality rates

Mortality rates measure the risk of death within a specific population, indicating the probability of death across a given time period; their value is calculated using the number of deaths observed and the EtR, which refers to the cumulative time that individuals within a specific population are at risk of dying. EtR accounts for the duration each person contributes to the population's overall risk of death; an analysis based on pension amounts can be achieved by weighting the death and exposure data by these values.

There are two distinct EtR measures, initial and central. The former, E_x, fully captures how long an individual is exposed to a risk at a given age and year, assuming they were alive at the end of the period. On the other hand, the latter, E_x^c, uses the precise exposure time to death [62].

E_x^c is the central EtR measure at age x, representing the observed waiting time in Poisson-type models; it is the elapsed time before death occurs and can be used to determine the force of mortality. The force of mortality illustrates the instantaneous rate of mortality at a precise age, measured on an annualised basis; it captures the speed of death for individuals at a particular age. The force of mortality, μ_x, can be calculated using central exposure and observed number of deaths, d_x, at age x:

$$\mu_x = \frac{\text{Number of deaths}}{\text{Total waiting time}} = \frac{d_x}{E_x^c} \tag{4.1}$$

E_x is the initial EtR measure at age x, representing the actuarial estimate using Binomial-type models. For observed deaths during that year, the initial EtR should extend from the central EtR measure to reflect the theoretical excess remaining in that particular status had they not died. The probability of death, q_x, represents the likelihood that a person will die at a particular age; for a person aged x, this can be calculated from the initial EtR as follows:

$$q_x = \frac{\text{Number of deaths}}{\text{EtR}} = \frac{d_x}{E_x} \tag{4.2}$$

Many alternative approaches for calculating the initial EtR can be found in actuarial literature; the predominant difference between them relates to the end date of exposure for deaths [63]. Methods using central EtR circumvent these issues. The starting date for an initial EtR calculation is taken from either the commencement date of the investigation period or pension, depending on which falls later. For alive members or those exiting due to reasons other than death, their exposure terminates on the end date of the investigation period or exit depending on which falls earlier. For members who die within the period, various approaches have been used to determine the end date of the initial EtR. The last day of exposure is typically extended to the day before their next birthday; some approaches prematurely stop the EtR at end the investigation period, especially when the scheme year does not coincide with the calendar year [63]. The principle of correspondence states that an alive individual should be included in the exposure for age x at time t only if their immediate death would be allocated to death data d_x at age x. Including exposure beyond the investigation period would violate this principle since deaths occurring after the end of the investigation period are excluded.

An approximate relationship between the central and initial EtR measures is depicted as:

$$E_x \approx E_x^c + 0.5 \times d_x \tag{4.3}$$

This formula assumes that deaths have occurred in the middle of the year on average; therefore, their survival would have resulted in an additional half a year of exposure. The relationship between the probability of death and the force of mortality is as shown [64]:

$$q_x \approx 1 - e^{-\mu_{x+0.5}} \tag{4.4}$$

By using the exact entry and exit dates of members, exact values for exposure within the specific population can be derived. However, calculating exposures when assuming entries and exits occur halfway through the year deepens understanding of whether the approximate relationships between mortality metrics still hold; in addition, this can significantly simplify the necessary calculations. By using the age nearest an individual's birthday at the start of the scheme year as a proxy for the age on their last birthday. Similarly, the member's age on their next birthday at the start of the scheme year can be used as a proxy for the age on their nearest birthday.

Definition of x	Rate interval	q estimate	μ estimate
Age last birthday	$[x,\ x+1]$	q_x	$\mu_{x+0.5}$
Age nearest birthday	$[x-0.5,\ x+0.5]$	$q_{x-0.5}$	μ_x
Age next birthday	$[x-1,\ x]$	q_{x-1}	$\mu_{x-0.5}$

Figure 4.1: Probability of death q and force of mortality μ for different age definitions.

These measures can be calculated with various age definitions, also known as rate intervals, affecting the applicable age intervals; it is important both the death and EtR measures use the same age definition. Some common probability of death and force of mortality estimates and the age definitions and rate intervals in which they apply are summarised in Figure 4.1.

The central death rate m_x is found by dividing the number of deaths d_x by the average amount of people alive at that age during the period; the average is usually estimated as the mean of populations at the start and end of the year if exact populations in the middle of the year are unavailable. The force of mortality is a continuous function relating to the instantaneous death rate at each age in a very small time period. Given that deaths would generally be expected to occur halfway through an interval on average, the relationship between the central death rate and force of mortality is as follows [65]:

$$m_x \approx \mu_{x+0.5}. \tag{4.5}$$

Various single-figure indices can summarise a population's mortality rates [66]. Although the simplest, the CMR is disadvantageously influenced by the balance of homogeneous demographic groups, complicating comparisons of populations with different demographic structures. The CMR can be calculated as such:

$$\text{CMR} = \frac{\text{Actual number of deaths across all ages}}{\text{Total central EtR across all ages}}$$

$$= \frac{\sum_x E^c_{x,t} m_{x,t}}{\sum_x E^c_{x,t}} \tag{4.6}$$

DSMR adjust the CMR to incorporate the underlying populations composition; although it reflects differences in $m_{x,t}$, discrepancies in the age distributions of populations are neglected. The DSMR is shown by:

$$\text{DSMR} = \text{CMR for standard population using specific mortality rate}$$

$$= \frac{\text{Standard population specific deaths}}{\text{Standard population exposure}}$$

$$= \frac{\sum_x {}^sE^c_{x,t} m_{x,t}}{\sum_x {}^sE^c_{x,t}} \tag{4.7}$$

Indirect standardisation advantageously allows the DSMR to be credibly approximated without age-specific mortality rates from the given population $m_{x,t}$; such quantities are often unreliable due to insufficient data. Calculating an ISMR requires an area comparability factor, F, comparing a specific population's structure to the standard population:

$$F = \frac{\text{CMR for standard population}}{\text{CMR for specific population using standard mortality rate}}$$

$$= \frac{(\text{Standard population actual deaths / Standard population exposure})}{(\text{Specific population standard deaths / Specific population exposure})}$$

$$= \frac{\left(\sum_x {}^sE^c_{x,t}\,{}^sm_{x,t} \;/\; \sum_x {}^sE^c_{x,t}\right)}{\left(\sum_x E^c_{x,t}\,{}^sm_{x,t} \;/\; \sum_x E^c_{x,t}\right)} \tag{4.8}$$

When $F < 1$, the specific population generally experiences heavier mortality than the standard population, whereas $F > 1$ signifies lighter mortality. The ISMR can then be determined as:

$$\text{ISMR} = F \times \text{CMR}$$

$$= \frac{(\text{Standard population actual deaths / Standard population exposure})}{(\text{Specific population standard deaths / Specific population actual deaths})}$$

$$= \frac{\left(\sum_x {}^sE^c_{x,t}\,{}^sm_{x,t} \;/\; \sum_x {}^sE^c_{x,t}\right)}{\left(\sum_x E^c_{x,t}\,{}^sm_{x,t} \;/\; \sum_x E^c_{x,t}m_{x,t}\right)} \tag{4.9}$$

Similarly to the area comparability factor, the SMR calculates the overall weight of a specific population mortality relative to the standard population as shown:

$$\text{SMR} = \frac{\text{Specific population actual deaths}}{\text{Specific population standard deaths}}$$

$$= \frac{\sum_x E^c_{x,t}m_{x,t}}{\sum_x E^c_{x,t}\,{}^sm_{x,t}} \tag{4.10}$$

In this case, SMR > 1 represents heavier mortality in the specific population, whereas SMR < 1 signifies the opposite.

Chapter 5

Mortality experience analysis applications

This chapter applies the information covered in previous chapters to a specific set of pension scheme data, LGPS EW; the analysis initially profiles the membership data, ensuring its suitability. The EtR methodology is justified and implemented in an AoE utilising the SAPS S3 series mortality tables. After examining the assumption's relative prudence, as recommended by the most recent scheme valuations, best estimate assumptions are derived as a baseline for comparison with other AoE techniques. The effects on the data's subdivision by IMD and region using postcode data are explored, along with the impact caused by IMD-based SAPS S4 tables. Next, mortality rates are graduated from the data in accordance with a formula from the Gompertz-Makeham family. The final sections use more experimental and rudimentary AoE approaches using ML regression and classification methods; initial improvements to model performance are attempted by supplementing membership data with external variables at the LSOA granularity level.

5.1 SAPS analysis (baseline)

5.1.1 Data overview

Local governments have numerous employees dedicated to serving their communities, primarily administrators and public service professionals. Together, these diverse groups collaborate to ensure the effective and efficient delivery of essential services, enhancing the community's well-being.

The LGPS EW dataset includes individuals involved in the scheme on 31st March in 2016 and 2020, in addition to members that exited the scheme from 1st April 2013 to 31st March 2020.

The different categories of individuals represented are as follows:

- Normal health pensioners, retired from active or deferred membership status;
- Ill health pensioners, retired early from active membership status due to ill health, usually with enhanced benefits;
- Spouse dependants, entitled to pensions following the death of a normal or ill health pensioner;
- Child dependants, entitled to pensions following the death of a normal or ill health pensioner.

All key data items were verified against GAD's specification requested for both completion and consistency as part of the 2016 [67] and 2020 [68] LGPS EW pension scheme valuations.

The first recorded death from COVID-19 in England and Wales occurred in early March 2020; 1,642 deaths were attributed to COVID-19 within this month overlapping the analysis, impacting on the results to a minor extent [69].

5.1.2 Data summaries

The Appendix (Chapter 7) contains additional visualisations of the pension scheme membership data.

Number of members

Figures 7.1 to 7.2 illustrate the number of pensioners and dependants in the scheme as of 31st March 2020, disaggregated by gender and region; Figures 7.3 to 7.6 display the corresponding numbers for each pension type. Equivalent visualisations are also provided for pensioners and dependants that exited during the intervaluation period from 1st April 2013 to 31st March 2020, segmented by gender in Figures 7.31 to 7.32 and pension type in Figures 7.33 to 7.36.

As expected from the LGPS EW scheme, regions outside of England and Wales have the lowest number and proportion of scheme members whilst the highest are from the North West and South East of England. The majority of members are female and similar proportions of gender are distributed regionally. Most members retired in normal health whilst there was a similar, lower number of ill health retirees and dependant spouses; in contrast, the number of dependant child scheme members is very small. The North West has a higher proportion of members with ill health, dependant spouses and dependant children, though the regional order remains relatively consistent across all categories. Similar patterns are observed in the 2020 valuation data and intervaluation period, suggesting a stability to membership composition during this period.

Pension amounts

Figures 7.7 to 7.8 depict the total pension amount of pensioner and dependant scheme members as of 31st March 2020, divided by gender and region; Figures 7.9 to 7.12 display the equivalent for each pension type. Comparable visualisations are also provided for exits from pensioner and dependant status during the intervaluation period 1st April 2013 to 31st March 2020; they are separated by gender in Figures 7.37 to 7.38 and pension type in Figures 7.39 to 7.42; the mean pensions for each group are represented in Figures 7.13 to 7.18 and Figures 7.43 to 7.48, respectively.

The highest sums and proportions of pension are located in the North West and South East of England, where most members are found; however, the average pension value per scheme member in 2020 terms is considerably higher in London. London has a markedly higher average pension in 2020 terms than other regions for both genders, with males receiving approximately £3,000 more. Despite substantially more females, the total male pensions in 2020 terms tend to outweigh their female counterparts across regions; even the area with the lowest average male pension shows more than the region with the greatest female average pension. The proportion of pensions in each region is similar for both genders with normal health retirement pensions comprising the majority of data. The pension size in 2020 terms from ill health retirements is substantially higher than dependant spouse pensions, despite a fairly equal split in numbers; the amount of dependant child pensions is minute relative to other categories. The average pension in 2020 terms from normal and ill health retirements is predictably similar; the amount is similar for those who have retired when expected and individuals retiring early from ill health. The gap between the average pension in London and other regions is only witnessed in the normal health retirement category, although the former still maintains higher regional values for the remaining categories. On average, spousal pensions broadly contain half the amount of member pensions; child pensions roughly contain half the value of spousal pensions. The North West accounts for a greater proportion of ill health, dependant spouse and dependant child pensions; the regional ordering remains similar across these categories. Average ill health retirement pensions in the Isle of Man are noticeably larger than other regions; the extremely low sample size in the movement data likely depicts increased fluctuations and variability for this region's values. Apart from these circumstances, similar patterns in the 2020 valuation data and intervaluation period are indicative of a stable membership composition.

IMD

Figures 7.19 to 7.20 display the mean IMD for pensioner and dependant scheme members as of 31st March 2020, segregated by gender and region; Figures 7.21 to 7.24 depict their corresponding values for each pension type. Equivalent visualisations are provided for exits from pensioner and dependant status during the intervaluation period from 1st April 2013 to 31st March 2020; these are segmented by gender in Figures 7.49 to 7.50 and pension type in Figures 7.51 to 7.54.

An IMD decile of 1 represents the most deprived and 10 the least deprived, as exemplified by the ONS and others [70]; London and Wales have the lowest average UK decile, indicating the highest levels of deprivation on average, whilst the South East of England shows the highest, depicting the lowest degree of deprivation. Disregarding London, the North of England generally has lower UK IMD decile means, denoting higher deprivation, than Scotland and Southern England. The average UK IMD decile for males is usually lower than females within the same region; although this suggests increased levels of deprivation for men, they counter-intuitively have higher pensions on average in actuality. In addition, both genders show a similar pattern regarding their UK IMD deciles across regions. Average UK IMD deciles are typically lower for ill health retirements than other categories, possibly a consequence of greater vulnerability to health issues in deprived areas. Spouse and child UK IMD averages show similar ranges across regions; normal health UK IMD averages are higher, likely due to fewer health issues in less deprived areas. The low average UK IMD decile for London, indicating high deprivation, is incongruous to the higher mean pension values, unless higher pensions are dramatically offset by increased living costs. The movement data typically shows lower UK IMD decile values, indicating higher deprivation levels than the data at 2020; this meets the expectation that more deprived individuals are more likely to exit the scheme through death.

Pension duration

Figures 7.25 to 7.26 display the mean pension duration of pensioner and dependant scheme members as of 31st March 2020, broken down by gender and region. Figures 7.27 to 7.30 show the corresponding values for each pension type. Equivalent visualisations are provided for exits from pensioner and dependant status during the intervaluation period from 1st April 2013 to 31st March 2020, segmented by gender in Figures 7.55 to 7.56 and by pension type in Figures 7.57 to 7.60.

Regions with older members tend to intuitively show a longer pension receipt duration on average. Males generally receive their pensions slightly longer than females whereas there is negligible difference between normal health members and spouses. As expected, ill health pensions have been received for substantially longer, as members are prematurely forced into retirement due to ill health issues. Child pensions are paid out for a shorter duration than the other categories, as anticipated.

The mean age at which members exit the scheme is six to seven years higher than the current mean membership age; furthermore, their age at the time of pension commencement indicates that exiting members have broadly received pensions for approximately seventeen to eighteen years. As anticipated, female members tend to delay their exit until an older age than their male counterparts, which aligns with the generally longer lifespans experienced by women; since women also usually start their pensions later than men, there is little gender difference in the length of time they are in receipt of their pensions, with men perhaps having a slight lead. Child pensions are typically received for a shorter period, approximately seven years on average. Spouse pensions usually span around thirteen years. Members receive ill health pensions for approximately twenty years on average, marginally longer than those receiving normal health pensions, which is roughly eighteen years.

Although absent from the Appendix (Chapter 7), similar maps were produced for other variables; the main insights gained are summarised below.

Age

Scheme members with unknown locations have the highest average age for both genders, potentially indicating administrative difficulties in engaging with and updating contact details for older generations. Otherwise, the average ages fall within reasonable ranges without seemingly obvious patterns by region or gender. Similar mean ages are found across normal and ill health retired members. The average ages for spouses are typically higher, which is probably attributed to the large proportion of female members in the scheme leading to a greater number of older male spouses. As expected, the mean age of a child dependant is under 18 years old.

SPA

The average SPA is similar across all regions, with males averaging higher values than females; the female SPA rose gradually to 65, equalising with males in November 2018. By 2020, the shared SPA reached 66, with further increases legislated to 67 by 2028 and 68 by 2046. Normal health, ill health and spousal scheme members show consistent SPA values across regions, reflecting a similar weighting between male and female SPA values. The SPA for children is broadly 68 across regions, as expected for individuals in this limited age bracket. SPA values average slightly higher in the at 2020 data in comparison to movement data; this is in line with expectations, resulting from the incremental SPA for females matching males, as well as a general increase across the whole population.

Pension commencement age

Locations outside of England and Wales show lower average starting ages for pensions; scheme members wanting to relocate with the means to do so, are usually also able to to retire earlier. The South East and East of England, with the highest two UK IMD deciles, generally have the latest retirement ages, indicating that people from areas of lower deprivation regularly work longer due to better health and capability. These patterns and values are similar across both genders. Retirements from normal health are taken between the ages of 59-61 on average, whilst retirements from ill health occur earlier from a wider range of years. Spouse pensions commence at later ages than normal health retirements on average, likely due to the predominance of female members. Understandably, child pensions begin from very young ages. Interestingly, the age when pensions commence is generally lower in data at 2020 data than in movement data, suggesting a preference for earlier retirement than the SPA; this trend is much more pronounced in the female, ill health and spouse categories.

Pension commencement age relative to SPA

Regions with the highest average pension commencement age are also the latest to retire relative to their SPA, with pensions typically starting a few years before SPA. Males tend to retire several years earlier than females, before reaching SPA since it was historically higher for men; in addition, they often attain greater pension values, facilitating earlier retirement. Normal health retirements generally occur a few years before reaching SPA, whilst ill health retirements happen several years earlier. Spouse pensions start closer to SPA than normal health retirements, even beyond this threshold in some regions. Child pensions, as expected, begin long before these individuals' SPA. The data at the 2020 valuation demonstrates a clear trend of earlier retirements age relative to SPA in comparison to the movement data; this could be a result of women's SPA increasing, a rising SPA for the general population or a tendency towards early retirement, which is particularly pronounced in the ill health and spouse memberships.

Exit timing

Given that the movement data spans a seven year period, it is reassuring that the majority of regions, especially the larger ones, roughly depict members exiting the scheme around the midpoint of the investigation on average across each gender and pension type.

5.1.3 EtR methodology

This section outlines the employed methodology determining the EtR for records within the dataset; examples provided are as follows:

1. Pension commenced before analysis period, remaining in the scheme throughout; in this scenario, the member contributes to exposure but not to exit count; central EtR = initial EtR.

2. Pension commenced during analysis period, leaving the scheme upon death in the final year; consequently, the member contributes to exposure and exit; central EtR < initial EtR.

3. Pension commenced before analysis period, leaving the scheme upon death before

the final year starts; thus, the member contributes to exposure and exit; central EtR < initial EtR.

4. Pension commenced before analysis period, leaving the scheme before the final year starts for any reason excluding death; as a result, the member contributes to exposure but not to exit count; central EtR = initial EtR.

In these examples, the EtR is calculated in years using the time difference between the start and end dates; multiplying by the revalued member pension at the end date of the analysis period provides an amounts weighted EtR.

Using the nearest age at the start of the year approximation for age on the last birthday simplifies the calculations, ensuring the age and year label change dates coincide. Assuming all entries and exits occur exactly halfway through the year limits a member's possible exposure values within a given year to 0, 0.5 or 1; upon conversion from central to initial EtR measures, this simplification fixes the value addition to 0.5 since the age change date coincides with the scheme year.

When exits occur, the following date is taken as the EtR end date, ensuring the actual exit date is included within the exposure; the same principle applies to the end of the analysis period.

5.1.4 Example 1: full exposure

The following assumptions are applied:

- Analysis period spans 01/04/2018 to 31/03/2020;
- Member was born on 01/06/1966;
- Member's pension, revalued to the end of the analysis period, is £5,000;
- Pension commenced before analysis period;
- Member remains in the scheme during the analysis period.

This case examines a member with an existing pension, prior to the analysis period, remaining in the scheme throughout; they contribute to exposure but not to exit count. The central EtR represents the duration of membership in the scheme during the analysis, which is the whole period. Since the member did not exit due to death, no initial EtR adjustment is needed; the central EtR equals the initial EtR. Table 5.1 presents a detailed breakdown of the exact central and initial EtR calculations; Table 5.2 replicates this for the approximate calculations.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	51	01/04/2018	01/06/2018	0.167	833
1	52	01/06/2018	01/04/2019	0.833	4,167
2	52	01/04/2019	01/06/2019	0.167	833
2	53	01/06/2019	01/04/2020	0.833	4,167
Initial EtR extension:					
None					

Figure 5.1: Allocation of exposure for each age and year using an exact method according to the actual dates when the scheme year and age labels change. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	52	01/04/2018	01/04/2019	1	5,000
2	53	01/04/2019	01/04/2020	1	5,000
Initial EtR extension:					
None					

Figure 5.2: Allocation of exposure for each age and year using an approximate method, determining the age label for each scheme year as the nearest age at the start of the year. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

5.1.5 Example 2: delayed exposure followed by death

The following assumptions are applied:

- Analysis period spans 01/04/2018 to 31/03/2020;
- Member was born on 01/06/1966;
- Member's pension, revalued to the end of the analysis period, is £5,000;
- Pension commenced on 01/05/2018;
- Member exited the scheme on 30/08/2019;
- Exit occurred due to death.

This scenario examines a member whose pension began during the analysis period, exiting the scheme in the final year upon their death; they contribute to exposure and exit count. The central EtR represents the time spent in the scheme during the analysis, from the member's start date to death; since their exit is caused by death during this period, the initial EtR is extended. For the precise method, the calculation extends to the end of the analysis period because this takes place before their next birthday. The approximate method extends the initial EtR to the date when the age label is subsequently recalculated, coinciding with the change in scheme year and end of the analysis period. Therefore, the central EtR is less than the initial EtR. Table 5.3 presents a detailed breakdown of the exact central and initial EtR calculations; Table 5.4 replicates this for the approximate calculations.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	51	01/05/2018	01/06/2018	0.083	417
1	52	01/06/2018	01/04/2019	0.833	4,167
2	52	01/04/2019	01/06/2019	0.167	833
2	53	01/06/2019	01/09/2019	0.250	1,250
Initial EtR extension:					
2	53	01/09/2019	01/04/2020	0.583	2,917

Figure 5.3: Allocation of exposure for each age and year using an exact method according to the actual dates when the scheme year and age labels change. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	52	30/09/2018	01/04/2019	0.5	2,500
2	53	01/04/2019	01/10/2019	0.5	2,500
Initial EtR extension:					
2	53	01/10/2019	01/04/2020	0.5	2,500

Figure 5.4: Allocation of exposure for each age and year using an approximate method, determining the age label for each scheme year as the nearest age at the start of the year. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

5.1.6 Example 3: partial exposure due to death

The following assumptions are applied:

- Analysis period spans 01/04/2018 to 31/03/2020;
- Member was born on 01/06/1966;
- Member's pension, revalued to the end of the analysis period, is £5,000;
- Pension commenced prior to the analysis period;
- Member exited the scheme on 30/08/2018;
- Exit occurred due to death.

This case examines a member with existing pension, prior to the analysis period, exiting the scheme before the final year upon their death; they contribute to exposure and exit count. The central EtR represents the time spent in the scheme during the analysis, from the start date to death; since their exit is caused by death, the initial EtR is extended. For the precise method, the calculation extends to the next birthday since this falls before the end of the analysis period. The approximate method extends the initial EtR to the date when the age label is subsequently recalculated, coinciding with the change in scheme year. Therefore, the central EtR is less than the initial EtR. Table 5.5 presents a detailed breakdown of the exact central and initial EtR calculations; Table 5.6 replicates this for the approximate calculations.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	51	01/04/2018	01/06/2018	0.167	833
1	52	01/06/2018	01/09/2018	0.250	1,250
Initial EtR extension:					
1	52	01/09/2018	01/04/2019	0.583	2,917
2	53	01/04/2019	01/06/2019	0.167	833

Figure 5.5: Allocation of exposure for each age and year using an exact method according to the actual dates when the scheme year and age labels change. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	52	01/04/2018	30/09/2018	0.5	2,500
Initial EtR extension:					
1	52	30/09/2018	01/04/2019	0.5	2,500

Figure 5.6: Allocation of exposure for each age and year using an approximate method, determining the age label for each scheme year as the nearest age at the start of the year. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

5.1.7 Example 4: partial exposure due to withdrawal

The following assumptions are applied:

- Analysis period spans 01/04/2018 to 31/03/2020;
- Member was born on 01/06/1966;
- Member's pension, revalued to the end of the analysis period, is £5,000;
- Pension commenced prior to the analysis period;
- Member exited the scheme on 30/08/2019;
- Exit occurred due to any reason excluding death.

This scenario examines a member whose pension began before the analysis period, exiting the scheme in the final year for any reason excluding death; they contribute to exposure but not to exit count. The central EtR represents the time spent in the scheme during the analysis, from the start of the analysis period to the member's exit; since their exit was not due to death, the initial EtR is unadjusted, equalling the central EtR. Table 5.7 presents a detailed breakdown of the exact central and initial EtR calculations; Table 5.8 replicates this for the approximate calculations.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	51	01/04/2018	01/06/2018	0.167	833
1	52	01/06/2018	01/04/2019	0.833	4,167
2	52	01/04/2019	01/06/2019	0.167	833
2	53	01/06/2019	01/09/2019	0.250	1,250
Initial EtR extension:					
None					

Figure 5.7: Allocation of exposure for each age and year using an exact method according to the actual dates when the scheme year and age labels change. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

Year number	Age	EtR start	EtR end	EtR	Weighted EtR
Central EtR:					
1	52	01/04/2018	01/04/2018	1	5,000
2	53	01/04/2019	01/10/2019	0.5	2,500
Initial EtR extension:					
None					

Figure 5.8: Allocation of exposure for each age and year using an approximate method, determining the age label for each scheme year as the nearest age at the start of the year. The upper section of the table displays the central EtR allocations; additional allocations required for the initial EtR are in the lower section.

5.1.8 EtR data

The previous section explained the methodology for deriving central and initial EtR figures; this section illustrates the resulting exposures within the data, demonstrating how they can be used to generate expected exits.

The initial EtR method uses the age on the last birthday definition, calculating the probability of death at each age by dividing the number of deaths by EtR, as demonstrated in formula 4.2. Expected exits are easier to quantify and summarise than mortality rates; thus, they are simpler to comprehend and interpret. Consequently, expected exits, $\hat{d}_x$, can be derived from mortality rate assumptions, $\hat{q}_x$, as follows:

$$\hat{d}_x = \hat{q}_x \times E_x \tag{5.1}$$

The central EtR method uses the age on the last birthday definition, calculating the force of mortality at each age by dividing the number of deaths by EtR:

$$\mu_{x+0.5} = \frac{d_x}{E_x^c} \tag{5.2}$$

The software being used requires mortality probabilities rather than forces; equation 4.4 helps derive expected exits, $\hat{d}_x$, which are based on the central EtR and the assumed mortality probabilities, $\hat{q}_x$, as illustrated:

$$\hat{d}_x = \hat{\mu}_{x+0.5} \times E_x^c \approx -ln(1 - \hat{q}_x) \times E_x^c \tag{5.3}$$

This method successfully eliminates the burden of deriving an initial EtR, avoiding the ambiguity over the exposure's termination date.

Table 5.9 splits deaths and exposure by lives across various categories based on pension type and gender; it presents exact and approximate exposures resulting from the application of both initial and central measures. Normal health males and females exhibit the most deaths, approximately three times greater than the corresponding ill health categories. Child categories have even less, exhibiting the fewest deaths overall. Normal health females have the highest exposure, followed by their male counterparts, contrary to the death breakdowns. Ill health categories have lower exposures with child categories maintaining the lowest. Spouse data is predominantly skewed towards females; predictably, the initial exposures are higher than central exposures. The slight discrepancy between exact and approximate exposures validates the occurrence of entries and exits halfway through the year on average; this consolidates the data quality and robustness of the calculations. Higher exact exposures could indicate that entries occur prior to the midpoint or exits happen after this point on average.

Category	d_x	E_x	$\hat{E}_x$	E_x^c	$\hat{E}_x^c$
Normal health males	68,659	2,468,506	2,461,996	2,437,552	2,427,667
Normal health females	64,507	3,965,010	3,953,543	3,935,913	3,921,289
Ill health males	23,275	533,766	534,175	523,182	522,538
Ill health females	21,401	662,724	662,962	653,010	652,261
Spouse males	9,057	225,633	226,178	221,572	221,650
Spouse females	40,791	705,992	707,378	687,467	686,983
Child males	39	24,794	25,039	24,771	25,019
Child females	38	25,184	25,443	25,167	25,424

Figure 5.9: Actual number of exits (d_x), exact (E_x) and approximate ($\hat{E}_x$) initial EtR and exact (E_x^c) and approximate ($\hat{E}_x^c$) central EtR, from 1st April 2013 to 31st March 2020 for each category.

Category	$\hat{E}_x^c + 0.5 \times d_x$	Ratio v $\hat{E}_x$	$E_x^c + 0.5 \times d_x$	Ratio v E_x
Normal health males	2,461,996	100.0%	2,471,882	100.1%
Normal health females	3,953,543	100.0%	3,968,166	100.1%
Ill health males	534,175	100.0%	534,819	100.2%
Ill health females	662,962	100.0%	663,710	100.1%
Spouse males	226,178	100.0%	226,100	100.2%
Spouse females	707,378	100.0%	707,863	100.3%
Child males	25,039	100.0%	24,791	100.0%
Child females	25,443	100.0%	25,186	100.0%

Figure 5.10: A table showing the approximate central EtR for lives ($\hat{E}_x^c$) plus half the deaths (d_x) equals the approximate initial EtR ($\hat{E}_x$) meaning the ratio equals 100%. It also shows the equivalent calculation, using exact central (E_x^c) and initial (E_x) EtR values, leading to an unequal result whereby the ratio does not equal 100%.

Table 5.10 proves the approximate central exposure plus half the deaths equals the approximate initial exposure, as expected. Although the exact central exposure plus half the deaths does not equal the exact initial exposure, a margin of error is anticipated due to misalignment of actual entries and exits with the scheme year's midpoint. The close proximity of this central and initial exposure check, in combination with the small differential between the exact and approximate exposures, emphasises the calculations' reasonableness.

Tables 5.11 and 5.12 display the equivalent tables weighted by pension amounts. The allocated differences to various pension types are similar to those observed for lives, with the exception of a notably larger proportion allocated to males; this shift indicates a gender imbalance whereby male pensions are generally greater in value within this population.

Table 5.13 presents the baseline pensioner mortality assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations respectively [71, 72]; each valuation utilised the subsequent increment in the SAPS series of pension scheme tables, from S1 in 2013 through to S3 in 2020. The mortality improvement assumptions applied at the 2013, 2016 and 2020 valuations were ONS 2014, ONS 2016 and ONS 2020 respectively; these were aligned with the UK's most recent ONS principal population projections at the time of each valuation.

The discrepancies between the tables in each category and valuation complicate comparisons across various columns and rows; however, scaling factors can be inferred, providing insight into each category's assumptions relative to that table's underlying population. The assumptions for normal health males border 100%, expressing similarity between their expected mortality experience and CMI population in every normal health male amounts table; in contrast, scaling factors for normal health females are usually lower than 100%, predicting longer lifespans on average than the populations underlying their respective SAPS tables. On the other hand, ill health males and females have scaling factors above 100%, suggesting a higher death rate than anticipated from the SAPS populations underlying their tables. The male spouse category exhibited heavier mortality rates than the normal health male amounts SAPS tables; these rates reduce slightly in relation to the dependant male amounts SAPS table in the 2020 valuation. The spouse female category shows a similar pattern, whereby moving to a normal health female from a dependant female amounts table increases the scaling factor; changing from a normal health female amounts table to the heavy version reduces the scaling factor. No assumptions were generated for child categories due to insufficient death experience data.

Category	d_x	E_x	$\hat{E}_x$	E_x^c	$\hat{E}_x^c$
Normal health males	461,626,826	20,206,012,254	20,141,173,583	19,998,589,086	19,910,360,171
Normal health females	211,641,570	14,679,818,643	14,613,012,506	14,584,531,689	14,507,191,721
Ill health males	147,091,489	3,807,814,871	3,810,853,309	3,741,058,297	3,737,307,564
Ill health females	93,699,818	3,054,413,132	3,055,886,705	3,012,088,193	3,009,036,796
Spouse males	11,188,474	386,664,214	387,609,370	381,578,219	382,015,133
Spouse females	139,475,485	2,568,006,884	2,573,975,801	2,504,587,865	2,504,238,058
Child males	62,740	39,253,574	39,749,728	39,214,507	39,718,359
Child females	49,997	40,108,402	40,616,340	40,087,732	40,591,341

Figure 5.11: Actual exit amounts (d_x), exact (E_x) and approximate ($\hat{E}_x$) initial EtR amounts and exact (E_x^c) and approximate ($\hat{E}_x^c$) central EtR amounts, from 1st April 2013 to 31st March 2020 for each category.

Category	$\hat{E}_x^c + 0.5 \times d_x$	Ratio v $\hat{E}_x$	$E_x^c + 0.5 \times d_x$	Ratio v E_x
Normal health males	20,141,173,583	100.0%	20,229,402,499	100.1%
Normal health females	14,613,012,506	100.0%	14,690,352,474	100.1%
Ill health males	3,810,853,309	100.0%	3,814,604,042	100.2%
Ill health females	3,055,886,705	100.0%	3,058,938,102	100.1%
Spouse males	387,609,370	100.0%	387,172,456	100.1%
Spouse females	2,573,975,801	100.0%	2,574,325,608	100.2%
Child males	39,749,728	100.0%	39,245,877	100.0%
Child females	40,616,340	100.0%	40,112,731	100.0%

Figure 5.12: A table showing the amounts approximate central EtR ($\hat{E}_x^c$) plus half the deaths (d_x) equals the approximate initial EtR ($\hat{E}_x$) meaning the ratio equals 100%. It also shows the equivalent calculation, using the exact central (E_x^c) and initial (E_x) EtR values, leading to an unequal result whereby the ratio does not equal 100%.

Category	2013 table	2013 scaling	2016 table	2016 scaling	2020 table	2020 scaling
Normal health males	S1NMA	99%	S2NMA	101%	S3NMA_M	99%
Normal health females	S1NFA	93%	S2NFA	92%	S3NFA_M	96%
Ill health males	S1IMA	104%	S2IMA	107%	S3IMA	117%
Ill health females	S1IFA	106%	S2IFA	106%	S3IFA	133%
Spouse males	S1NMA	120%	S2NMA	132%	S3DMA	96%
Spouse females	S1DFA	101%	S2NFA	106%	S3NFA_H	97%

Figure 5.13: Baseline pensioner mortality assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations respectively. These standard tables are from the SAPS series tables, published by the CMI.

5.1.9 Exact initial EtR

This section is a breakdown of the AoE conducted against the SAPS tables using the exact initial EtR with age at the last birthday [7]. Tables illustrating results from other combinations of central and approximate techniques can be found in Sections 7.3 to 7.5. Although various methods of calculating EtR produce discrepancies in the anticipated exits, key items such as average ages at exit, optimal scaling factors for best fit and ratios between actual and expected exits reveal very similar outcomes; this validates that all approaches produce reasonable mortality experience analysis.

Table 5.14 compares the mean exit ages, weighted by pension amounts, for different member categories to assumptions set at the 2013, 2016 and 2020 pension scheme valuations.

The normal health categories exhibit similar results for the actual mean exit ages for each gender and an increase of a few years in comparison to the ill health groups; the experience indicates that males with health issues appear to live longer than their female counterparts, but this is reversed when comparing male and female dependants. Female spouses consistently show the oldest average exit age by a substantial margin.

For males and females of normal health, the actual average exit ages are slightly higher than in the 2020, 2016 and 2013 assumptions, highlighting the accuracy of these predictions; in contrast, males and females with health issues display significantly lower average exit ages than their respective assumptions, which could have been particularly conservative. For male spouses, the actual mean exit age is very close to the 2020 assumption and lower than those made in 2016 and 2013; female spouses depict actual average exit age that is slightly lower than all the yearly assumptions.

For normal health females, the mortality assumptions gradually increase from 2013 to 2020, whereas ill health females initially declined in 2016 before rising in 2020; conversely, the assumed mean exit age increased for all other categories in 2016, subsequently decreasing in the 2020 valuation.

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
Normal health males	81.10	80.72	80.91	80.77
Normal health females	81.00	80.97	80.50	80.08
Ill health males	75.63	77.23	77.36	77.08
Ill health females	73.47	78.21	78.01	78.12
Spouse males	78.13	78.22	80.02	79.84
Spouse females	87.69	88.37	88.55	88.20

Figure 5.14: Actual mean exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected mean exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	Year	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
Normal health males	2013	80.97	80.54	80.71	80.61
Normal health males	2014	81.11	80.75	80.96	80.84
Normal health males	2015	81.20	80.77	80.96	80.82
Normal health males	2016	81.14	80.66	80.83	80.70
Normal health males	2017	81.22	80.73	80.90	80.75
Normal health males	2018	81.01	80.81	80.97	80.81
Normal health males	2019	81.05	80.79	81.01	80.83
Normal health females	2013	80.23	80.87	80.29	80.02
Normal health females	2014	81.27	81.06	80.56	80.24
Normal health females	2015	81.17	81.06	80.58	80.20
Normal health females	2016	81.09	80.91	80.45	80.02
Normal health females	2017	81.05	80.99	80.53	80.07
Normal health females	2018	80.80	80.98	80.53	80.04
Normal health females	2019	81.25	80.92	80.53	80.01
Ill health males	2013	74.45	75.62	75.86	75.74
Ill health males	2014	75.01	76.34	76.55	76.37
Ill health males	2015	75.25	76.82	76.98	76.75
Ill health males	2016	75.05	77.18	77.31	77.05
Ill health males	2017	76.57	77.82	77.89	77.60
Ill health males	2018	75.90	78.29	78.31	77.99
Ill health males	2019	77.05	78.74	78.76	78.41
Ill health females	2013	71.94	76.27	76.10	76.57
Ill health females	2014	72.73	77.11	76.95	77.29
Ill health females	2015	72.75	77.77	77.60	77.80
Ill health females	2016	73.54	78.20	78.03	78.14
Ill health females	2017	74.35	78.76	78.57	78.59
Ill health females	2018	73.76	79.29	79.08	79.02
Ill health females	2019	74.76	79.83	79.60	79.46
Spouse males	2013	76.62	76.79	78.69	78.57
Spouse males	2014	75.08	77.45	79.37	79.23
Spouse males	2015	78.15	78.00	79.86	79.71
Spouse males	2016	78.05	77.67	79.45	79.30
Spouse males	2017	79.29	78.33	80.09	79.92
Spouse males	2018	78.75	78.87	80.59	80.41
Spouse males	2019	79.63	79.38	81.11	80.92
Spouse females	2013	87.48	87.80	87.96	87.62
Spouse females	2014	86.84	88.11	88.29	87.95
Spouse females	2015	87.64	88.24	88.43	88.07
Spouse females	2016	87.93	88.50	88.66	88.33
Spouse females	2017	88.19	88.65	88.82	88.48
Spouse females	2018	87.68	88.68	88.85	88.51
Spouse females	2019	88.12	88.66	88.89	88.54

Figure 5.15: Actual mean exit ages from 1st April 2013 to 31st March 2020 for each category and year, and expected mean exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Table 5.15 presents a more comprehensive annual breakdown; despite inevitable fluctuations, the actual average exit ages for all categories gradually increase over time, as expected with improvements to life expectancy. The assumptions for 2013, 2016 and 2020 were usually similar to the actual values for normal health, with a slight predisposition to be younger than the actual mean age. Conversely, the assumptions regarding ill health and spouse categories were generally consistently higher than the actual values, often markedly so for the former. 2018 exhibits a significantly high mortality rate across most groups, unusually displaying a notable downward fluctuation in average exit age, which is incongrous to the higher values witnessed in 2017 and 2019.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
Normal health males	20,206,012,254	461,626,826	475,516,618	455,415,784	453,384,307
Normal health females	14,679,818,643	211,641,570	213,967,473	205,989,728	205,354,699
Ill health males	3,807,814,871	147,091,489	145,984,942	136,698,530	127,942,382
Ill health females	3,054,413,132	93,699,818	89,605,762	79,371,938	76,180,941
Spouse males	386,664,214	11,188,474	11,142,259	10,518,490	9,686,881
Spouse females	2,568,006,884	139,475,485	141,625,038	135,471,489	125,295,485

Figure 5.16: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Table 5.16 displays the initial EtR, the actual exit amounts observed over the inter-valuation period and the expected exit amounts based on the assumptions from the 2013, 2016 and 2020 valuations. The expected exits based on the 2013 and 2016 assumptions are lower than the actual exits for all categories across the whole period; nevertheless, the exits based on the 2020 assumptions had a propensity to be much closer to, even exceeding, the actual exits for female spouses and both normal health categories; the results are illustrated in Table 5.17, where the ratio of actual to expected exits is less than one. The disparity between actual and expected exits for the ill health categories is reduced with the adoption of more recent valuation assumptions, given that the 2013 expected exits portrayed significant underestimation. However, it is unsurprising that 2013 assumptions show greater divergence since these were based on even older data; the 2016 and 2020 valuations benefit from more current data, overlapping with the actual exit data.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.01	1.02
Normal health females	0.99	1.03	1.03
Ill health males	1.01	1.08	1.15
Ill health females	1.05	1.18	1.23
Spouse males	1.00	1.06	1.16
Spouse females	0.98	1.03	1.11

Figure 5.17: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	Year	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
Normal health males	2013	2,863,256,463	60,156,878	67,035,788	63,720,505	65,715,932
Normal health males	2014	2,951,373,636	66,990,292	69,650,593	66,333,889	68,657,635
Normal health males	2015	3,011,746,941	64,487,375	70,675,226	68,368,193	69,985,698
Normal health males	2016	2,764,492,973	66,015,153	63,594,683	62,140,224	60,950,048
Normal health males	2017	2,823,842,762	68,110,071	65,530,085	63,636,678	61,901,957
Normal health males	2018	2,870,812,872	65,710,707	67,315,469	64,996,274	62,727,323
Normal health males	2019	2,920,486,606	70,156,348	71,714,774	66,220,021	63,445,713
Normal health females	2013	1,858,675,890	25,814,524	27,388,397	26,210,797	27,013,778
Normal health females	2014	1,977,965,613	28,694,138	29,353,538	28,154,388	29,129,838
Normal health females	2015	2,096,792,719	28,212,963	30,825,372	29,950,006	30,795,395
Normal health females	2016	1,998,629,079	30,128,051	28,540,607	27,981,631	27,534,387
Normal health females	2017	2,124,273,832	32,857,702	30,487,709	29,672,798	28,973,681
Normal health females	2018	2,248,124,392	31,734,810	32,276,384	31,221,036	30,286,084
Normal health females	2019	2,375,357,117	34,199,382	35,095,466	32,799,072	31,621,537
Ill health males	2013	604,603,684	19,803,276	21,555,388	20,002,836	19,386,032
Ill health males	2014	592,823,300	20,885,047	21,728,410	20,175,949	19,635,464
Ill health males	2015	576,845,454	20,351,910	21,498,468	20,323,746	19,569,739
Ill health males	2016	533,583,862	21,794,575	20,146,236	19,266,974	17,765,116
Ill health males	2017	517,533,286	22,038,598	20,245,864	19,243,335	17,581,235
Ill health males	2018	499,509,130	21,275,445	20,049,508	18,952,291	17,172,395
Ill health males	2019	482,916,155	20,942,638	20,761,068	18,733,400	16,832,402
Ill health females	2013	465,001,162	11,778,968	12,233,266	10,809,367	10,677,667
Ill health females	2014	463,151,341	12,346,080	12,714,745	11,241,866	11,185,277
Ill health females	2015	459,696,973	12,564,365	13,053,231	11,683,037	11,571,411
Ill health females	2016	419,762,345	13,892,182	12,187,653	10,999,652	10,430,923
Ill health females	2017	418,358,388	14,577,684	12,619,254	11,298,928	10,632,574
Ill health females	2018	415,648,567	14,127,456	12,997,617	11,546,795	10,776,902
Ill health females	2019	412,794,355	14,413,083	13,799,995	11,792,294	10,906,186
Spouse males	2013	45,333,919	1,032,894	1,184,604	1,077,496	1,035,884
Spouse males	2014	49,748,966	1,564,100	1,353,608	1,248,693	1,204,983
Spouse males	2015	53,432,820	1,278,867	1,508,306	1,431,565	1,364,799
Spouse males	2016	52,651,614	1,542,505	1,444,837	1,379,565	1,259,586
Spouse males	2017	57,312,944	1,797,194	1,648,077	1,585,130	1,433,780
Spouse males	2018	61,894,774	1,877,980	1,851,318	1,789,429	1,604,262
Spouse males	2019	66,289,177	2,094,934	2,151,509	2,006,611	1,783,586
Spouse females	2013	381,962,648	17,964,934	20,335,379	19,139,211	18,411,619
Spouse females	2014	389,817,028	22,407,052	21,243,456	20,187,775	19,446,459
Spouse females	2015	389,075,830	19,311,757	21,243,987	20,495,182	19,543,647
Spouse females	2016	346,048,812	19,751,019	19,214,345	18,719,732	17,034,959
Spouse females	2017	350,658,082	21,222,412	19,603,549	18,997,474	17,123,065
Spouse females	2018	353,980,137	18,785,934	19,712,576	18,984,571	16,958,150
Spouse females	2019	356,464,348	20,032,377	20,271,746	18,947,543	16,777,587

Figure 5.18: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category and year, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Tables 5.18 and 5.19 provide the equivalent divisions for each year. For males and females of normal health, the EtR and actual exit amounts generally increase between 2013 and 2019, culminating in 2020 with consistently higher assumptions than 2016 and 2013; this indicates incremental adjustments of expected exits over time. Males and females with health issues display a different pattern over the years, with decreasing EtR values. Males show a slight downward trend to their actual exit fluctuations, whilst females present the opposite, peaking in 2017. The 2020 assumptions for ill health categories are greater than in earlier valuations, though the differences are less pronounced. For male and female spouses, EtR and actual exit values typically rise, witnessing significant surges in 2019. The 2020 assumptions for spouses surpass those from 2016 and 2013 across all categories; this reflects incremental adjustments of expected exits over time.

For males and females of normal health, the ratio of actual to expected exit amounts from the 2020 assumptions mainly starts lower in the earlier years (2013-2015), increasing in the later years (2016-2019); the ratios for the 2016 and 2013 assumptions follow a similar pattern at slightly higher values. Ill health males and females exhibit a trend where the ratio, from the 2020 assumptions, is consistently lower in the earlier years, increasing significantly in the later years; this is also true for ratios from the 2013 and 2016 assumptions, though with a more pronounced upward pattern. For male and female spouses, the 2020 assumptions ratio starts lower in the earlier years, increasing over time; correspondingly, the ratios from 2013 and 2016 assumptions depict this pattern with greater values. Although the overall ratio of actual to expected exits sensibly fits some categories, the expected exit assumptions misalign with actual exit data year by year; during the early years, this ratio typically presents as relatively low, gradually increasing over time.

Figure 5.20 illustrates the actual exit amounts by age, from 1st April 2013 to 31st March 2020 for each category, alongside expected exit amounts based on assumptions, as recommended by GAD during the 2013, 2016 and 2020 valuations. These visual representations evidently illustrate that the 2013 valuation assumptions provide the weakest fit

Category	Year	2020 assumption	2016 assumption	2013 assumption
Normal health males	2013	0.90	0.94	0.92
Normal health males	2014	0.96	1.01	0.98
Normal health males	2015	0.91	0.94	0.92
Normal health males	2016	1.04	1.06	1.08
Normal health males	2017	1.04	1.07	1.10
Normal health males	2018	0.98	1.01	1.05
Normal health males	2019	0.98	1.06	1.11
Normal health females	2013	0.94	0.98	0.96
Normal health females	2014	0.98	1.02	0.99
Normal health females	2015	0.92	0.94	0.92
Normal health females	2016	1.06	1.08	1.09
Normal health females	2017	1.08	1.11	1.13
Normal health females	2018	0.98	1.02	1.05
Normal health females	2019	0.97	1.04	1.08
Ill health males	2013	0.92	0.99	1.02
Ill health males	2014	0.96	1.04	1.06
Ill health males	2015	0.95	1.00	1.04
Ill health males	2016	1.08	1.13	1.23
Ill health males	2017	1.09	1.15	1.25
Ill health males	2018	1.06	1.12	1.24
Ill health males	2019	1.01	1.12	1.24
Ill health females	2013	0.96	1.09	1.10
Ill health females	2014	0.97	1.10	1.10
Ill health females	2015	0.96	1.08	1.09
Ill health females	2016	1.14	1.26	1.33
Ill health females	2017	1.16	1.29	1.37
Ill health females	2018	1.09	1.22	1.31
Ill health females	2019	1.04	1.22	1.32
Spouse males	2013	0.87	0.96	1.00
Spouse males	2014	1.16	1.25	1.30
Spouse males	2015	0.85	0.89	0.94
Spouse males	2016	1.07	1.12	1.22
Spouse males	2017	1.09	1.13	1.25
Spouse males	2018	1.01	1.05	1.17
Spouse males	2019	0.97	1.04	1.17
Spouse females	2013	0.88	0.94	0.98
Spouse females	2014	1.05	1.11	1.15
Spouse females	2015	0.91	0.94	0.99
Spouse females	2016	1.03	1.06	1.16
Spouse females	2017	1.08	1.12	1.24
Spouse females	2018	0.95	0.99	1.11
Spouse females	2019	0.99	1.06	1.19

Figure 5.19: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category and year, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

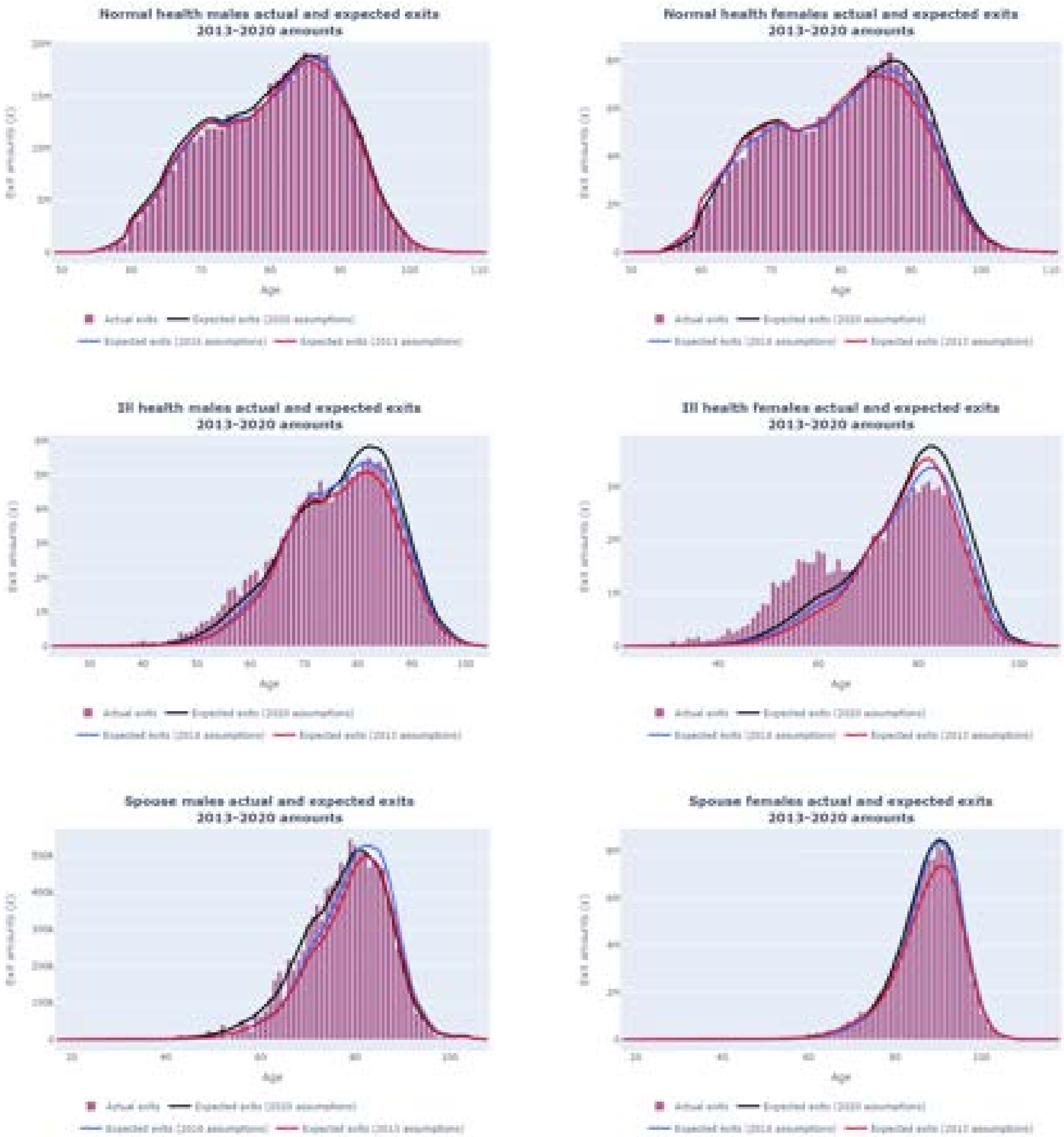


Figure 5.20: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

for the data during this period; although there is a reasonable shape to expected exits for normal health and spouse categories; for those of poor health, there is significant underestimation of exits at younger ages and overestimation at older ages. Despite offering the suitable match to actual exit behaviour at younger ages, the 2020 assumption exhibits the most prominent degree of overestimation at older ages. This assumption seems to be the best fit for data concerning females of normal health, paralleling the peak in actual exits at higher ages; this is also applicable to male spouses, where the 2020 assumption best replicates the gradient of exits increasing by age, without compromising the fit at higher ages.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.96	1.02	1.01
Normal health females	0.95	0.95	0.96
Ill health males	1.18	1.15	1.20
Ill health females	1.39	1.25	1.30
Spouse males	0.96	1.40	1.39
Spouse females	0.96	1.09	1.12

Figure 5.21: Proposed optimal scaling factors for the unscaled mortality base and improvement tables, used in the 2013, 2016 and 2020 valuation assumptions for each category, to get the total expected exit amounts to equal the actual exit amounts.

The valuation assumptions for 2013, 2016 and 2020 do not apply the optimal scaling factors to fit the 2013-2020 experience data; this is partly since the analyses used to determine each assumption were based on data from various, shorter time periods. There are also numerous reasons why valuation assumptions with contingencies preventing an optimal fit might be adopted, as discussed in Chapter 3. Table 5.21 illustrates what the optimal scaling factors would be to ensure total expected exits match the total actual exits for each category. Table 5.22 presents the optimal scaling factors for each category to minimise the least square residuals of actual and expected exit amounts across each age, offering a more robust fit at each data point. A significant discrepancy between optimal scaling factors, produced by both metrics, would suggest the shape of the underlying expected mortality rates inaccurately represents the actual data. The ill health categories exhibit a substantial differential, particularly for females. Although other categories show reasonably consistent results, the similar values depicted do not guarantee a close fit between actual and expected data.

The optimal scaling factors derived from the least squares method can be compared against valuation assumption scaling factors in Table 5.13. For males of normal health, the valuation assumptions and optimal scaling factors display minimal variation; this is most prominent in 2020, where the optimal scaling factor is lower than the valuation assumption. Normal health females exhibit stable optimal scaling factors, though the valuation assumptions vary more, presenting as lower in 2013 and 2016. The ill health valuation assumptions depict a significantly increased scaling factor in 2020; this is less pronounced in the optimal scaling factors. For male and female spouses, the optimal and valuation scaling factors align well in 2016 and 2020, though the latter is substantially lower in 2013.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.03	1.02
Normal health females	0.95	0.95	0.97
Ill health males	1.13	1.09	1.12
Ill health females	1.19	1.06	1.05
Spouse males	0.98	1.35	1.34
Spouse females	0.94	1.06	1.12

Figure 5.22: Proposed optimal scaling factors for the unscaled mortality base and improvement tables, used in the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

Table 5.23 offers a more detailed breakdown, presenting optimal scaling factors to minimise the least square residuals of actual and expected exit amounts across each age for each year. The normal health categories contain the most abundant data, stabilising factors over time; in contrast, the ill health and spouse categories have less data, exhibiting more substantial discrepancies between actual and expected exits from year to year.

Category	Year	2020 assumption	2016 assumption	2013 assumption
Normal health males	2013	0.90	0.96	0.91
Normal health males	2014	0.97	1.03	0.97
Normal health males	2015	0.92	0.96	0.92
Normal health males	2016	1.04	1.08	1.08
Normal health males	2017	1.04	1.09	1.10
Normal health males	2018	0.97	1.02	1.04
Normal health males	2019	0.97	1.07	1.10
Normal health females	2013	0.89	0.90	0.88
Normal health females	2014	0.95	0.95	0.93
Normal health females	2015	0.88	0.88	0.86
Normal health females	2016	1.03	1.01	1.04
Normal health females	2017	1.04	1.02	1.06
Normal health females	2018	0.94	0.94	0.98
Normal health females	2019	0.94	0.97	1.02
Ill health males	2013	1.05	1.02	1.01
Ill health males	2014	1.08	1.05	1.03
Ill health males	2015	1.06	1.01	1.01
Ill health males	2016	1.20	1.12	1.18
Ill health males	2017	1.23	1.17	1.23
Ill health males	2018	1.18	1.12	1.20
Ill health males	2019	1.12	1.12	1.21
Ill health females	2013	1.13	1.00	0.96
Ill health females	2014	1.12	1.00	0.95
Ill health females	2015	1.09	0.96	0.92
Ill health females	2016	1.29	1.12	1.12
Ill health females	2017	1.33	1.17	1.19
Ill health females	2018	1.22	1.08	1.11
Ill health females	2019	1.16	1.07	1.12
Spouse males	2013	0.87	1.26	1.19
Spouse males	2014	1.06	1.47	1.38
Spouse males	2015	0.83	1.16	1.11
Spouse males	2016	1.04	1.42	1.42
Spouse males	2017	1.09	1.49	1.50
Spouse males	2018	0.98	1.32	1.35
Spouse males	2019	0.94	1.32	1.35
Spouse females	2013	0.85	0.98	0.99
Spouse females	2014	0.98	1.12	1.14
Spouse females	2015	0.86	0.97	1.00
Spouse females	2016	0.98	1.09	1.17
Spouse females	2017	1.03	1.16	1.25
Spouse females	2018	0.89	1.00	1.10
Spouse females	2019	0.95	1.10	1.22

Figure 5.23: Proposed optimal scaling factors for the unscaled mortality base and improvement tables, used in the 2013, 2016 and 2020 valuation assumptions for each category and year, to minimise the least square residuals of actual and expected exit amounts across each age.

5.2 IMD analysis

This section further partitions the categories analysed in Section 5.1, in accordance to their IMD values; the CMI mapping tool aligns UK postcodes with IMD deciles [70]. Figure 5.24 illustrates the IMD decile locations, adjusted to cover the UK, from 2014 for Wales and 2015 for England. Decile 1, representative of the most deprived areas in the UK, progresses to decile 10, representative of the least deprived. Additional categories were created for incomplete records lacking a corresponding postcode or IMD value in the CMI mapping tool.

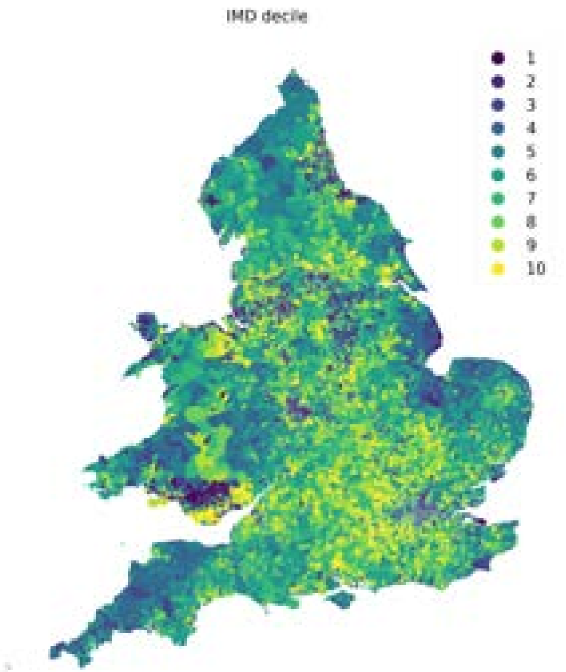


Figure 5.24: Locations of IMD deciles across England and Wales.

Table 5.25 presents the exits and exposure on a lives basis for each category. Males and females of normal health exhibit higher numbers of deaths and exposure, corresponding to higher IMD scores; conversely, men and women of ill health show a decreasing trend in deaths and exposure as IMD scores increase. The data for normal and ill health categories show slightly more female deaths than males at higher IMD values, though the reverse is true at lower IMD scores. However, female exposure is substantially higher than their male counterparts at each respective IMD. The spouse categories present a less distinct pattern, apart from a slight trend towards increased death and exposure data at higher IMD values. Spousal data for men is substantially smaller than for females. Categories without an assigned IMD tend to have the least data for each gender and pension type, which is somewhat reassuring; however, group sizes are higher than desired since they are usually similar in magnitude to their IMD counterparts.

Table 5.26 displays the equivalent exposure and actual exits in 2020 pension terms, alongside supplementary expected exits from 2013, 2016 and 2020 valuation assumptions for each category. A notable difference of this data is its significant weight towards male members in normal health, ill health males and female spouse categories. The general patterns across IMD scores largely remain unchanged in comparison to the lives table, with the exception for ill health female categories where the reverse occurs, portraying increasing deaths and exposure pension amounts in correlation to increases in IMD value.

Expected exits tend to underestimate actual exits at lower IMD values and overestimate them at higher IMD values. The expected exits arising from the valuation assumptions gradually shift upwards, suggesting that the assumptions' prudence level has decreased with time; this is likely influenced by the adopted scaling factor applied to the mortality table.

Table 5.27 shows the average exit ages weighted by pension amounts across categories and their comparisons to assumptions set at 2013, 2016 and 2020 pension scheme valuations.

There is a general trend where the actual average exit age increases alongside IMD

Category	d_x	E_x
N-health male no IMD	5,056	146,348
N-health male IMD 01	5,580	153,458
N-health male IMD 02	5,769	172,020
N-health male IMD 03	5,855	187,208
N-health male IMD 04	6,211	213,473
N-health male IMD 05	6,369	233,407
N-health male IMD 06	6,653	250,267
N-health male IMD 07	6,873	265,859
N-health male IMD 08	6,914	280,081
N-health male IMD 09	6,628	278,559
N-health male IMD 10	6,751	287,825
N-health female no IMD	4,234	209,322
N-health female IMD 01	4,645	205,933
N-health female IMD 02	5,159	252,387
N-health female IMD 03	5,221	289,131
N-health female IMD 04	5,928	339,880
N-health female IMD 05	6,012	379,699
N-health female IMD 06	6,274	407,136
N-health female IMD 07	6,708	447,567
N-health female IMD 08	6,776	469,338
N-health female IMD 09	6,725	473,094
N-health female IMD 10	6,825	491,522
I-health male no IMD	1,441	32,950
I-health male IMD 01	3,378	71,055
I-health male IMD 02	2,789	61,969
I-health male IMD 03	2,583	56,739
I-health male IMD 04	2,309	53,926
I-health male IMD 05	2,176	51,185
I-health male IMD 06	2,010	47,100
I-health male IMD 07	1,913	45,657
I-health male IMD 08	1,782	43,496
I-health male IMD 09	1,594	38,221
I-health male IMD 10	1,300	31,468
I-health female no IMD	1,272	40,058
I-health female IMD 01	2,423	69,523
I-health female IMD 02	2,315	66,668
I-health female IMD 03	2,191	63,759
I-health female IMD 04	2,028	64,226
I-health female IMD 05	1,970	64,170
I-health female IMD 06	1,916	61,803
I-health female IMD 07	1,953	62,429
I-health female IMD 08	1,930	62,126
I-health female IMD 09	1,791	57,267
I-health female IMD 10	1,612	50,696
Spouse male no IMD	696	17,276
Spouse male IMD 01	806	16,476
Spouse male IMD 02	781	17,400
Spouse male IMD 03	796	18,188
Spouse male IMD 04	834	19,838
Spouse male IMD 05	854	21,492
Spouse male IMD 06	864	21,163
Spouse male IMD 07	893	22,667
Spouse male IMD 08	842	23,996
Spouse male IMD 09	884	24,030
Spouse male IMD 10	807	23,106
Spouse female no IMD	3,334	56,603
Spouse female IMD 01	3,591	61,767
Spouse female IMD 02	3,772	60,593
Spouse female IMD 03	3,447	60,783
Spouse female IMD 04	3,635	62,670
Spouse female IMD 05	3,738	65,316
Spouse female IMD 06	3,787	66,845
Spouse female IMD 07	3,862	68,320
Spouse female IMD 08	3,953	69,768
Spouse female IMD 09	3,908	67,653
Spouse female IMD 10	3,764	65,675

Figure 5.25: Actual number of exits (d_x) and exact initial EtR, from 1st April 2013 to 31st March 2020 for each category.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
N-health male no IMD	1,058,566,134	33,760,502	32,706,022	31,435,551	31,841,258
N-health male IMD 01	791,404,617	24,415,996	17,881,011	17,115,216	17,025,019
N-health male IMD 02	980,253,761	28,110,395	22,445,802	21,490,300	21,384,544
N-health male IMD 03	1,201,989,843	31,124,891	27,501,562	26,326,827	26,182,892
N-health male IMD 04	1,499,127,890	35,200,804	34,169,287	32,695,449	32,513,230
N-health male IMD 05	1,781,332,125	40,036,760	39,905,796	38,174,117	37,987,053
N-health male IMD 06	2,049,541,512	44,651,260	47,556,774	45,526,056	45,242,476
N-health male IMD 07	2,311,771,755	49,152,279	52,873,057	50,619,443	50,321,434
N-health male IMD 08	2,603,756,175	53,843,495	60,251,873	57,675,647	57,327,573
N-health male IMD 09	2,757,568,919	55,878,862	63,595,960	60,892,692	60,536,323
N-health male IMD 10	3,170,699,522	65,451,582	76,629,474	73,464,487	73,022,505
N-health female no IMD	707,230,473	14,941,758	13,876,903	13,231,678	13,337,288
N-health female IMD 01	546,855,873	11,556,789	8,521,152	8,208,324	8,169,443
N-health female IMD 02	761,917,584	14,003,200	11,979,281	11,516,350	11,436,178
N-health female IMD 03	936,667,926	15,117,727	14,117,108	13,584,485	13,514,826
N-health female IMD 04	1,180,130,943	18,526,243	17,228,265	16,592,475	16,530,660
N-health female IMD 05	1,385,778,461	19,200,357	19,672,361	18,971,400	18,911,641
N-health female IMD 06	1,545,039,718	21,310,878	21,490,992	20,722,285	20,663,266
N-health female IMD 07	1,732,562,119	23,076,981	24,534,357	23,617,812	23,518,881
N-health female IMD 08	1,869,194,179	23,299,523	25,828,635	24,903,466	24,827,187
N-health female IMD 09	1,934,851,892	24,590,689	27,428,901	26,412,682	26,296,225
N-health female IMD 10	2,079,589,474	26,017,425	29,289,518	28,228,773	28,149,105
I-health male no IMD	229,148,415	9,114,681	10,273,319	9,593,229	9,083,398
I-health male IMD 01	402,421,665	17,753,787	14,062,330	13,148,894	12,336,728
I-health male IMD 02	365,298,348	15,338,057	13,048,523	12,183,702	11,409,783
I-health male IMD 03	357,472,806	14,997,069	13,220,968	12,360,295	11,567,293
I-health male IMD 04	361,248,994	13,841,399	13,445,404	12,598,390	11,788,039
I-health male IMD 05	365,845,502	13,640,692	13,905,209	13,037,647	12,205,062
I-health male IMD 06	349,957,864	13,764,024	13,304,010	12,488,294	11,691,236
I-health male IMD 07	364,293,909	12,804,865	14,278,703	13,375,443	12,491,243
I-health male IMD 08	360,507,601	12,962,099	14,342,855	13,425,284	12,521,743
I-health male IMD 09	338,673,799	11,985,594	13,639,999	12,788,167	11,919,719
I-health male IMD 10	312,945,969	10,889,222	12,463,622	11,699,185	10,928,139
I-health female no IMD	188,023,402	6,133,579	6,197,551	5,485,928	5,393,823
I-health female IMD 01	233,149,555	8,144,414	6,736,751	5,986,065	5,773,698
I-health female IMD 02	247,216,254	8,147,342	7,153,743	6,362,503	6,129,151
I-health female IMD 03	255,408,117	8,658,452	7,467,402	6,627,789	6,374,509
I-health female IMD 04	283,620,336	8,827,740	8,126,421	7,210,282	6,909,231
I-health female IMD 05	301,600,895	8,947,303	8,702,839	7,683,431	7,329,505
I-health female IMD 06	308,063,208	8,952,057	8,577,142	7,609,403	7,282,670
I-health female IMD 07	310,842,998	9,100,160	9,022,314	7,981,774	7,627,752
I-health female IMD 08	324,903,286	9,482,984	9,578,640	8,476,287	8,113,742
I-health female IMD 09	309,488,262	8,814,493	9,213,039	8,149,258	7,797,234
I-health female IMD 10	292,096,818	8,491,294	8,829,920	7,799,219	7,449,625
Spouse male no IMD	28,293,341	902,267	856,284	817,053	769,574
Spouse male IMD 01	22,757,818	909,522	676,535	642,494	590,744
Spouse male IMD 02	25,458,179	883,154	730,118	687,251	632,445
Spouse male IMD 03	28,693,965	876,444	840,485	799,388	735,639
Spouse male IMD 04	32,846,524	999,120	939,460	886,519	814,981
Spouse male IMD 05	36,451,965	1,079,657	1,043,146	984,295	905,853
Spouse male IMD 06	37,399,788	1,011,784	1,051,730	986,069	906,791
Spouse male IMD 07	40,641,179	1,137,968	1,127,875	1,054,340	969,614
Spouse male IMD 08	44,530,929	1,147,099	1,235,245	1,156,631	1,062,323
Spouse male IMD 09	44,414,479	1,134,614	1,304,239	1,234,587	1,133,566
Spouse male IMD 10	45,176,048	1,106,845	1,337,141	1,269,863	1,165,350
Spouse female no IMD	197,502,876	10,898,202	11,245,402	10,751,670	10,175,776
Spouse female IMD 01	161,430,257	9,124,683	7,381,323	7,022,935	6,530,194
Spouse female IMD 02	165,014,117	9,854,911	8,149,553	7,780,410	7,208,214
Spouse female IMD 03	181,517,846	9,612,026	9,093,053	8,675,730	8,034,600
Spouse female IMD 04	200,955,155	10,982,607	10,616,364	10,142,431	9,391,970
Spouse female IMD 05	224,666,938	11,890,616	12,256,924	11,728,192	10,828,764
Spouse female IMD 06	247,543,714	13,170,509	13,565,757	12,975,838	11,984,098
Spouse female IMD 07	262,576,307	14,317,146	14,723,104	14,088,361	13,011,088
Spouse female IMD 08	288,486,622	15,330,291	16,266,320	15,567,431	14,327,987
Spouse female IMD 09	304,988,173	16,313,398	17,713,196	16,965,767	15,626,597
Spouse female IMD 10	333,324,879	17,981,095	20,614,041	19,772,724	18,176,198

Figure 5.26: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
N-health male no IMD	81.07	83.00	83.13	83.03
N-health male IMD 01	79.03	80.13	80.32	80.18
N-health male IMD 02	79.78	80.28	80.47	80.33
N-health male IMD 03	80.17	80.43	80.62	80.48
N-health male IMD 04	80.38	80.54	80.73	80.58
N-health male IMD 05	80.67	80.25	80.44	80.29
N-health male IMD 06	81.11	80.59	80.77	80.63
N-health male IMD 07	81.59	80.39	80.58	80.43
N-health male IMD 08	81.51	80.65	80.83	80.68
N-health male IMD 09	81.66	80.51	80.70	80.55
N-health male IMD 10	82.36	81.01	81.19	81.04
N-health female no IMD	79.97	83.47	83.01	82.64
N-health female IMD 01	80.19	81.34	80.90	80.50
N-health female IMD 02	80.31	81.59	81.12	80.70
N-health female IMD 03	80.67	81.21	80.75	80.33
N-health female IMD 04	80.78	80.96	80.49	80.08
N-health female IMD 05	81.13	80.65	80.19	79.78
N-health female IMD 06	80.83	80.51	80.04	79.62
N-health female IMD 07	81.41	80.79	80.31	79.88
N-health female IMD 08	81.43	80.49	80.01	79.59
N-health female IMD 09	81.49	80.76	80.29	79.86
N-health female IMD 10	81.48	80.63	80.17	79.75
I-health male no IMD	77.05	79.11	79.11	78.83
I-health male IMD 01	74.53	75.85	76.12	75.89
I-health male IMD 02	74.71	76.13	76.40	76.17
I-health male IMD 03	75.33	76.68	76.87	76.62
I-health male IMD 04	75.65	76.83	76.98	76.71
I-health male IMD 05	76.08	77.10	77.22	76.95
I-health male IMD 06	75.32	77.10	77.21	76.94
I-health male IMD 07	75.91	77.54	77.63	77.33
I-health male IMD 08	76.20	77.73	77.82	77.52
I-health male IMD 09	76.15	78.03	78.06	77.74
I-health male IMD 10	76.13	77.92	77.93	77.60
I-health female no IMD	73.00	79.54	79.25	79.24
I-health female IMD 01	74.79	77.84	77.70	77.87
I-health female IMD 02	74.54	77.83	77.72	77.90
I-health female IMD 03	73.30	77.99	77.86	78.02
I-health female IMD 04	72.75	77.89	77.73	77.87
I-health female IMD 05	73.73	78.12	77.91	78.00
I-health female IMD 06	72.38	77.52	77.36	77.53
I-health female IMD 07	73.12	78.21	77.99	78.07
I-health female IMD 08	73.88	78.33	78.11	78.19
I-health female IMD 09	73.44	78.45	78.23	78.28
I-health female IMD 10	73.27	78.68	78.46	78.50
Spouse male no IMD	77.27	79.03	81.04	80.87
Spouse male IMD 01	77.31	78.43	80.12	79.95
Spouse male IMD 02	77.19	78.05	79.78	79.60
Spouse male IMD 03	78.50	78.50	80.27	80.09
Spouse male IMD 04	78.56	78.20	79.99	79.81
Spouse male IMD 05	76.58	78.18	79.98	79.80
Spouse male IMD 06	78.95	77.90	79.69	79.51
Spouse male IMD 07	77.55	77.72	79.51	79.31
Spouse male IMD 08	78.35	77.86	79.72	79.53
Spouse male IMD 09	79.21	78.30	80.04	79.87
Spouse male IMD 10	79.57	78.49	80.28	80.10
Spouse female no IMD	87.77	88.73	88.93	88.59
Spouse female IMD 01	85.49	87.07	87.24	86.77
Spouse female IMD 02	85.74	87.73	87.90	87.47
Spouse female IMD 03	86.46	87.77	87.94	87.52
Spouse female IMD 04	87.17	88.14	88.32	87.94
Spouse female IMD 05	87.52	88.28	88.46	88.10
Spouse female IMD 06	87.89	88.35	88.53	88.17
Spouse female IMD 07	88.05	88.51	88.69	88.34
Spouse female IMD 08	88.22	88.40	88.58	88.23
Spouse female IMD 09	88.63	88.63	88.81	88.49
Spouse female IMD 10	89.17	89.00	89.18	88.89

Figure 5.27: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

score, with the exception of ill health females; this illustrates a correlation between IMD and mortality rates. This observation helps justify the subdivision of data by IMD, due to a lack of homogeneous outcomes across different IMD subgroups. There is a subtle tendency in ill health categories for females where increased IMD values result in lower average exit ages; this lack of an established pattern when subdividing by IMD may be less effective for this category.

Females usually hold older actual and expected exit ages on average relative to males of the same health and IMD. Individuals of normal health consistently display higher actual average exit ages than those with impaired health; the actual average exit age is typically lower than the assumptions made in 2020, 2016 and 2013, which is consistent irrespective of gender and health status. Over time, the assumptions for exit ages have increased between 2013 to 2020, except for normal health males. Assuming that people will die later, therefore receiving their pensions for longer adds an element of prudence to the mortality curve with age.

Table 5.28 similarly produces a gradual decline regarding the ratio of actual to expected exit values between 2013 and 2020 assumptions across most categories, indicating consistent increments of expected exits over time. Utilising a higher quantity of deaths is less prudent, since the assumption results in fewer pension payments and a reduction to the scheme's remaining liability. Categories with lower IMD scores persistently have greater values relative to higher IMD categories, suggesting that individuals residing in the most deprived areas have higher death rates. Additionally, females of normal and ill health tend to have slightly higher values than males in corresponding IMD categories, indicating that female assumptions are more prudent since more actual exits occur relative to expected exits. Excluding ill health categories, the non-IMD ratio of actual to expected exits typically falls around the middle of the IMD range, suggesting a fairly even distribution of exits within the data across potential IMD values; the behaviour of such a category falls under the assumption that it realistically consists of random IMD values; this suggests there is no particular bias towards any category, with individuals

Category	2020 assumption	2016 assumption	2013 assumption
N-health male no IMD	1.03	1.07	1.06
N-health male IMD 01	1.37	1.43	1.43
N-health male IMD 02	1.25	1.31	1.31
N-health male IMD 03	1.13	1.18	1.19
N-health male IMD 04	1.03	1.08	1.08
N-health male IMD 05	1.00	1.05	1.05
N-health male IMD 06	0.94	0.98	0.99
N-health male IMD 07	0.93	0.97	0.98
N-health male IMD 08	0.89	0.93	0.94
N-health male IMD 09	0.88	0.92	0.92
N-health male IMD 10	0.85	0.89	0.90
N-health female no IMD	1.08	1.13	1.12
N-health female IMD 01	1.36	1.41	1.41
N-health female IMD 02	1.17	1.22	1.22
N-health female IMD 03	1.07	1.11	1.12
N-health female IMD 04	1.08	1.12	1.12
N-health female IMD 05	0.98	1.01	1.02
N-health female IMD 06	0.99	1.03	1.03
N-health female IMD 07	0.94	0.98	0.98
N-health female IMD 08	0.90	0.94	0.94
N-health female IMD 09	0.90	0.93	0.94
N-health female IMD 10	0.89	0.92	0.92
I-health male no IMD	0.89	0.95	1.00
I-health male IMD 01	1.26	1.35	1.44
I-health male IMD 02	1.18	1.26	1.34
I-health male IMD 03	1.13	1.21	1.30
I-health male IMD 04	1.03	1.10	1.17
I-health male IMD 05	0.98	1.05	1.12
I-health male IMD 06	1.03	1.10	1.18
I-health male IMD 07	0.90	0.96	1.03
I-health male IMD 08	0.90	0.97	1.04
I-health male IMD 09	0.88	0.94	1.01
I-health male IMD 10	0.87	0.93	1.00
I-health female no IMD	0.99	1.12	1.14
I-health female IMD 01	1.21	1.36	1.41
I-health female IMD 02	1.14	1.28	1.33
I-health female IMD 03	1.16	1.31	1.36
I-health female IMD 04	1.09	1.22	1.28
I-health female IMD 05	1.03	1.16	1.22
I-health female IMD 06	1.04	1.18	1.23
I-health female IMD 07	1.01	1.14	1.19
I-health female IMD 08	0.99	1.12	1.17
I-health female IMD 09	0.96	1.08	1.13
I-health female IMD 10	0.96	1.09	1.14
Spouse male no IMD	1.05	1.10	1.17
Spouse male IMD 01	1.34	1.42	1.54
Spouse male IMD 02	1.21	1.29	1.40
Spouse male IMD 03	1.04	1.10	1.19
Spouse male IMD 04	1.06	1.13	1.23
Spouse male IMD 05	1.04	1.10	1.19
Spouse male IMD 06	0.96	1.03	1.12
Spouse male IMD 07	1.01	1.08	1.17
Spouse male IMD 08	0.93	0.99	1.08
Spouse male IMD 09	0.87	0.92	1.00
Spouse male IMD 10	0.83	0.87	0.95
Spouse female no IMD	0.97	1.01	1.07
Spouse female IMD 01	1.24	1.30	1.40
Spouse female IMD 02	1.21	1.27	1.37
Spouse female IMD 03	1.06	1.11	1.20
Spouse female IMD 04	1.03	1.08	1.17
Spouse female IMD 05	0.97	1.01	1.10
Spouse female IMD 06	0.97	1.02	1.10
Spouse female IMD 07	0.97	1.02	1.10
Spouse female IMD 08	0.94	0.98	1.07
Spouse female IMD 09	0.92	0.96	1.04
Spouse female IMD 10	0.87	0.91	0.99

Figure 5.28: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

from polarising deprivation levels being equally difficult to contact, thus balancing each other out. The unknown IMD categories for ill health imitate the highest IMD scores, implying that postcode data was insufficient for individuals of ill health from the least deprived areas or a tendency to change residence to outside the UK.

Charts from 5.29 to 5.39 illustrate expected exits using valuation assumptions between 2013 and 2020, typically falling below actual exit values at lower IMD scores. In contrast, higher IMD categories produce expected lines that typically surpass actual exit bars by a substantial amount. There is a reasonable familiarity in the fit between actual and expected exits for most categories at mid-ranging IMD values, with the noticeable exception for females of ill health; in this case, the shape of the expected exit mortality tables underestimates exits at younger ages and overestimates them at older ages.

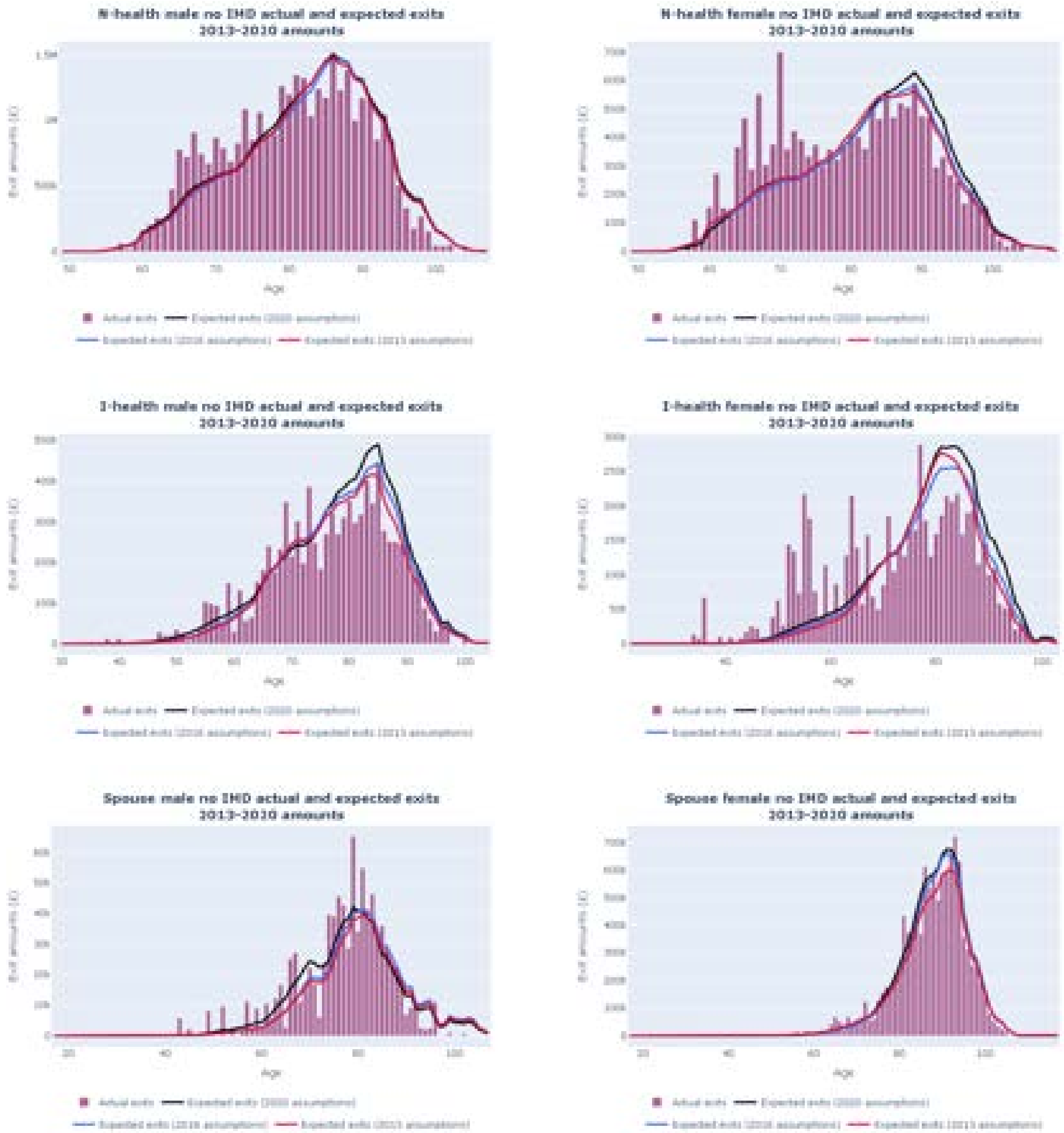


Figure 5.29: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

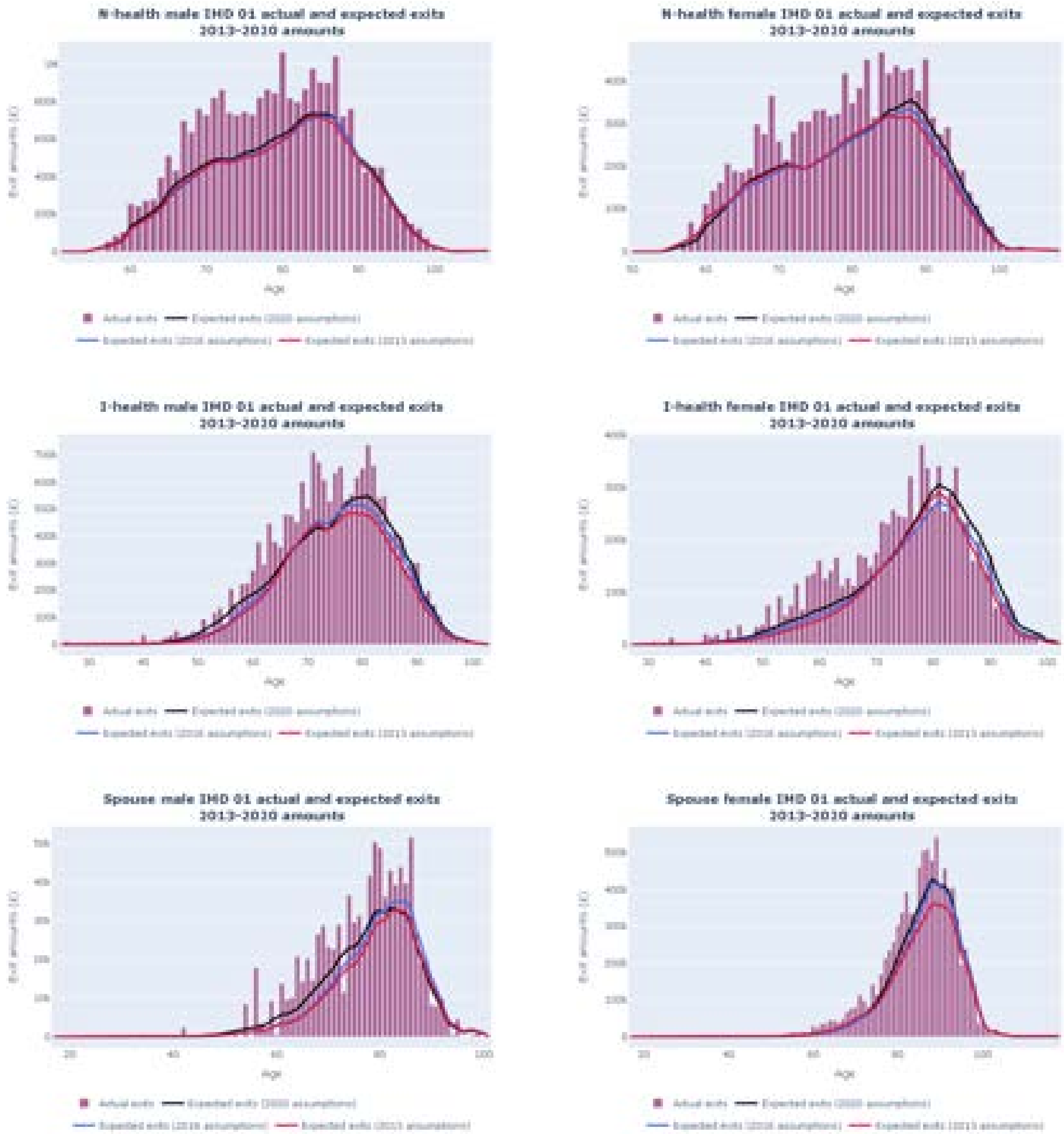


Figure 5.30: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

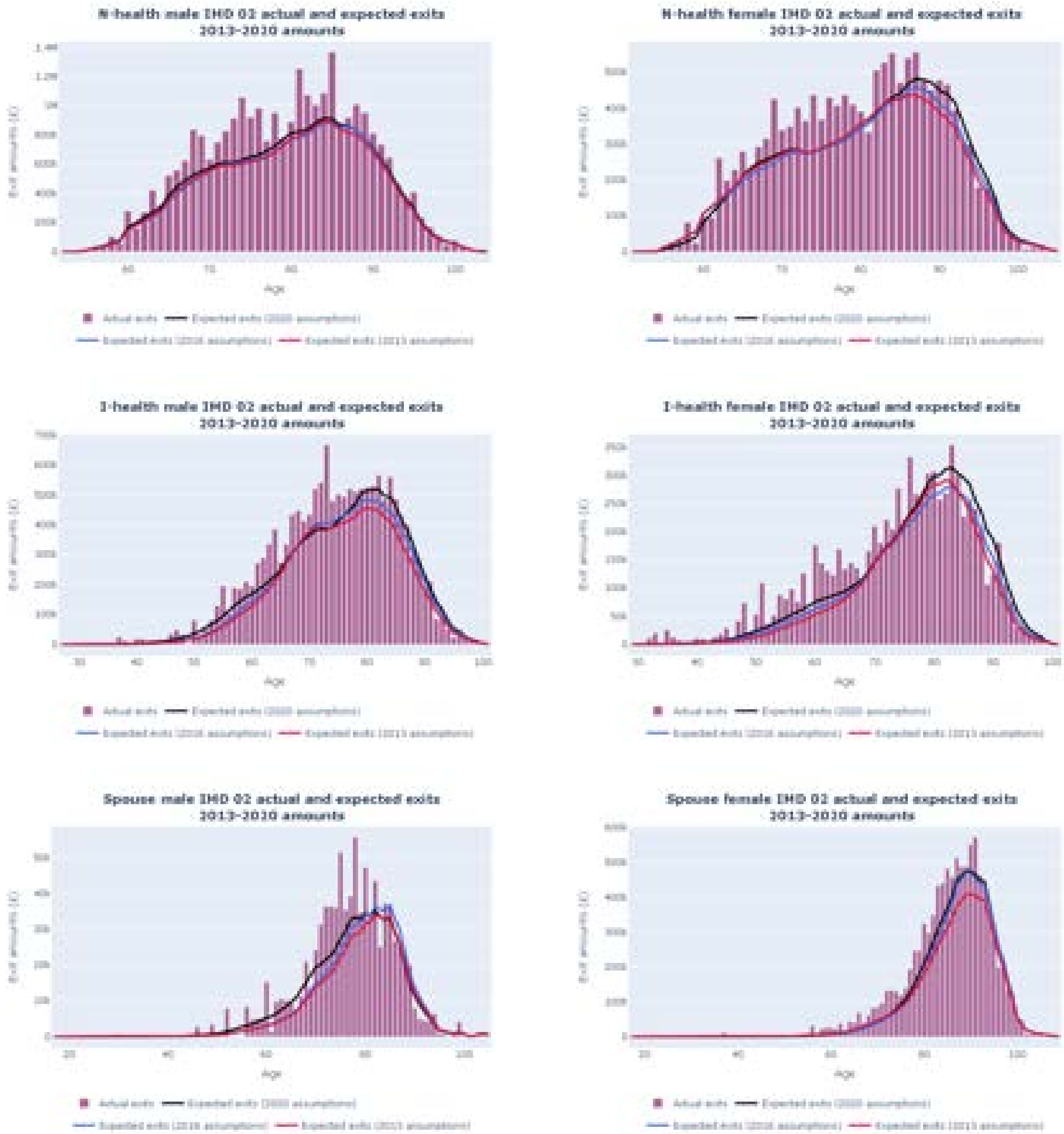


Figure 5.31: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

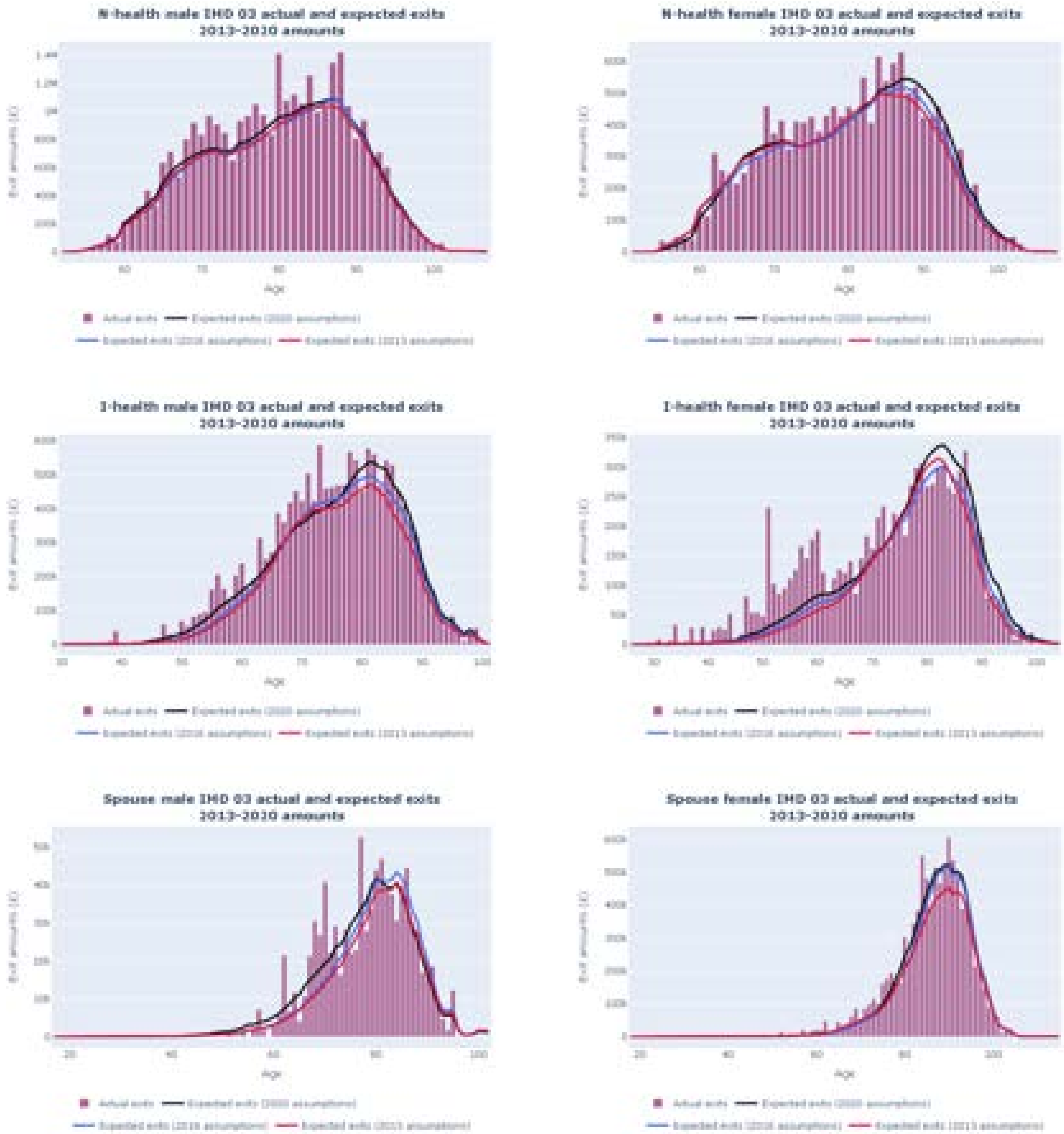


Figure 5.32: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

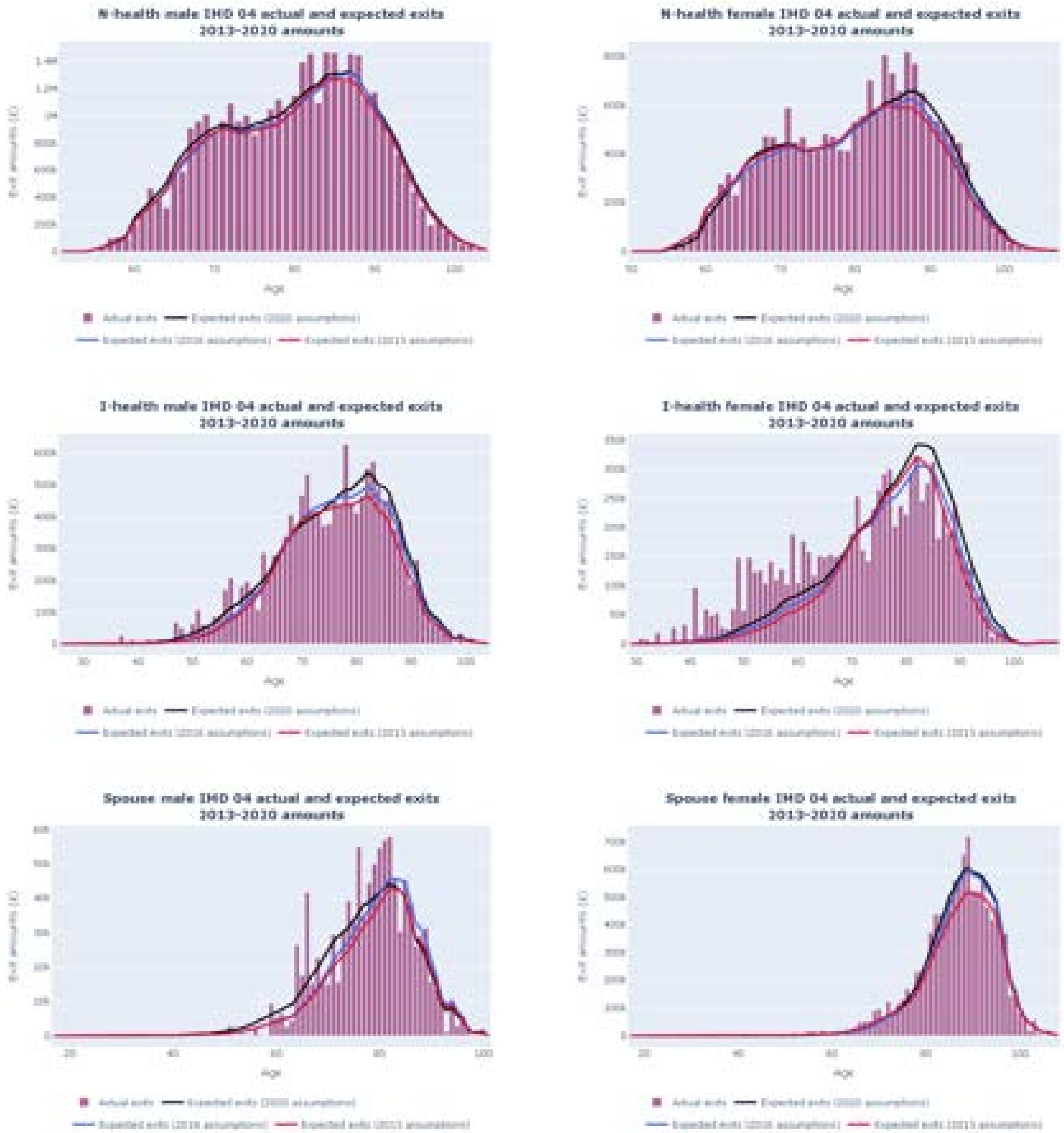


Figure 5.33: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

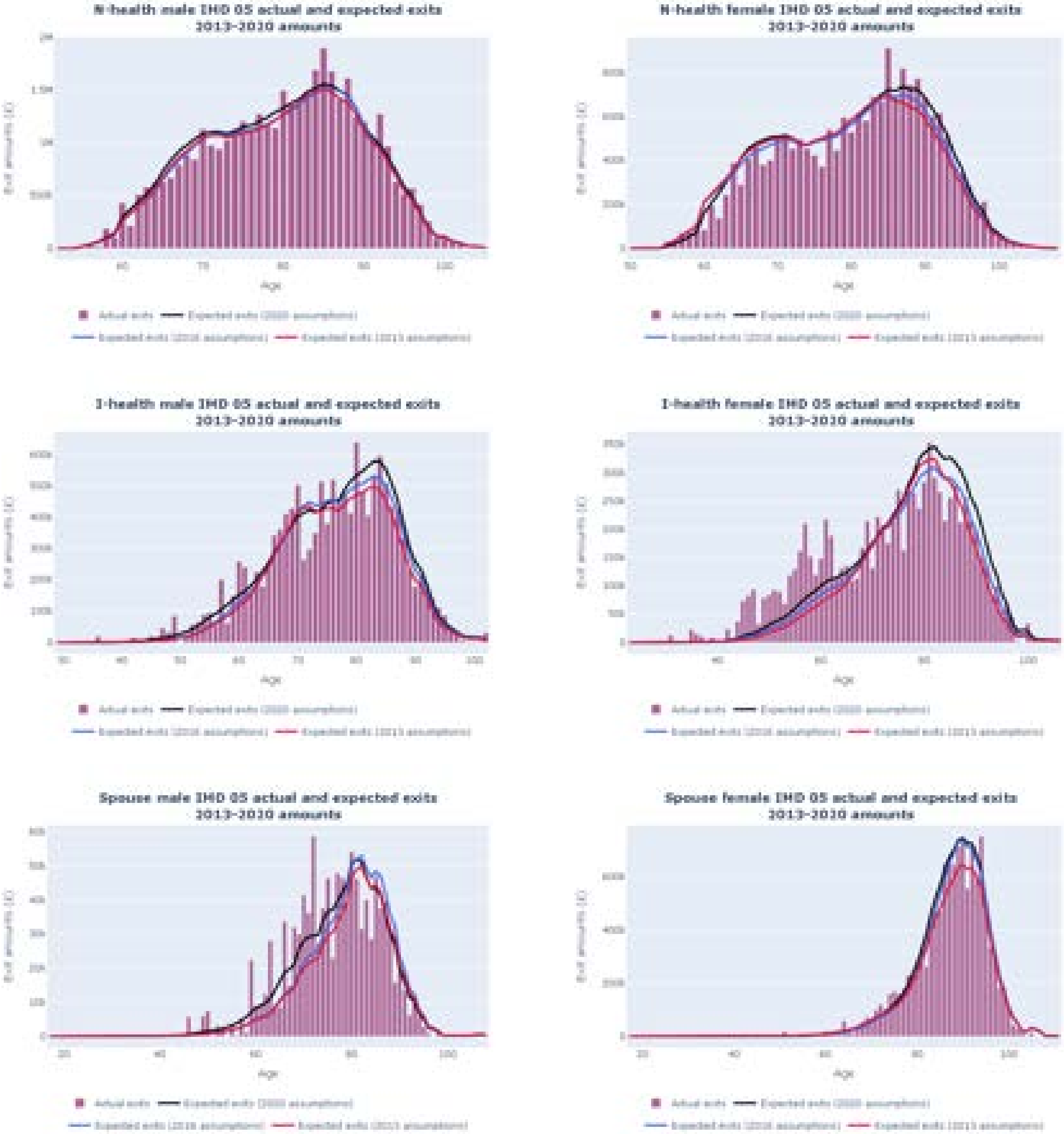


Figure 5.34: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

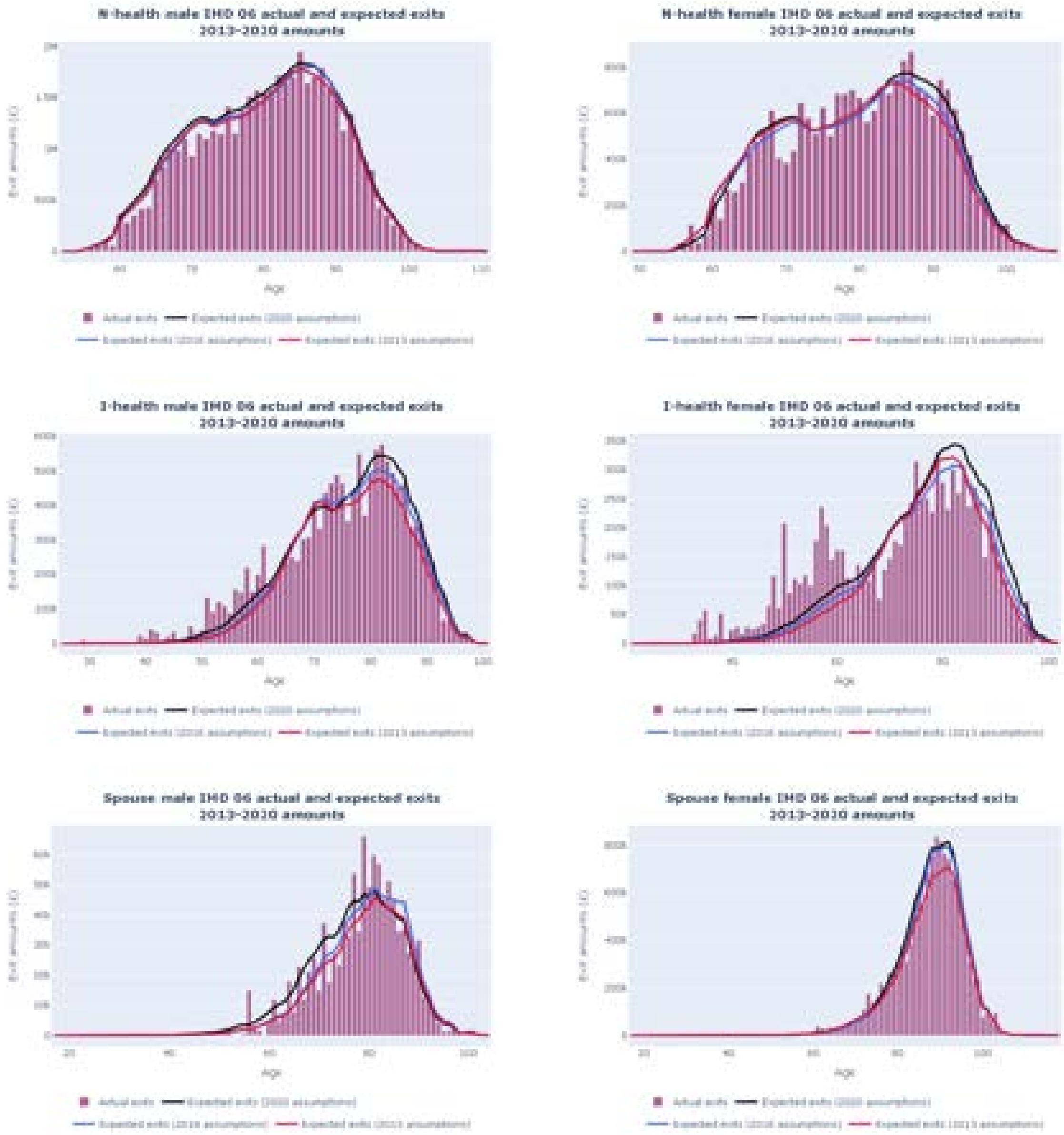


Figure 5.35: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

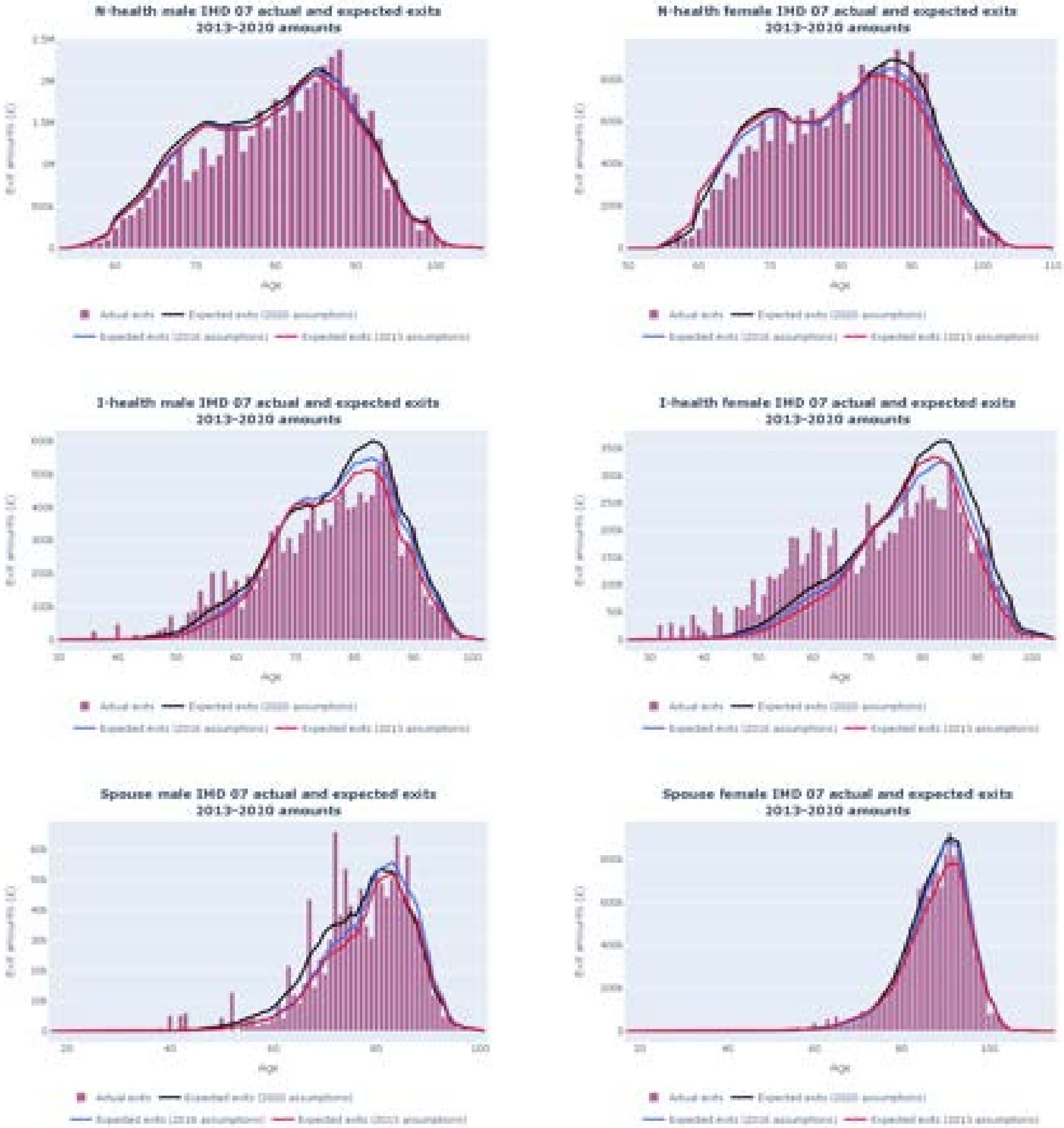


Figure 5.36: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

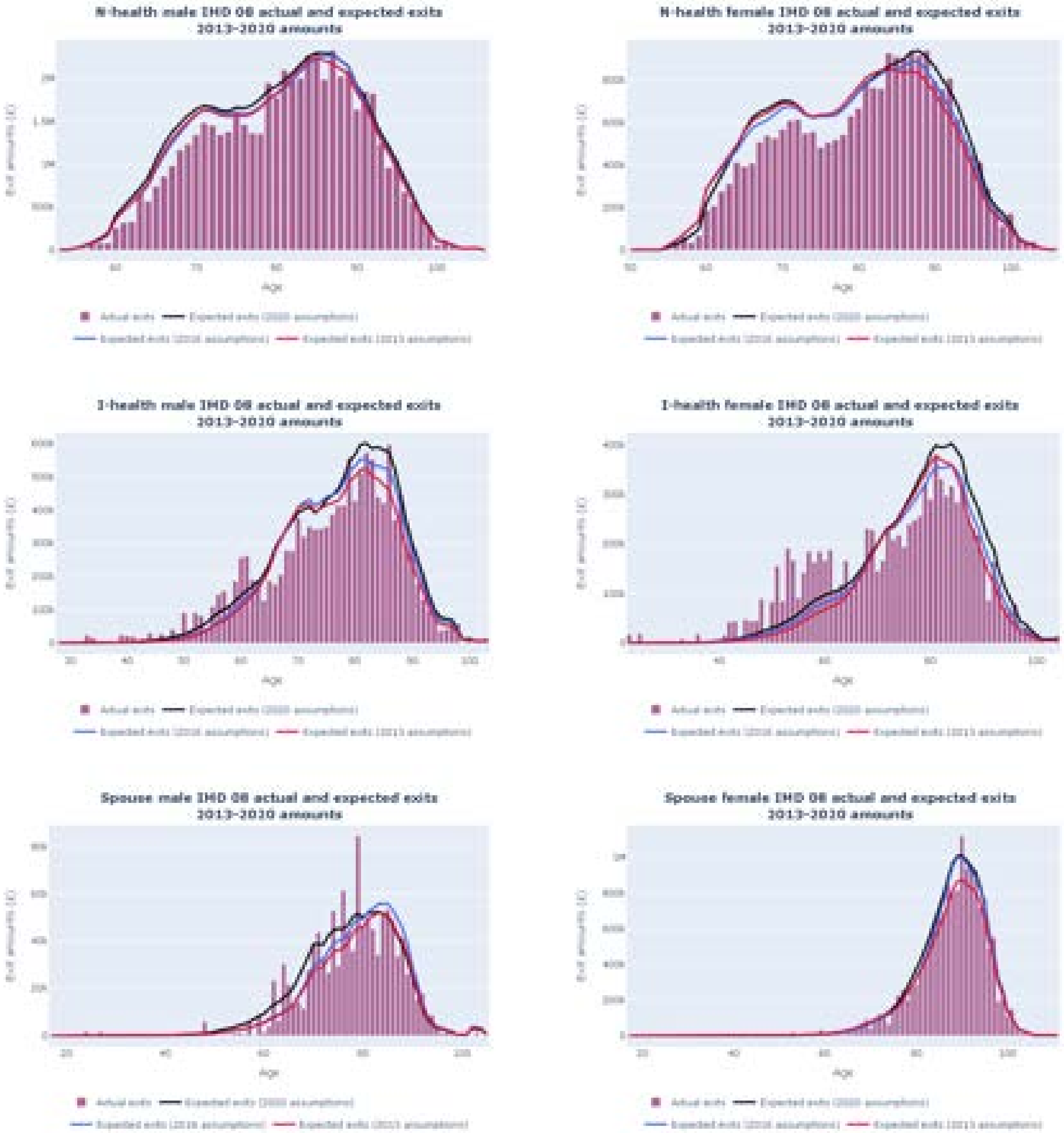


Figure 5.37: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

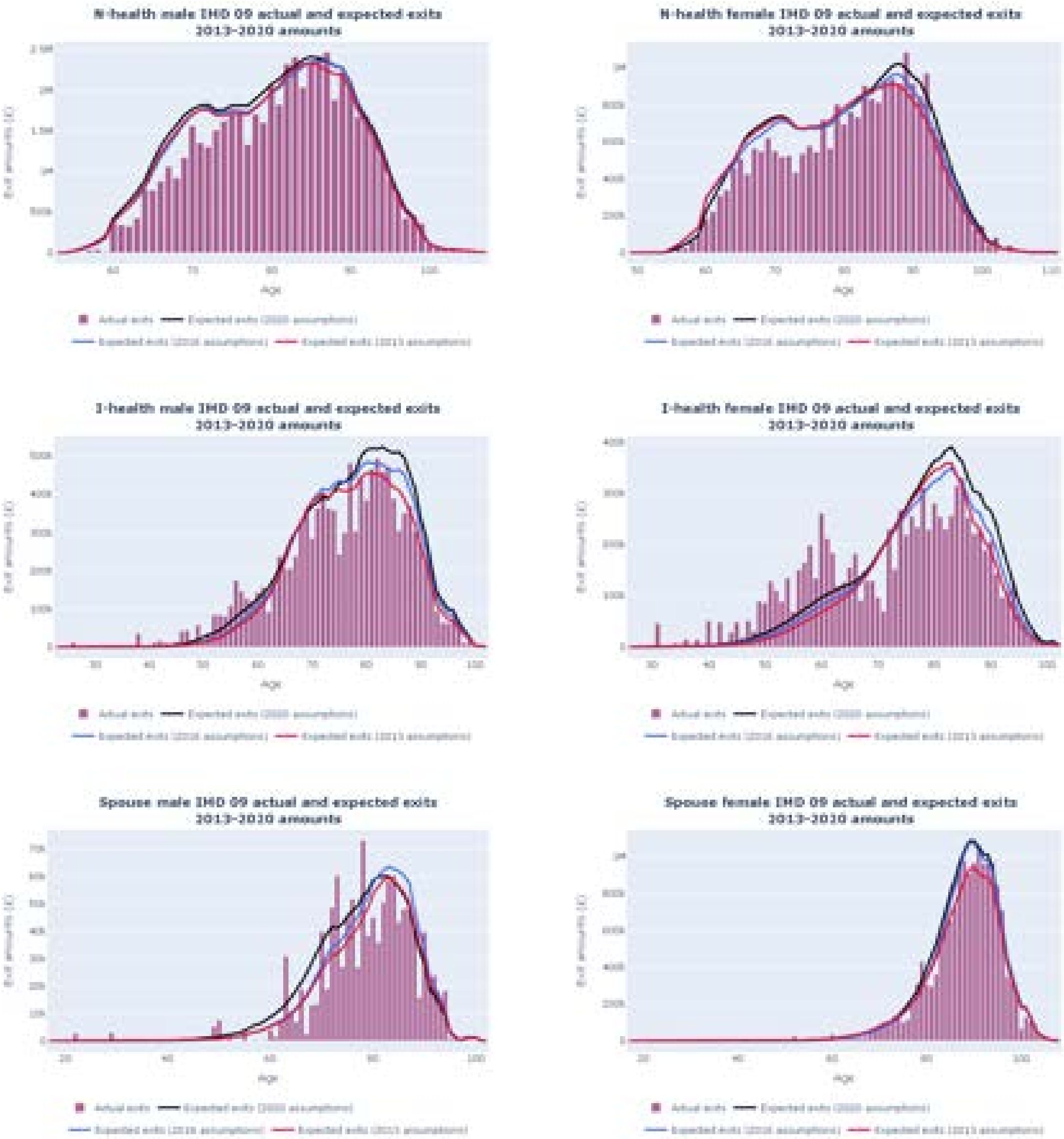


Figure 5.38: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

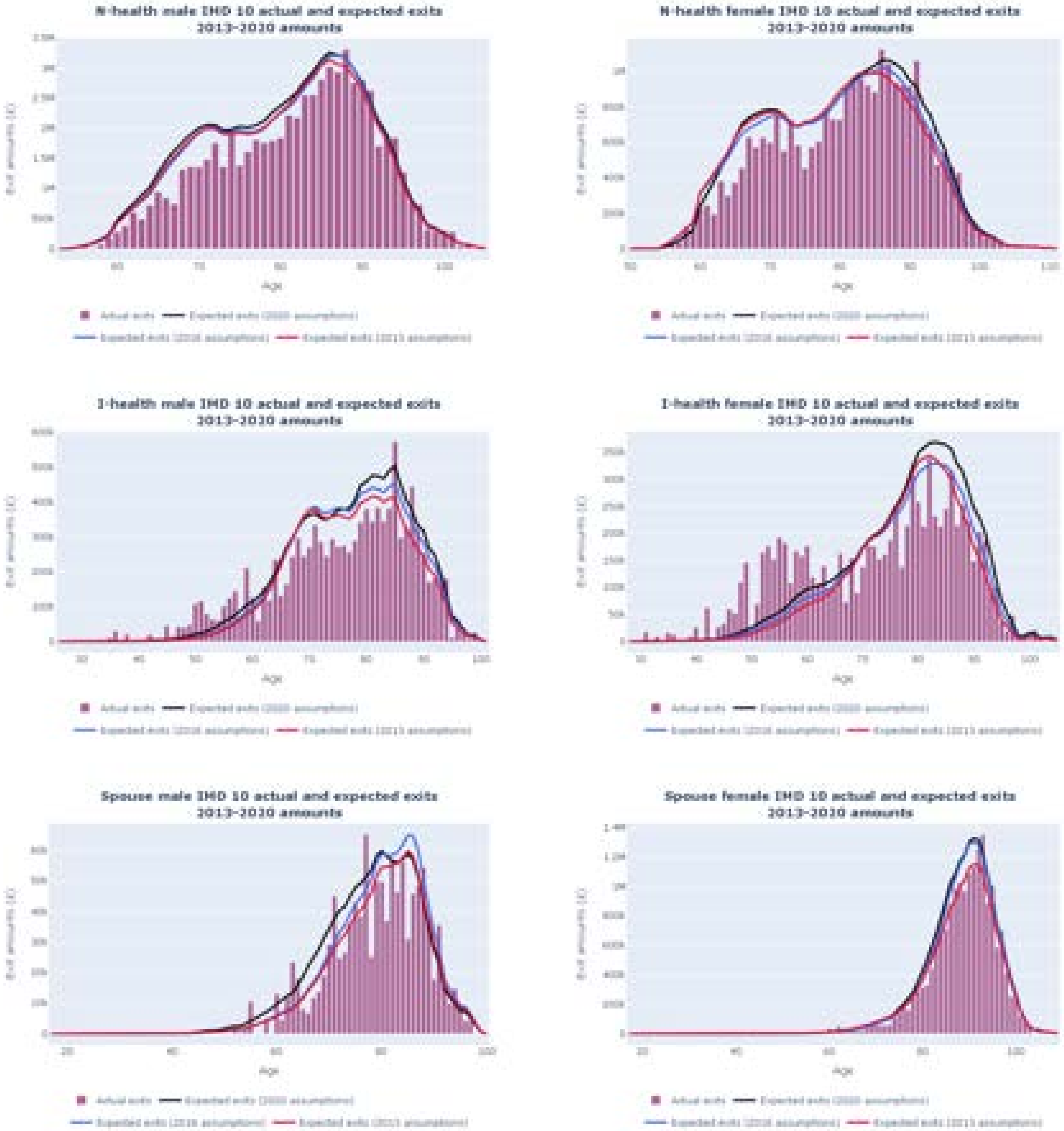


Figure 5.39: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

5.2.1 IMD subdivision compared to baseline

By utilising postcode data to refine groupings by IMD, optimal scaling factors for the SAPS mortality tables can be determined; these factors should offer a more tailored fit to the data within their respective categories. For each IMD, the model fits can be evaluated with and without IMD-specific scaling to assess whether it significantly enhances model performance, as well as the extent of its influence.

Table 5.40 illustrates the optimal scaling factors required to match total expected and actual exits for each category. Table 5.41 displays the optimal scaling factors for each category, minimising the least square residuals of actual and expected exit amounts across each age. Only the ill health categories exhibit a relatively large disparity between the two metrics, especially for females, whereas the rest show reasonably consistent results. The optimal scaling factors clearly establish a downward trend with increasing IMD scores for both measures; these observed factors without IMD subdivision, as in Section 5.1, resemble the mid-range IMD values.

An F-test compares the performance of models with and without IMD-specific scaling; the generic formula for this test, applicable when two models (one and two) to be compared have different numbers of parameters [73, 74], is:

$$\mathrm{F} = \frac{(RSS_1 - RSS_2)/(DoF_1 - DoF_2)}{RSS_2/DoF_2}. \tag{5.4}$$

The DoF is the number of parameters subtracted from the number of data points, which is four for males and five for females [7].

However, since the models' only differential is scaling factor, the F statistic is simplified as follows:

$$\mathrm{F} = \frac{RSS_1}{RSS_2}. \tag{5.5}$$

The F-statistic can then be converted to a probability and compared against a chosen critical value, such as $\alpha = 0.05$, determining whether the more complex model has statis-

Category	2020 assumption	2016 assumption	2013 assumption
N-health male no IMD	1.02	1.08	1.05
N-health male IMD 01	1.35	1.44	1.42
N-health male IMD 02	1.24	1.32	1.30
N-health male IMD 03	1.12	1.19	1.18
N-health male IMD 04	1.02	1.09	1.07
N-health male IMD 05	0.99	1.06	1.04
N-health male IMD 06	0.93	0.99	0.98
N-health male IMD 07	0.92	0.98	0.97
N-health male IMD 08	0.88	0.94	0.93
N-health male IMD 09	0.87	0.93	0.91
N-health male IMD 10	0.85	0.90	0.89
N-health female no IMD	1.03	1.04	1.04
N-health female IMD 01	1.30	1.30	1.32
N-health female IMD 02	1.12	1.12	1.14
N-health female IMD 03	1.03	1.02	1.04
N-health female IMD 04	1.03	1.03	1.04
N-health female IMD 05	0.94	0.93	0.94
N-health female IMD 06	0.95	0.95	0.96
N-health female IMD 07	0.90	0.90	0.91
N-health female IMD 08	0.87	0.86	0.87
N-health female IMD 09	0.86	0.86	0.87
N-health female IMD 10	0.85	0.85	0.86
I-health male no IMD	1.04	1.02	1.04
I-health male IMD 01	1.48	1.44	1.50
I-health male IMD 02	1.38	1.35	1.40
I-health male IMD 03	1.33	1.30	1.35
I-health male IMD 04	1.20	1.18	1.22
I-health male IMD 05	1.15	1.12	1.16
I-health male IMD 06	1.21	1.18	1.22
I-health male IMD 07	1.05	1.02	1.07
I-health male IMD 08	1.06	1.03	1.08
I-health male IMD 09	1.03	1.00	1.05
I-health male IMD 10	1.02	1.00	1.04
I-health female no IMD	1.32	1.19	1.21
I-health female IMD 01	1.61	1.44	1.50
I-health female IMD 02	1.51	1.36	1.41
I-health female IMD 03	1.54	1.38	1.44
I-health female IMD 04	1.44	1.30	1.35
I-health female IMD 05	1.37	1.23	1.29
I-health female IMD 06	1.39	1.25	1.30
I-health female IMD 07	1.34	1.21	1.26
I-health female IMD 08	1.32	1.19	1.24
I-health female IMD 09	1.27	1.15	1.20
I-health female IMD 10	1.28	1.15	1.21
Spouse male no IMD	1.01	1.46	1.41
Spouse male IMD 01	1.29	1.87	1.85
Spouse male IMD 02	1.16	1.70	1.68
Spouse male IMD 03	1.00	1.45	1.43
Spouse male IMD 04	1.02	1.49	1.47
Spouse male IMD 05	0.99	1.45	1.43
Spouse male IMD 06	0.92	1.35	1.34
Spouse male IMD 07	0.97	1.42	1.41
Spouse male IMD 08	0.89	1.31	1.30
Spouse male IMD 09	0.84	1.21	1.20
Spouse male IMD 10	0.79	1.15	1.14
Spouse female no IMD	0.94	1.07	1.08
Spouse female IMD 01	1.20	1.38	1.41
Spouse female IMD 02	1.17	1.34	1.38
Spouse female IMD 03	1.03	1.17	1.21
Spouse female IMD 04	1.00	1.15	1.18
Spouse female IMD 05	0.94	1.07	1.11
Spouse female IMD 06	0.94	1.08	1.11
Spouse female IMD 07	0.94	1.08	1.11
Spouse female IMD 08	0.91	1.04	1.08
Spouse female IMD 09	0.89	1.02	1.05
Spouse female IMD 10	0.85	0.96	1.00

Figure 5.40: Proposed optimal scaling factors for the unscaled mortality base and improvement tables, used for the 2013, 2016 and 2020 valuation assumptions for each category, to ensure that total expected exit amounts equal actual exit amounts.

Category	2020 assumption	2016 assumption	2013 assumption
N-health male no IMD	0.98	1.03	1.00
N-health male IMD 01	1.34	1.42	1.40
N-health male IMD 02	1.24	1.31	1.29
N-health male IMD 03	1.12	1.19	1.17
N-health male IMD 04	1.04	1.10	1.08
N-health male IMD 05	1.00	1.06	1.04
N-health male IMD 06	0.94	1.00	0.99
N-health male IMD 07	0.94	0.99	0.98
N-health male IMD 08	0.90	0.96	0.95
N-health male IMD 09	0.89	0.94	0.93
N-health male IMD 10	0.86	0.91	0.90
N-health female no IMD	0.94	0.95	0.96
N-health female IMD 01	1.28	1.28	1.31
N-health female IMD 02	1.11	1.11	1.14
N-health female IMD 03	1.01	1.02	1.04
N-health female IMD 04	1.04	1.04	1.05
N-health female IMD 05	0.95	0.94	0.95
N-health female IMD 06	0.95	0.95	0.96
N-health female IMD 07	0.92	0.92	0.93
N-health female IMD 08	0.88	0.87	0.88
N-health female IMD 09	0.87	0.87	0.88
N-health female IMD 10	0.86	0.86	0.87
I-health male no IMD	0.98	0.96	0.98
I-health male IMD 01	1.45	1.39	1.43
I-health male IMD 02	1.35	1.30	1.34
I-health male IMD 03	1.29	1.24	1.28
I-health male IMD 04	1.17	1.12	1.16
I-health male IMD 05	1.12	1.07	1.10
I-health male IMD 06	1.15	1.10	1.13
I-health male IMD 07	0.99	0.95	0.98
I-health male IMD 08	1.01	0.97	1.00
I-health male IMD 09	0.98	0.94	0.97
I-health male IMD 10	0.95	0.91	0.93
I-health female no IMD	1.04	0.93	0.90
I-health female IMD 01	1.49	1.32	1.31
I-health female IMD 02	1.38	1.22	1.22
I-health female IMD 03	1.33	1.17	1.16
I-health female IMD 04	1.22	1.08	1.08
I-health female IMD 05	1.19	1.06	1.06
I-health female IMD 06	1.19	1.05	1.04
I-health female IMD 07	1.12	0.99	0.99
I-health female IMD 08	1.14	1.01	1.00
I-health female IMD 09	1.07	0.95	0.95
I-health female IMD 10	1.06	0.94	0.94
Spouse male no IMD	1.06	1.48	1.43
Spouse male IMD 01	1.27	1.73	1.71
Spouse male IMD 02	1.17	1.59	1.58
Spouse male IMD 03	1.00	1.36	1.34
Spouse male IMD 04	1.06	1.47	1.46
Spouse male IMD 05	0.96	1.29	1.28
Spouse male IMD 06	0.96	1.36	1.35
Spouse male IMD 07	0.96	1.34	1.33
Spouse male IMD 08	0.92	1.28	1.27
Spouse male IMD 09	0.84	1.17	1.16
Spouse male IMD 10	0.80	1.11	1.10
Spouse female no IMD	0.92	1.05	1.08
Spouse female IMD 01	1.15	1.30	1.38
Spouse female IMD 02	1.10	1.24	1.32
Spouse female IMD 03	0.98	1.12	1.18
Spouse female IMD 04	0.97	1.10	1.17
Spouse female IMD 05	0.92	1.04	1.11
Spouse female IMD 06	0.92	1.05	1.11
Spouse female IMD 07	0.93	1.06	1.12
Spouse female IMD 08	0.91	1.03	1.10
Spouse female IMD 09	0.90	1.02	1.08
Spouse female IMD 10	0.86	0.97	1.03

Figure 5.41: Proposed optimal scaling factors for the unscaled mortality base and improvement tables, used for the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

tically better performance than the simpler one. The null hypothesis posits that there is no significant difference between the simpler and more complex model; the alternative hypothesis suggests that the more complex model fits the data better. In this context, model one represents the simpler model with unspecified scaling; model two is more intricate, utilising IMD-specific scaling. If the latter performs significantly better, the F-statistic would be high; as a result, the probability, also known as the p-value, would fall below the critical threshold; if it exceeds the critical value, no significant difference between the two models can be concluded; thus, the simpler model excluding IMD-specific scaling would be preferred, as the additional complexity would not be justified.

Table 5.42 indicates that the significance level is weaker towards the middle IMD values; this is as expected since the optimal scaling factor for the non-IMD split is essentially the weighted average of the scaling factors deduced from the IMD splits. With the presence of a negative correlation between optimal scaling factor and IMD score, scaling factors for middling values are typically closer to the mean, before diverging upon reaching the more extreme IMD scores. The unknown IMD groups yield p-values greater than 0.05 in all cases, signifying inconclusive evidence that IMD-specific scaling factors are a better fit than unspecified factors; these results validate the hypothesis that data underlying these categories contain an amalgamation of IMD records with the average value residing in the middle range. Evidently, the IMD-specific scaling factors are a more suitable fit for the data at polarised IMD values; therefore, IMD is a determinant used to differentiate mortality between data records.

Figures 5.43 to 5.53 also demonstrate this trend. For IMD 1, expected exits without specific scaling substantially underestimate actual exits in contrast to when adopting IMD-specific scaling. The line for expected exits with unspecified scaling rises alongside IMD, making it comparable to the actual exits and expected exits with IMD scaling in the mid-range IMD scores; by IMD 10, the former vastly overestimates the actual exits and IMD-scaled expected exits.

Figures 5.54 to 5.59 display the locations of optimal IMD scaling factors across England

Category	DoF	Scale (IMD)	Scale (no IMD)	RSS (IMD)	RSS (no IMD)	F	p-value
N-health male no IMD	54	0.98	0.97	1,534,846,516,669	1,540,837,682,616	1.00	0.49432
N-health male IMD 01	52	1.34	0.97	428,431,598,447	1,747,037,424,702	4.08	0.00000
N-health male IMD 02	48	1.24	0.97	503,849,254,153	1,589,317,741,052	3.15	0.00006
N-health male IMD 03	52	1.12	0.97	543,129,207,815	1,064,981,907,470	1.96	0.00833
N-health male IMD 04	50	1.04	0.97	404,806,406,860	551,323,912,176	1.36	0.13905
N-health male IMD 05	49	1.00	0.97	612,460,200,038	649,919,045,893	1.06	0.41812
N-health male IMD 06	54	0.94	0.97	491,192,031,180	543,807,046,421	1.11	0.35492
N-health male IMD 07	50	0.94	0.97	1,977,717,113,732	2,067,599,851,871	1.05	0.43788
N-health male IMD 08	50	0.90	0.97	1,184,415,232,153	1,635,381,115,450	1.38	0.12870
N-health male IMD 09	51	0.89	0.97	1,756,828,845,366	2,543,594,382,743	1.45	0.09489
N-health male IMD 10	49	0.86	0.97	2,917,235,857,785	4,948,826,061,106	1.70	0.03363
N-health female no IMD	55	0.94	0.95	734,333,415,027	735,496,681,848	1.00	0.49767
N-health female IMD 01	54	1.28	0.95	89,002,584,369	330,878,404,373	3.72	0.00000
N-health female IMD 02	50	1.11	0.95	151,256,965,195	257,239,351,770	1.70	0.03163
N-health female IMD 03	52	1.01	0.95	134,312,543,442	159,251,053,833	1.19	0.27068
N-health female IMD 04	53	1.04	0.95	127,680,067,741	196,234,420,812	1.54	0.06043
N-health female IMD 05	54	0.95	0.95	156,873,290,506	157,093,921,851	1.00	0.49795
N-health female IMD 06	53	0.95	0.95	255,378,641,972	255,468,252,559	1.00	0.49949
N-health female IMD 07	55	0.92	0.95	228,867,525,138	241,327,075,528	1.05	0.42244
N-health female IMD 08	52	0.88	0.95	200,026,194,278	305,938,852,970	1.53	0.06436
N-health female IMD 09	57	0.87	0.95	274,354,058,274	416,345,985,794	1.52	0.05914
N-health female IMD 10	57	0.86	0.95	400,770,569,568	596,212,118,801	1.49	0.06837
I-health male no IMD	71	0.98	1.13	152,332,210,174	201,635,085,316	1.32	0.11994
I-health male IMD 01	73	1.45	1.13	253,345,848,135	664,161,407,078	2.62	0.00003
I-health male IMD 02	71	1.35	1.13	178,155,882,358	346,238,485,936	1.94	0.00285
I-health male IMD 03	68	1.29	1.13	131,296,598,752	219,303,832,791	1.67	0.01804
I-health male IMD 04	71	1.17	1.13	136,323,364,022	143,619,034,208	1.05	0.41338
I-health male IMD 05	70	1.12	1.13	190,411,094,074	191,173,198,929	1.00	0.49336
I-health male IMD 06	73	1.15	1.13	149,023,247,507	150,403,403,849	1.01	0.48435
I-health male IMD 07	69	0.99	1.13	145,218,386,488	223,748,989,744	1.54	0.03737
I-health male IMD 08	71	1.01	1.13	139,181,029,355	200,044,799,460	1.44	0.06440
I-health male IMD 09	76	0.98	1.13	133,546,219,936	221,144,391,959	1.66	0.01463
I-health male IMD 10	72	0.95	1.13	158,132,351,620	260,177,174,158	1.65	0.01811
I-health female no IMD	77	1.04	1.19	212,079,383,727	226,854,312,207	1.07	0.38419
I-health female IMD 01	71	1.49	1.19	116,569,282,102	181,092,838,088	1.55	0.03273
I-health female IMD 02	67	1.38	1.19	101,986,884,093	131,108,317,607	1.29	0.15319
I-health female IMD 03	74	1.33	1.19	155,292,968,702	171,878,688,787	1.11	0.33182
I-health female IMD 04	75	1.22	1.19	171,309,896,175	172,535,737,036	1.01	0.48773
I-health female IMD 05	79	1.19	1.19	148,486,757,278	148,507,620,086	1.00	0.49975
I-health female IMD 06	76	1.19	1.19	193,724,568,635	193,733,986,938	1.00	0.49992
I-health female IMD 07	74	1.12	1.19	178,232,757,009	183,629,388,912	1.03	0.44913
I-health female IMD 08	76	1.14	1.19	175,360,643,054	178,912,616,191	1.02	0.46529
I-health female IMD 09	72	1.07	1.19	209,307,080,522	227,479,330,877	1.09	0.36244
I-health female IMD 10	73	1.06	1.19	214,792,656,403	235,497,099,400	1.10	0.34763
Spouse male no IMD	83	1.06	0.98	2,684,350,066	2,835,815,427	1.06	0.40158
Spouse male IMD 01	76	1.27	0.98	1,683,612,119	3,093,248,630	1.84	0.00436
Spouse male IMD 02	80	1.17	0.98	1,757,940,674	2,425,790,703	1.38	0.07600
Spouse male IMD 03	78	1.00	0.98	2,131,030,278	2,138,721,200	1.00	0.49367
Spouse male IMD 04	80	1.06	0.98	2,528,617,400	2,730,325,715	1.08	0.36613
Spouse male IMD 05	82	0.96	0.98	3,521,154,647	3,542,914,254	1.01	0.48891
Spouse male IMD 06	83	0.96	0.98	2,342,741,780	2,351,534,181	1.00	0.49321
Spouse male IMD 07	79	0.96	0.98	3,651,587,517	3,661,868,221	1.00	0.49503
Spouse male IMD 08	84	0.92	0.98	3,848,154,421	4,065,186,565	1.06	0.40104
Spouse male IMD 09	76	0.84	0.98	4,964,546,893	6,091,866,123	1.23	0.18720
Spouse male IMD 10	77	0.80	0.98	3,474,670,749	5,483,314,840	1.58	0.02348
Spouse female no IMD	91	0.92	0.94	99,689,508,947	101,117,379,800	1.01	0.47303
Spouse female IMD 01	91	1.15	0.94	65,728,970,001	162,896,156,750	2.48	0.00001
Spouse female IMD 02	85	1.10	0.94	88,696,468,194	161,881,089,946	1.83	0.00303
Spouse female IMD 03	91	0.98	0.94	90,878,930,936	97,717,765,194	1.08	0.36502
Spouse female IMD 04	86	0.97	0.94	79,258,583,629	84,157,096,280	1.06	0.39080
Spouse female IMD 05	84	0.92	0.94	97,928,419,053	100,637,168,229	1.03	0.45040
Spouse female IMD 06	87	0.92	0.94	71,747,203,115	74,231,379,156	1.03	0.43712
Spouse female IMD 07	90	0.93	0.94	64,610,616,816	65,059,034,475	1.01	0.48695
Spouse female IMD 08	89	0.91	0.94	134,708,123,568	144,343,736,058	1.07	0.37261
Spouse female IMD 09	84	0.90	0.94	170,279,750,678	195,218,975,034	1.15	0.26624
Spouse female IMD 10	85	0.86	0.94	220,521,288,083	356,615,898,149	1.62	0.01395

Figure 5.42: F-test comparing models with and without IMD-specific scaling factors for each category.

and Wales; optimal scaling factors for their respective tables are typically depicted as high for ill-health categories, particularly for females, but low for normal health and spouses. The most prominent pockets of high-value scaling factors are visible in select regions with the lowest IMD scores, such as South Wales, North West England, North East England and London.

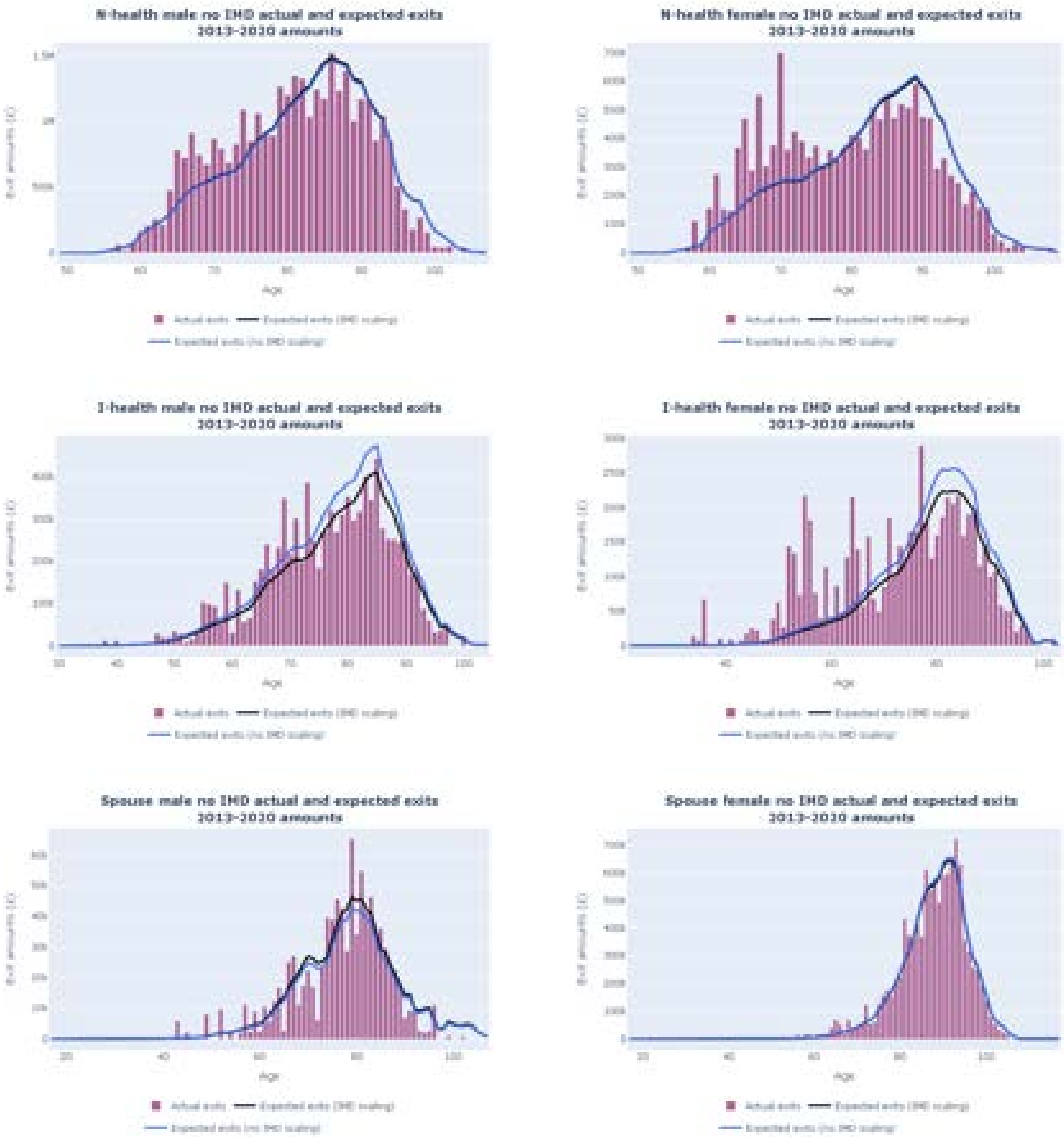


Figure 5.43: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

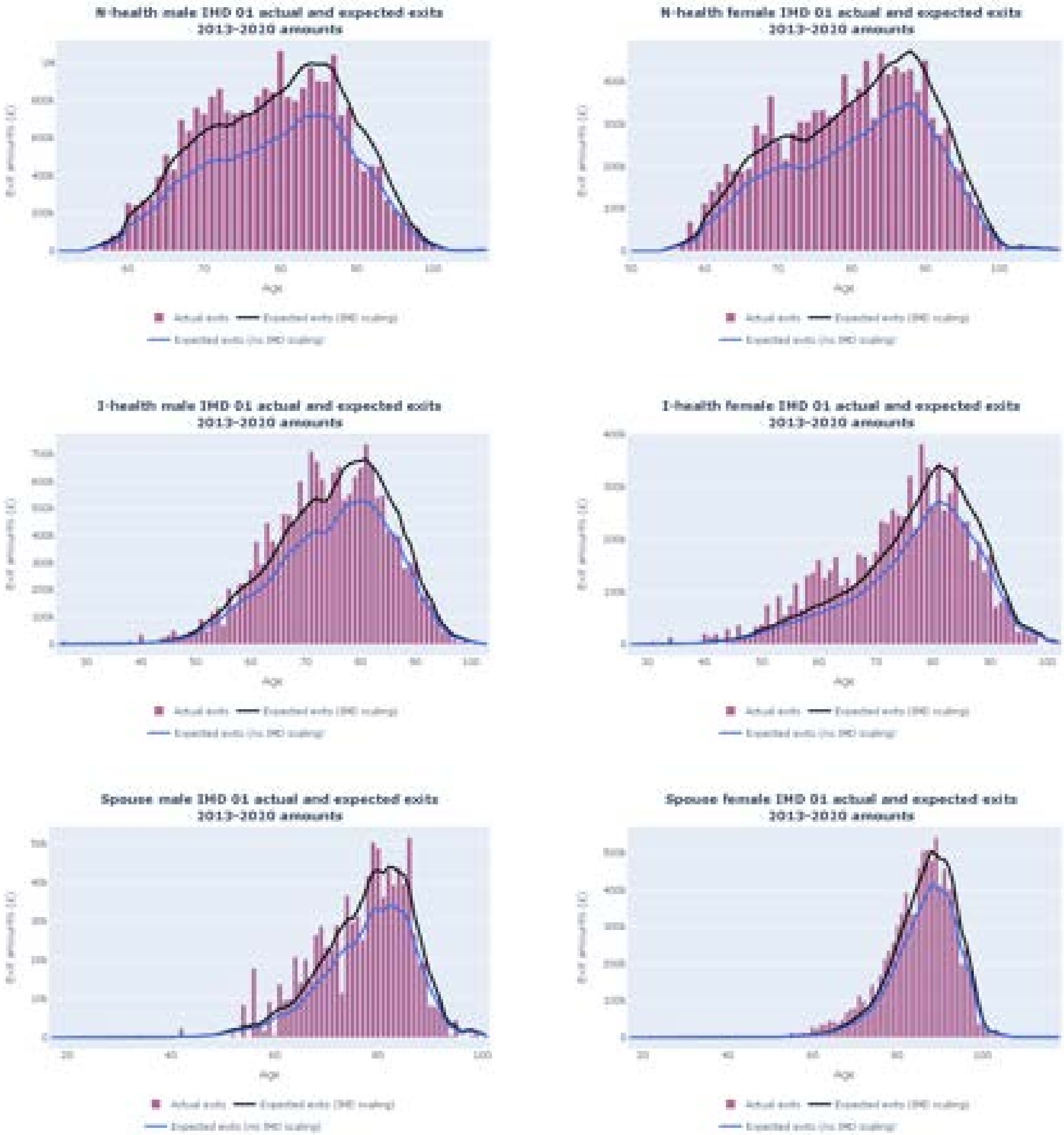


Figure 5.44: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

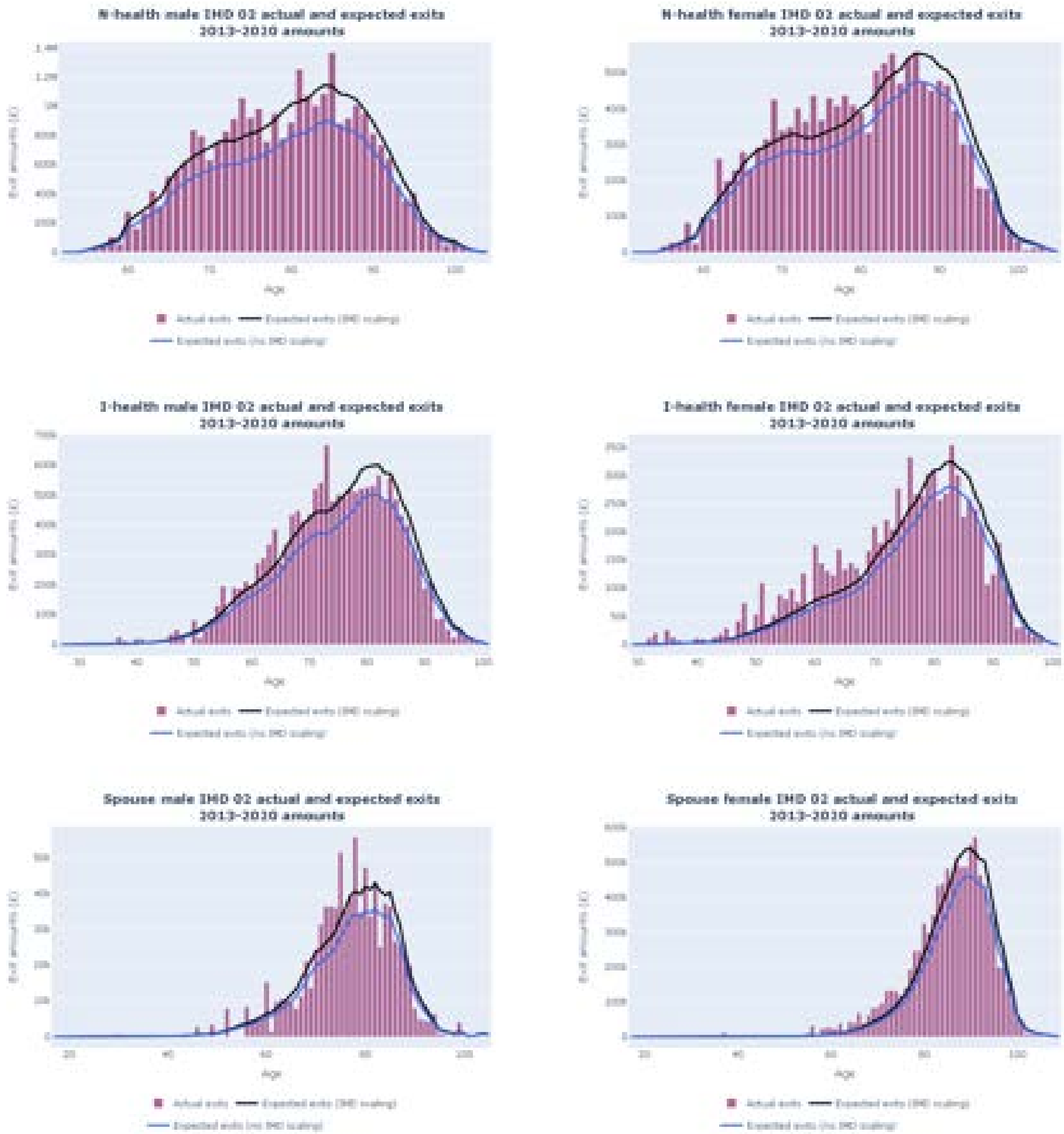


Figure 5.45: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

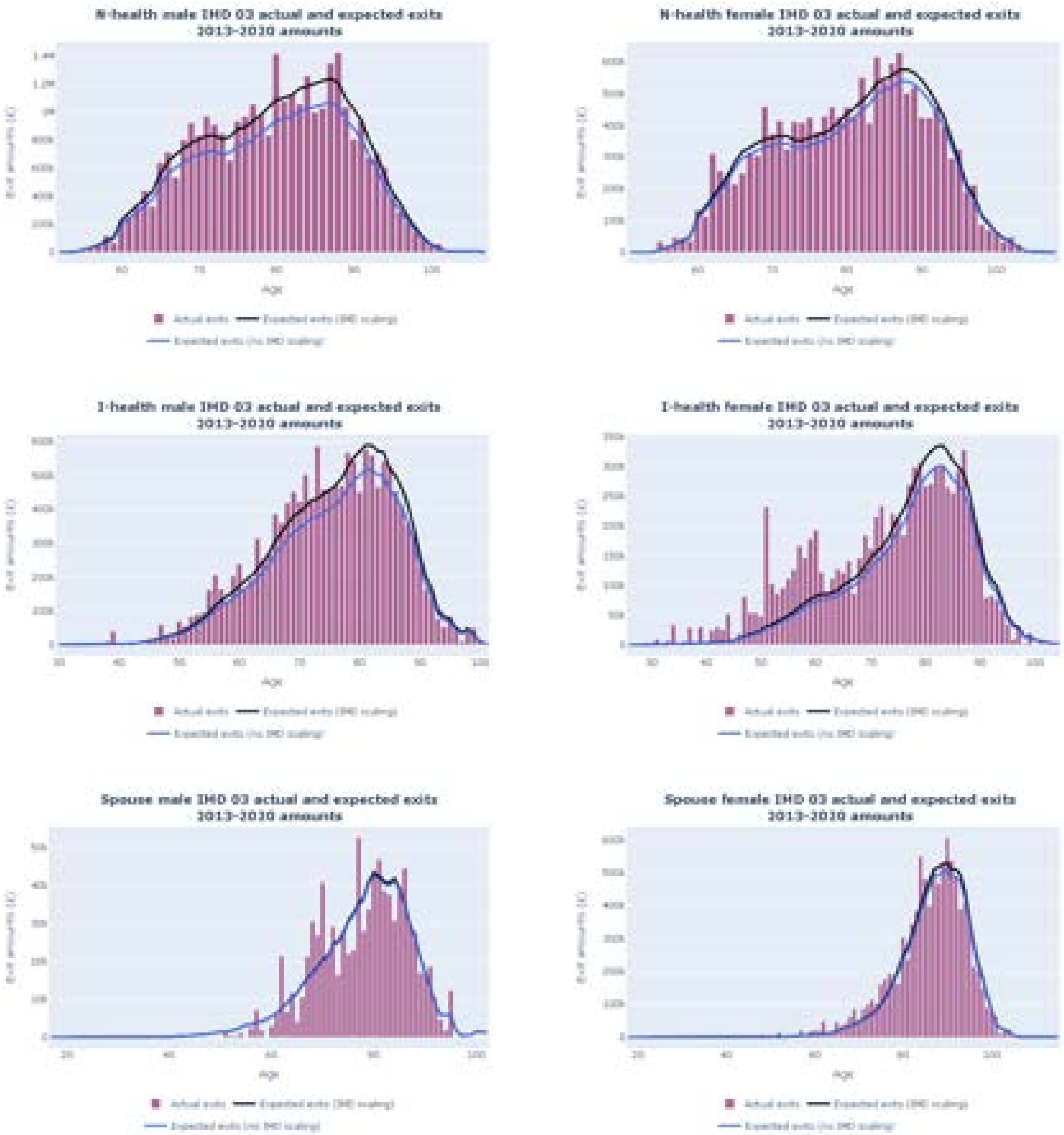


Figure 5.46: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

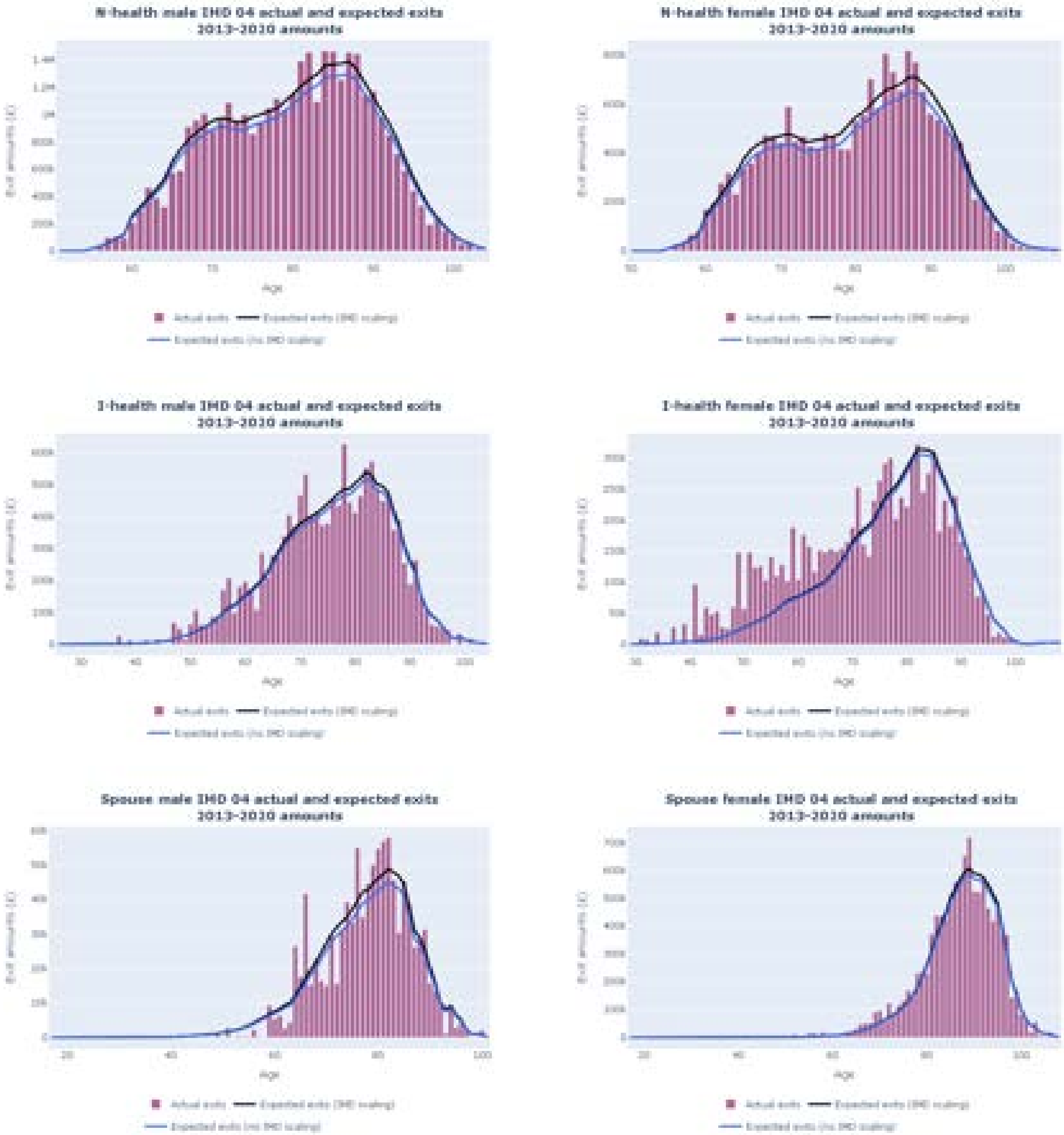


Figure 5.47: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

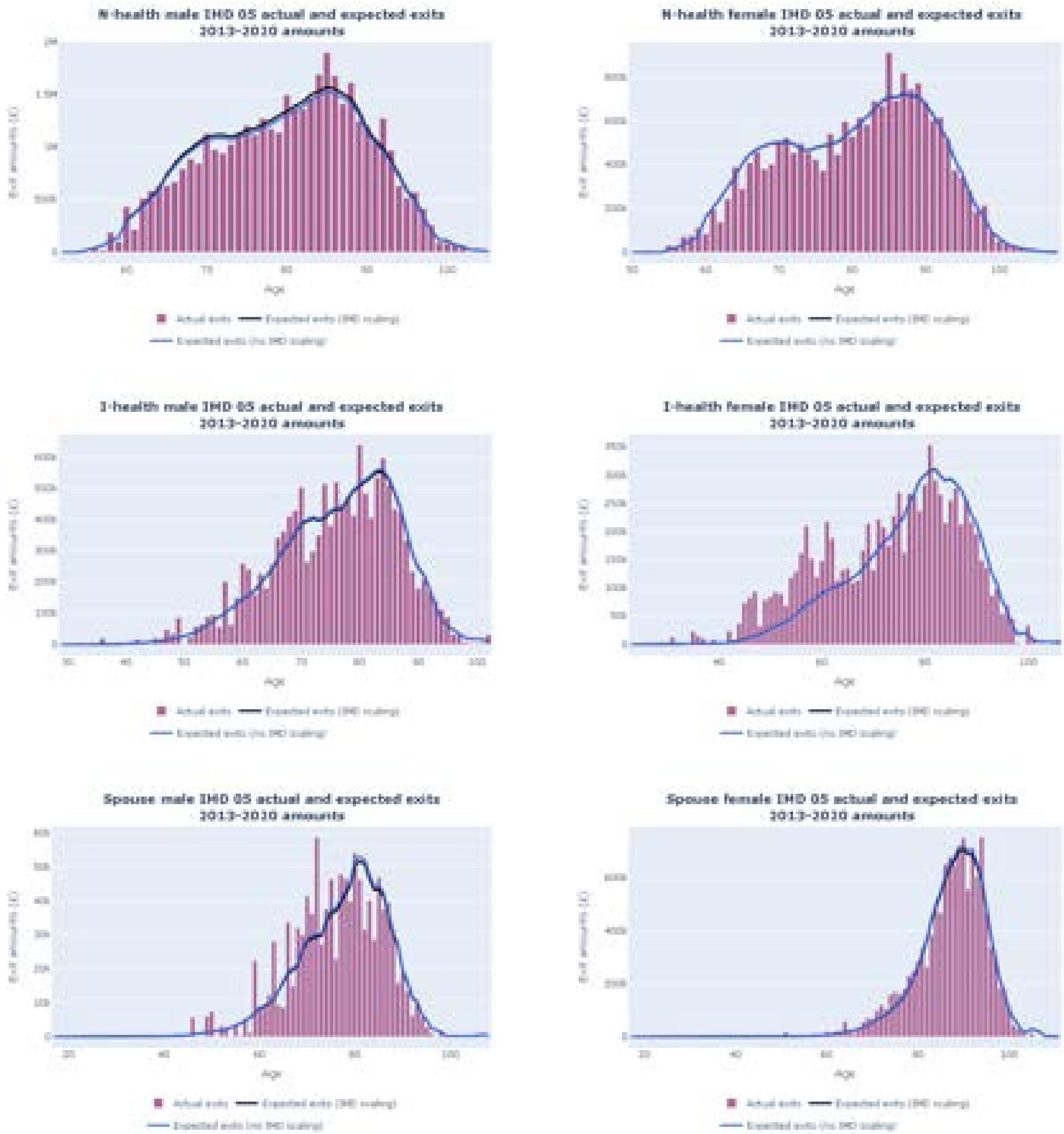


Figure 5.48: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

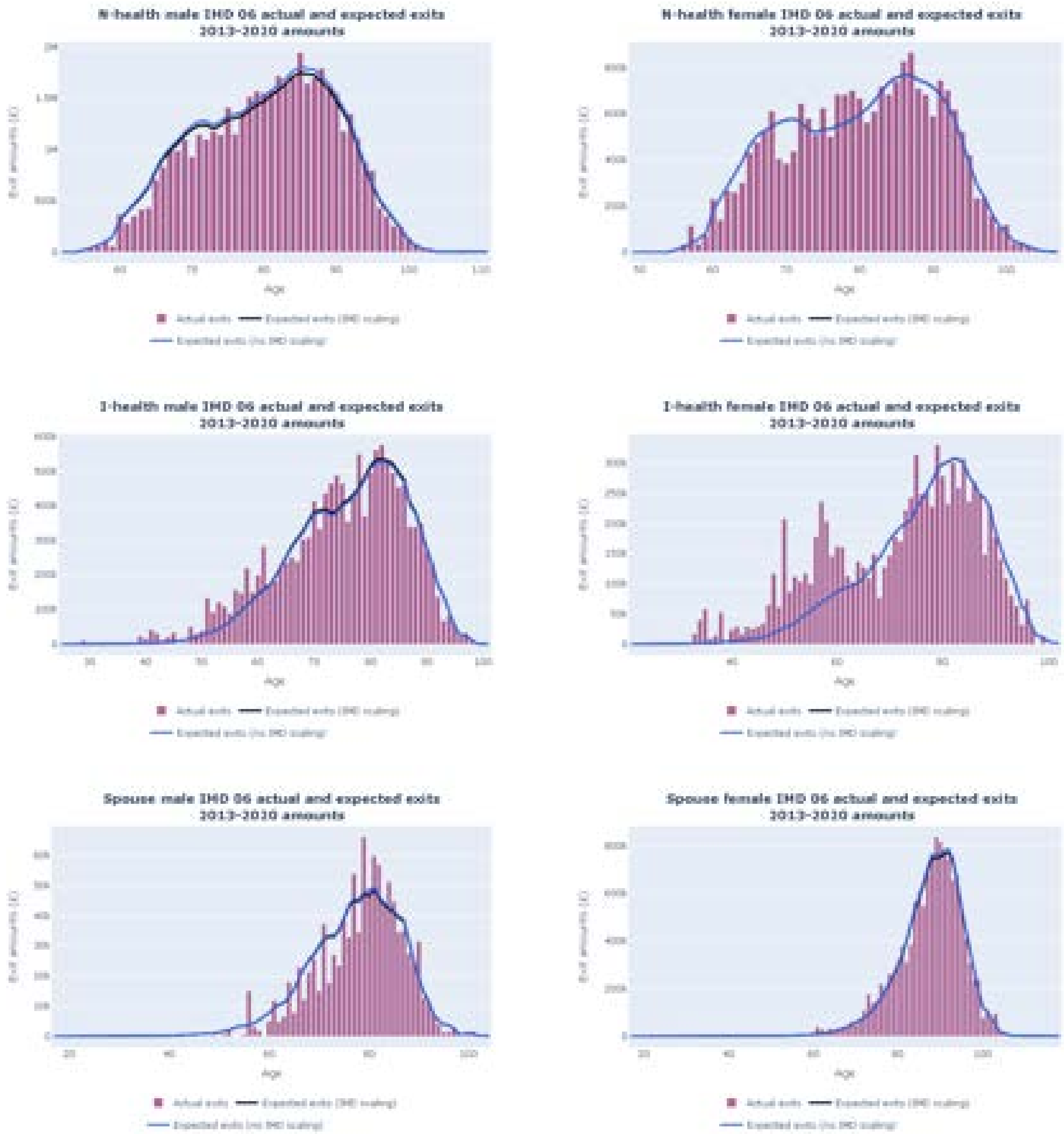


Figure 5.49: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

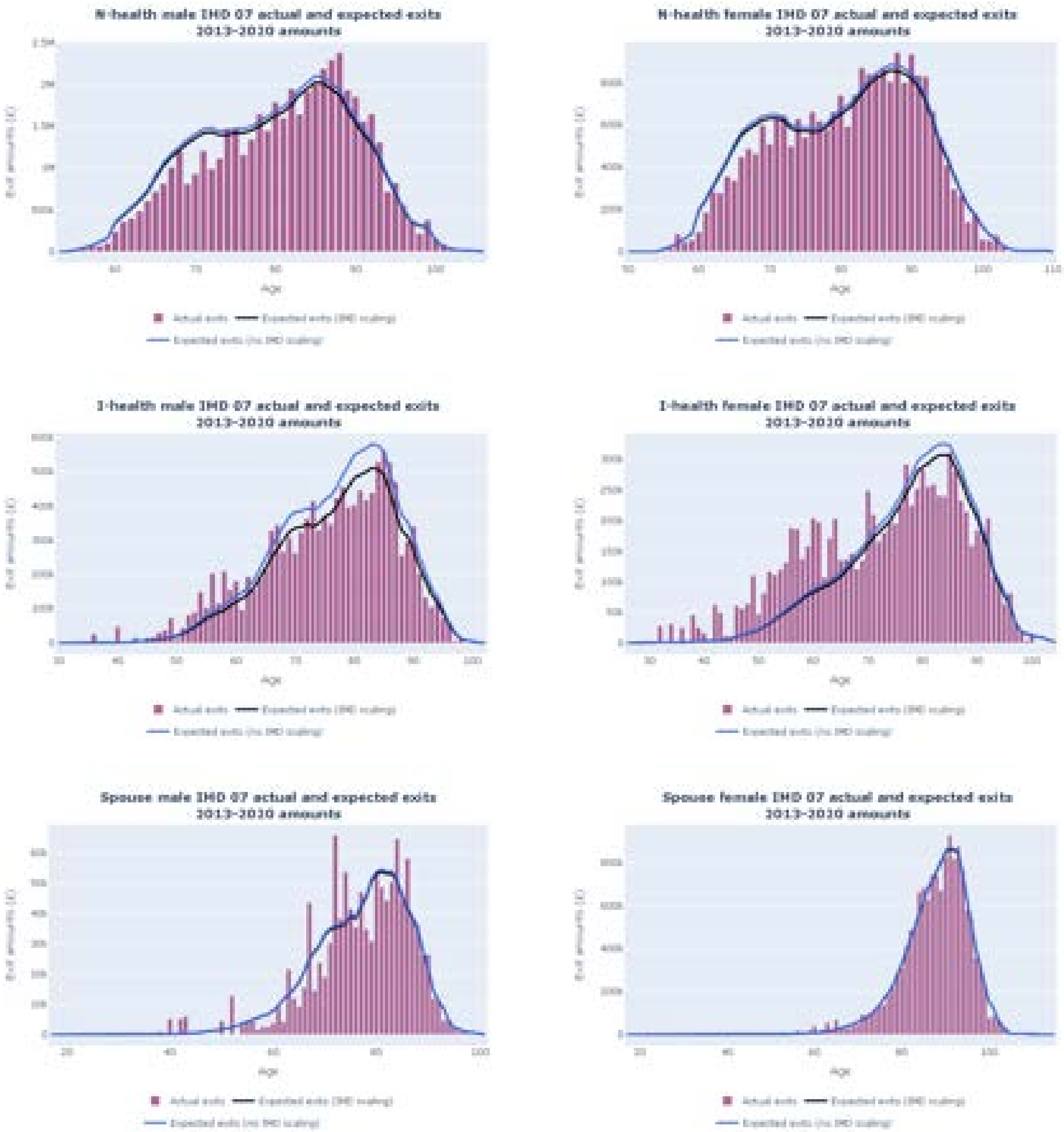


Figure 5.50: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

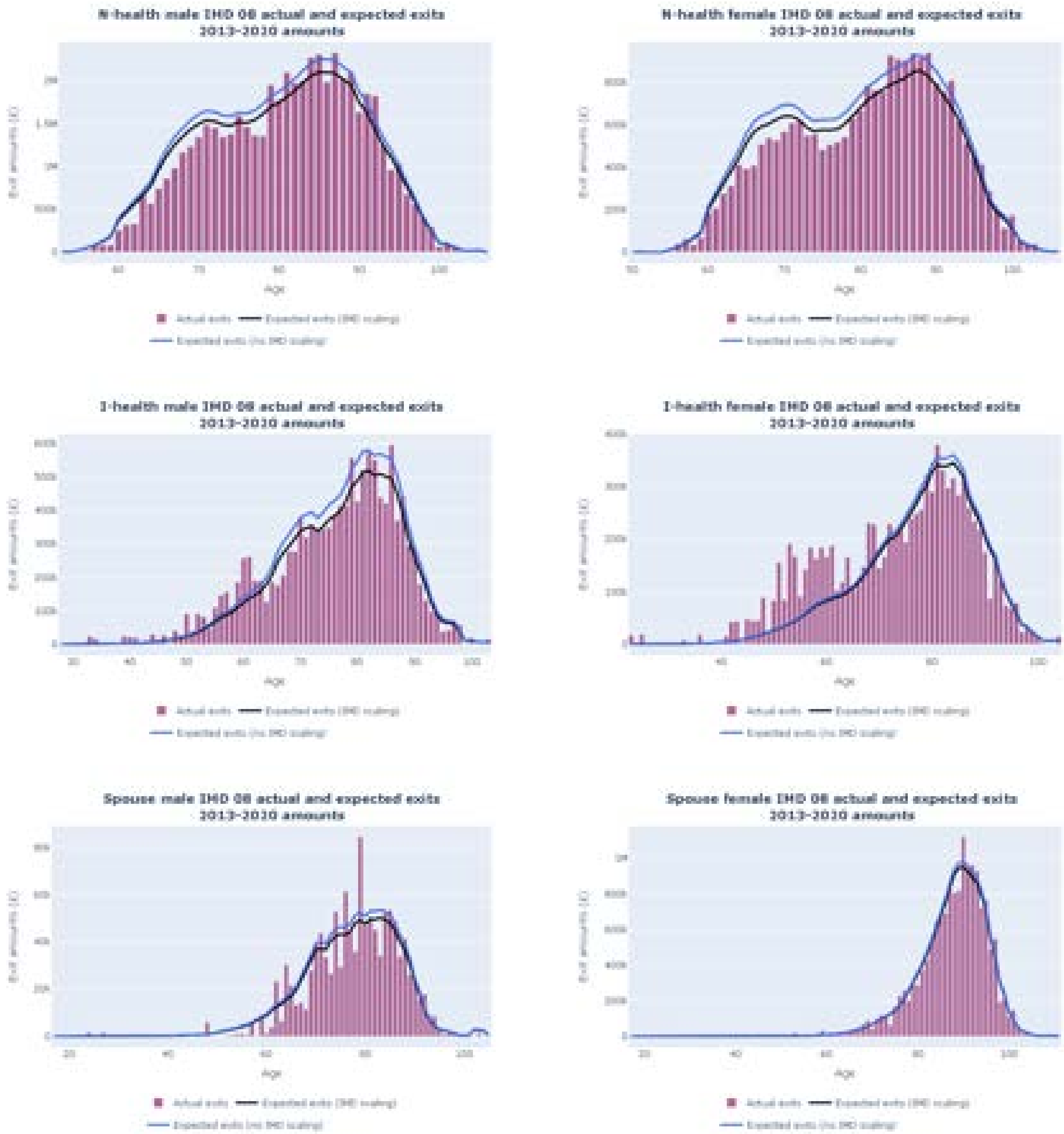


Figure 5.51: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

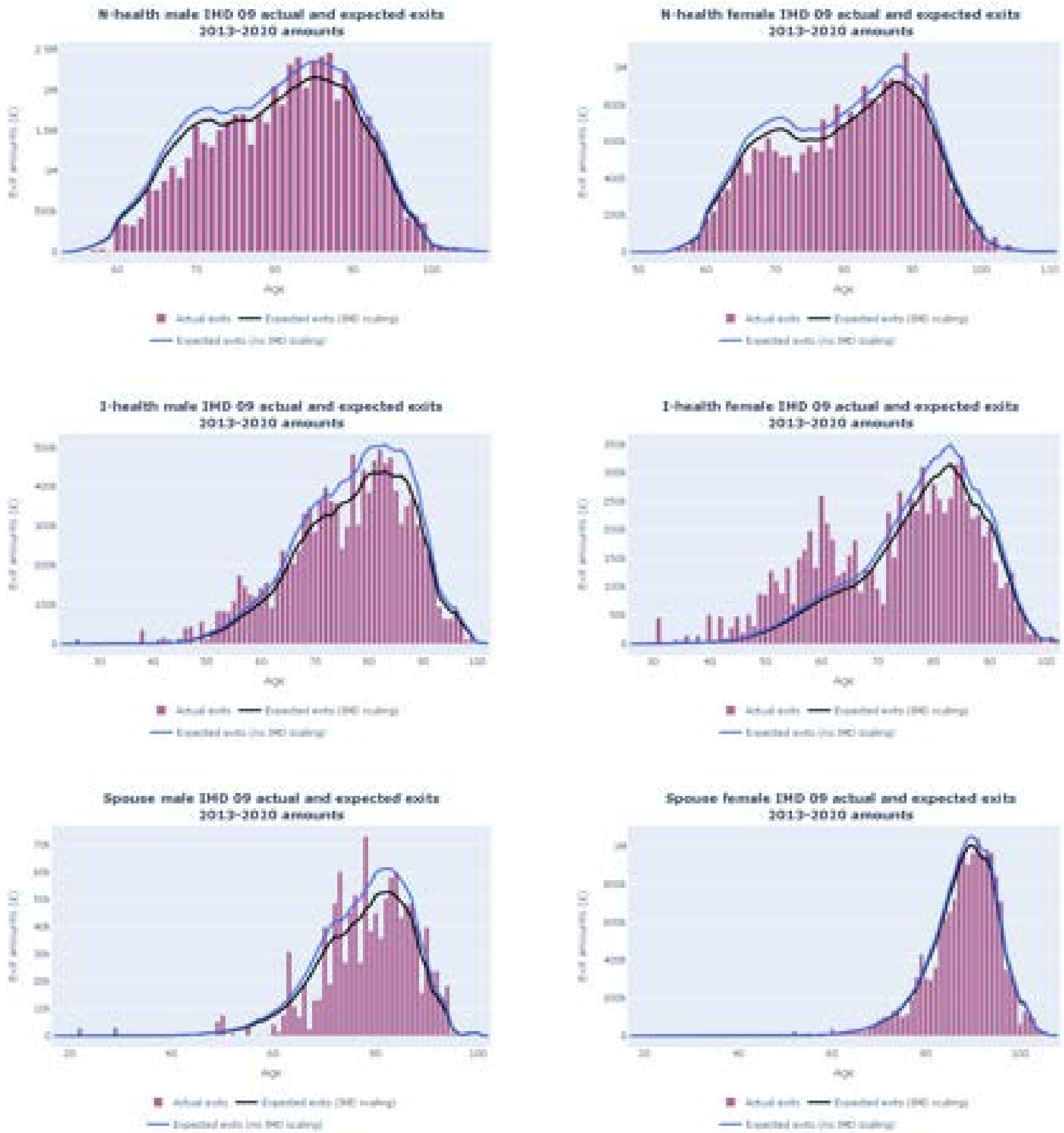


Figure 5.52: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.

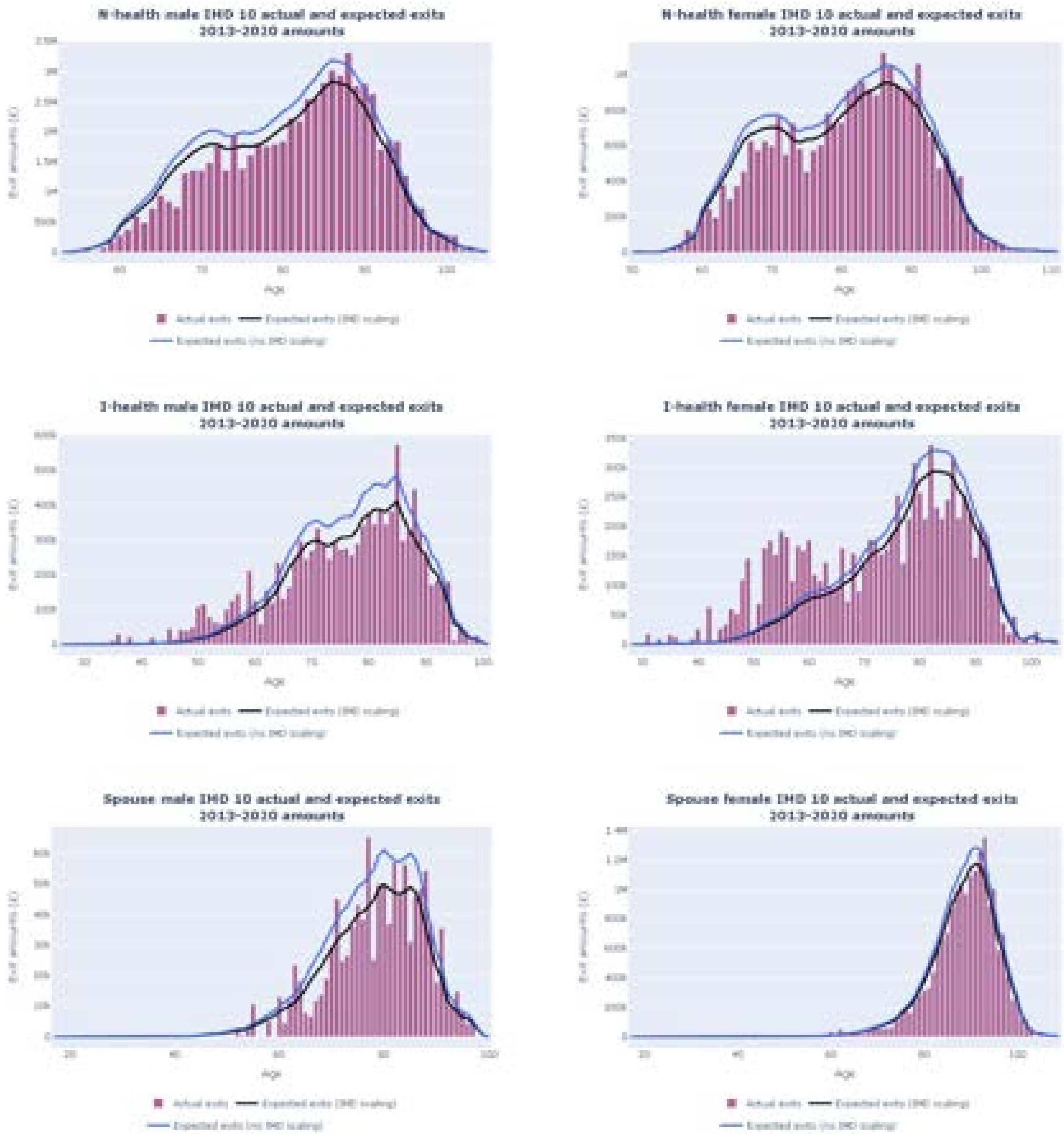
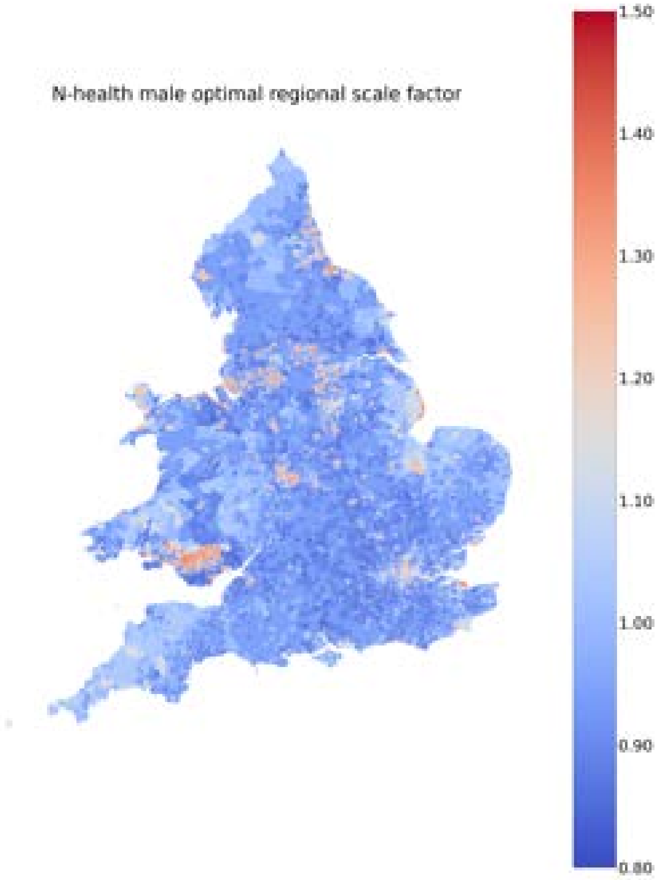
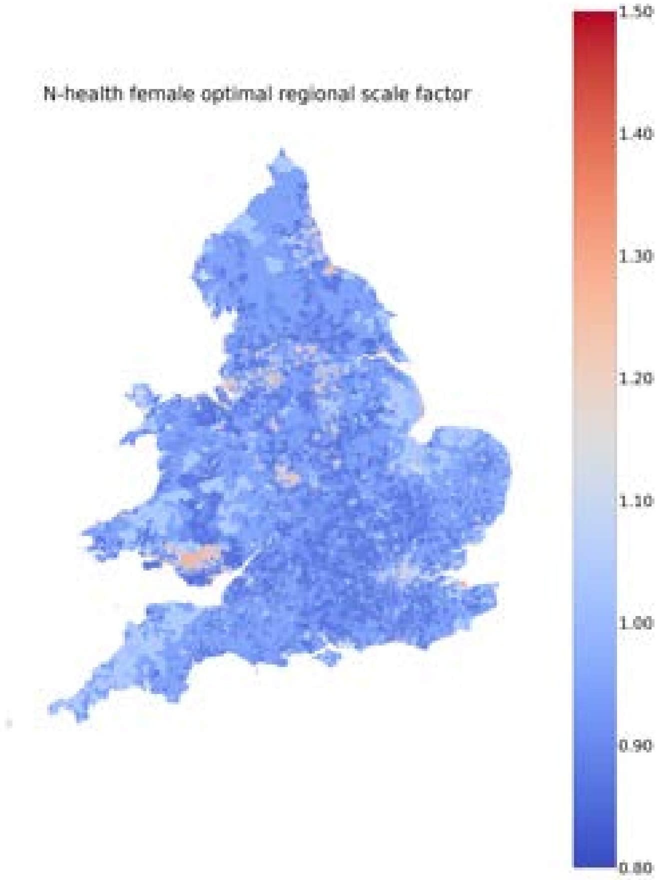


Figure 5.53: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without IMD specificity.



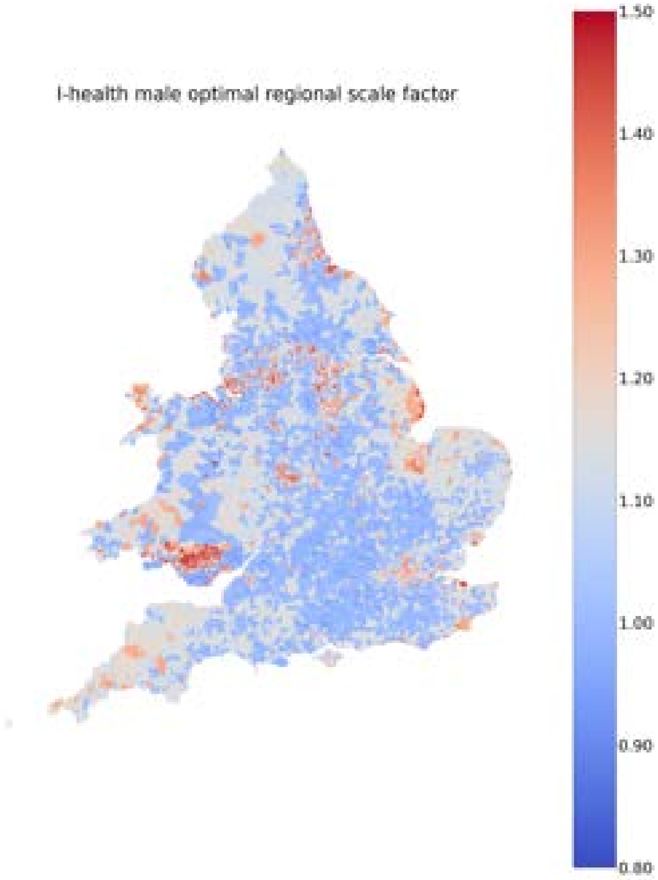
IMD	N-health male optimal regional scale factor
No IMD	0.98
1	1.34
2	1.24
3	1.12
4	1.04
5	1.00
6	0.94
7	0.94
8	0.90
9	0.89
10	0.86

Figure 5.54: Optimal scaling factor by IMD decile for normal health male pensioners between 1st April 2013 and 31st March 2020.



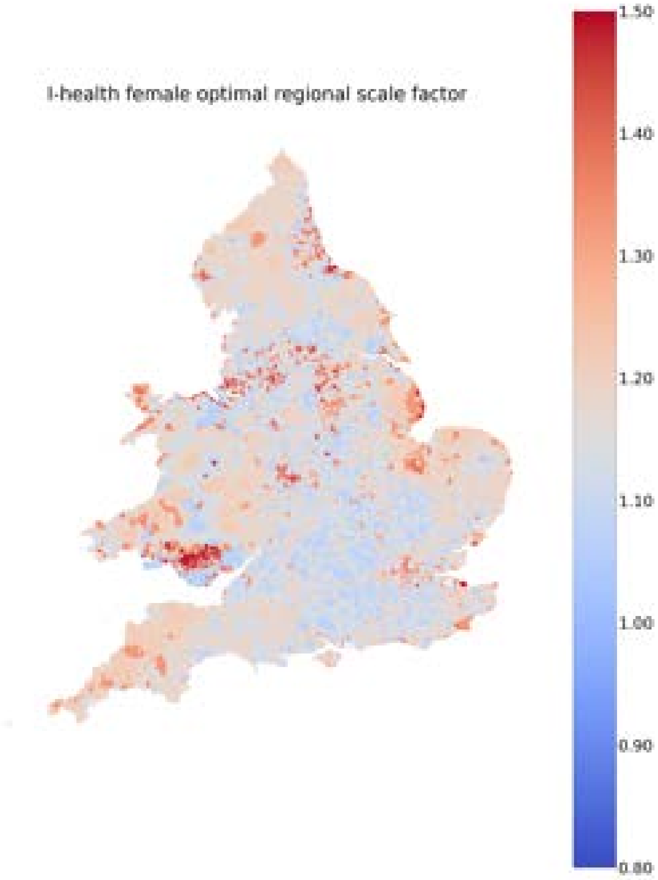
IMD	N-health female optimal regional scale factor
No IMD	0.94
1	1.28
2	1.11
3	1.01
4	1.04
5	0.95
6	0.95
7	0.92
8	0.88
9	0.87
10	0.86

Figure 5.55: Optimal scaling factor by IMD decile for normal health female pensioners between 1st April 2013 and 31st March 2020.



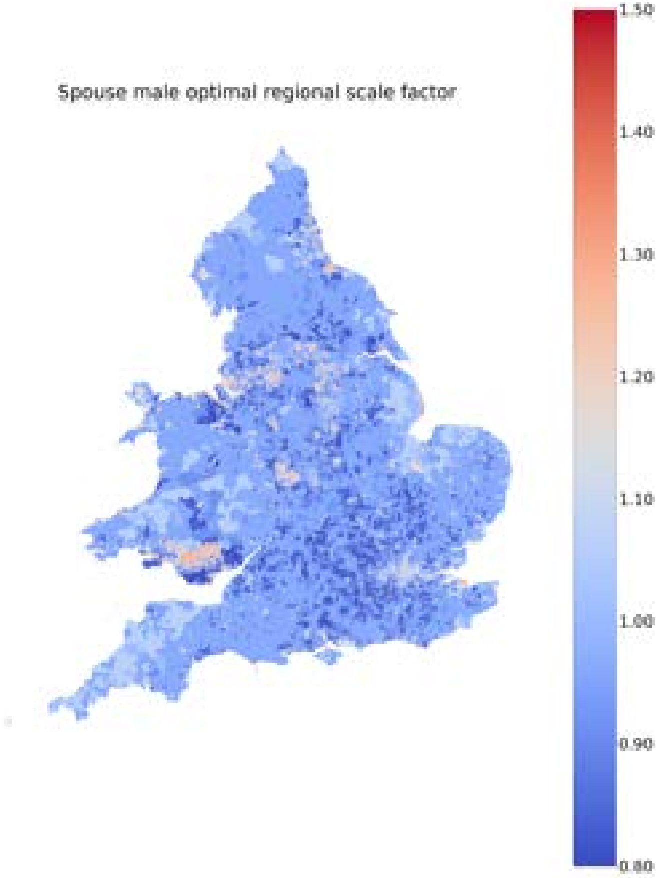
IMD	I-health male optimal regional scale factor
No IMD	0.98
1	1.45
2	1.35
3	1.29
4	1.17
5	1.12
6	1.15
7	0.99
8	1.01
9	0.98
10	0.95

Figure 5.56: Optimal scaling factor by IMD decile for ill health male pensioners between 1st April 2013 and 31st March 2020.



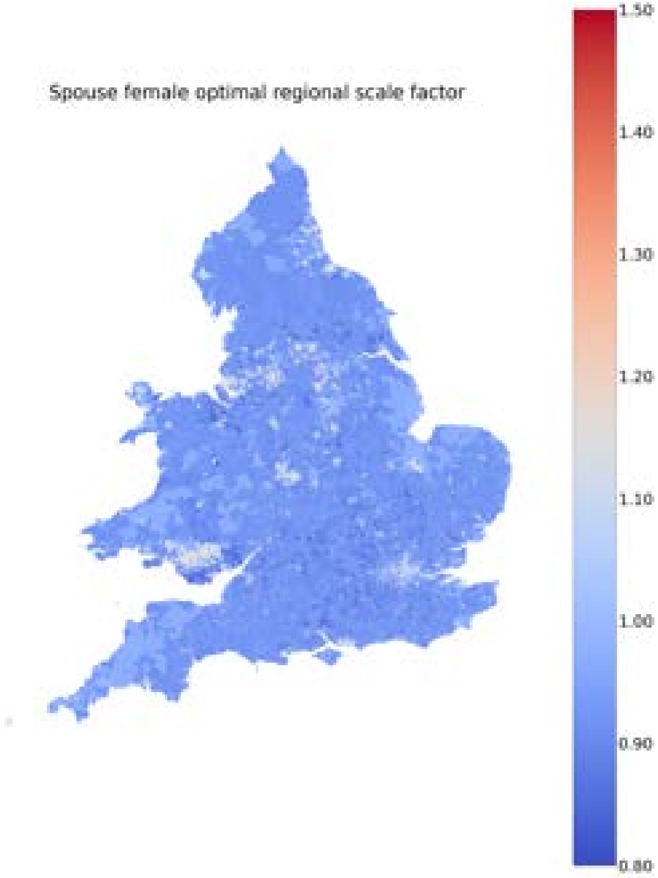
IMD	I-health female optimal regional scale factor
No IMD	1.04
1	1.49
2	1.38
3	1.33
4	1.22
5	1.19
6	1.19
7	1.12
8	1.14
9	1.07
10	1.06

Figure 5.57: Optimal scaling factor by IMD decile for ill health female pensioners between 1st April 2013 and 31st March 2020.



IMD	Spouse male optimal regional scale factor
No IMD	1.06
1	1.27
2	1.17
3	1.00
4	1.06
5	0.96
6	0.96
7	0.96
8	0.92
9	0.84
10	0.80

Figure 5.58: Optimal scaling factor by IMD decile for male dependants between 1st April 2013 and 31st March 2020.



IMD	Spouse female optimal regional scale factor
No IMD	0.92
1	1.15
2	1.10
3	0.98
4	0.97
5	0.92
6	0.92
7	0.93
8	0.91
9	0.90
10	0.86

Figure 5.59: Optimal scaling factor by IMD decile for female dependants between 1st April 2013 and 31st March 2020.

5.2.2 IMD SAPS tables

The CMI released a new S4 series of SAPS mortality tables in 2024, which included novel IMD-based tables [9, 10]; there are now four IMD tables per category for male pensioners, female pensioners and female dependants. Group 1 exhibits the highest mortality and Group 4 exhibits the lowest; member or dependant records can be allocated to these on the basis of pension type, gender, IMD and pension valued as of 1st January 2017.

Figure 5.60 illustrates the IMD value for each Scottish Data Zone and LSOA in England and Wales; Figures 5.61 to 5.64 display the number of data records and pension amounts associated with each LSOA and Data Zone between 1st April 2013 and 31st March 2020. The membership distribution is fairly consistent across all member types, with the greatest concentration located in Wales, North West England and South West England; weaker areas of concentration are detected in Scotland, London and the Midlands.

Figures 5.65 to 5.72 depict the proportion of members and pensions by IMD and pension band, classified by colour. Those with the lowest combined value of IMD and pension are allocated to Group 1; increasing IMD or pension band results in higher allocations, with the greatest combined values of IMD and pension assigned to Group 4. Pension bands differentiating groups vary for each member type; for male pensioners, female pensioners and female dependants, higher proportions of members are allocated to Groups 1 and 2 on a lives basis, shifting towards Groups 3 to 4 when weighted by pension amounts. Although male dependants demonstrate this shift to an extent, the majority of members are assigned to Groups 1 and 2 for lives and amounts.

Figures 5.73 to 5.76 illustrate the proportion of members in each IMD for each member type, weighted by lives and amounts. The "U" category represents incomplete data records where a postcode was matched without a corresponding IMD value in the CMI dataset; blank values indicate entries where the postcode was incompatible with the CMI dataset. For all member types, the proportion of members on a lives basis increases, peaking around IMD 8, before gradually declining; conversely, the membership proportions continually rise, reaching the highest IMD value of 10 on an amounts basis.

Figures 5.77 to 5.80 allocate these proportions to the SAPS S4 IMD tables with the highest typically residing in Groups 2 and 3; proportions shift to higher group values when weighted by amounts rather than lives, skewing the distribution towards males instead of females.

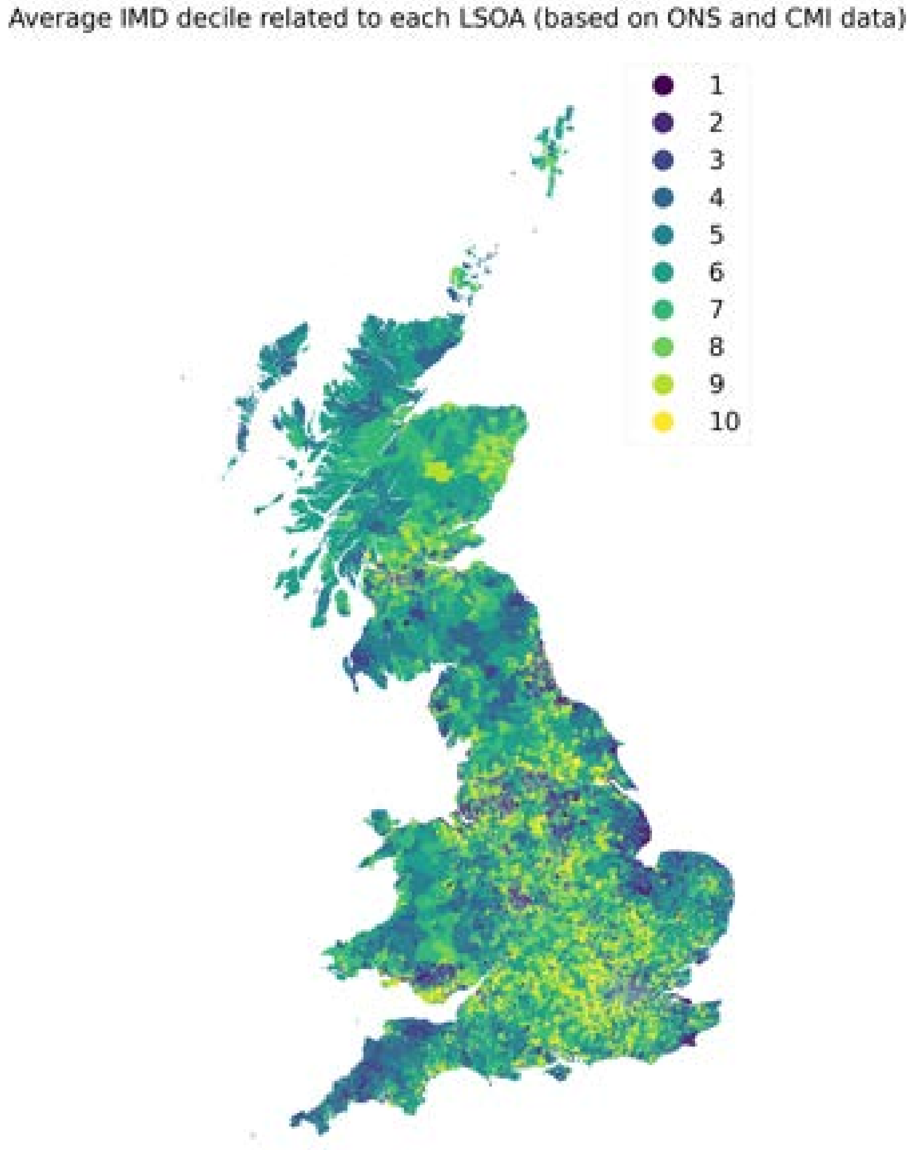


Figure 5.60: IMD value for each Scottish Data Zone and LSOA in England and Wales.

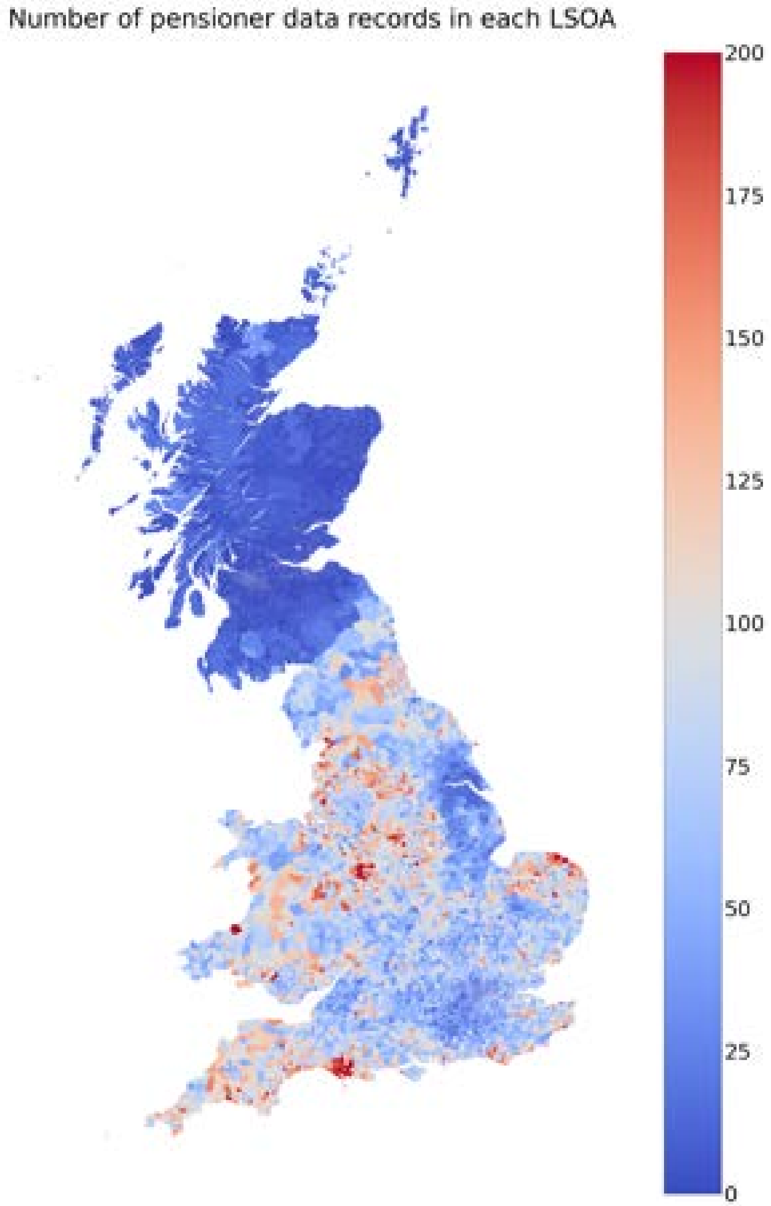


Figure 5.61: Number of pensioner members, between 1st April 2013 and 31st March 2020, by Scottish Data Zone and LSOA in England and Wales.

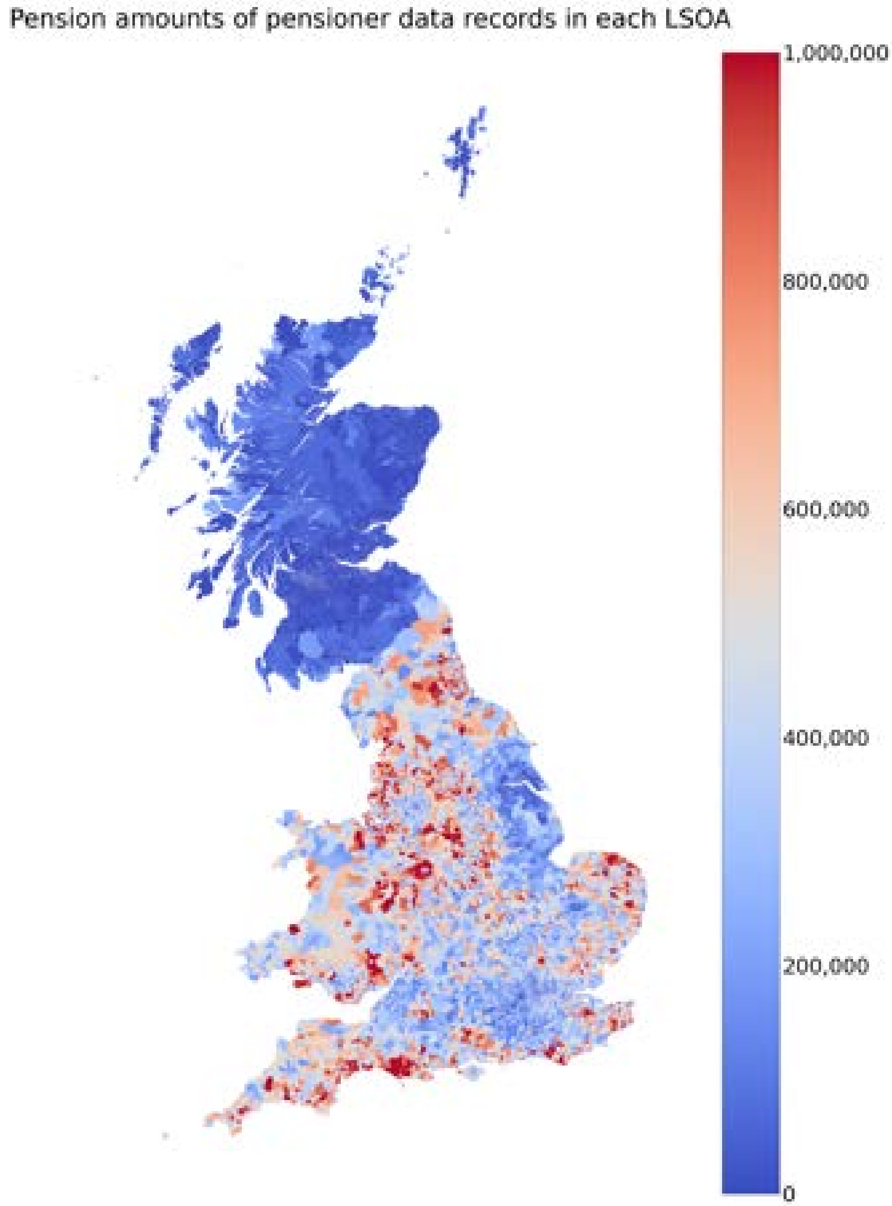


Figure 5.62: Pension amounts (in 2020 terms) of pensioner members, between 1st April 2013 and 31st March 2020, by Scottish Data Zone and LSOA in England and Wales.

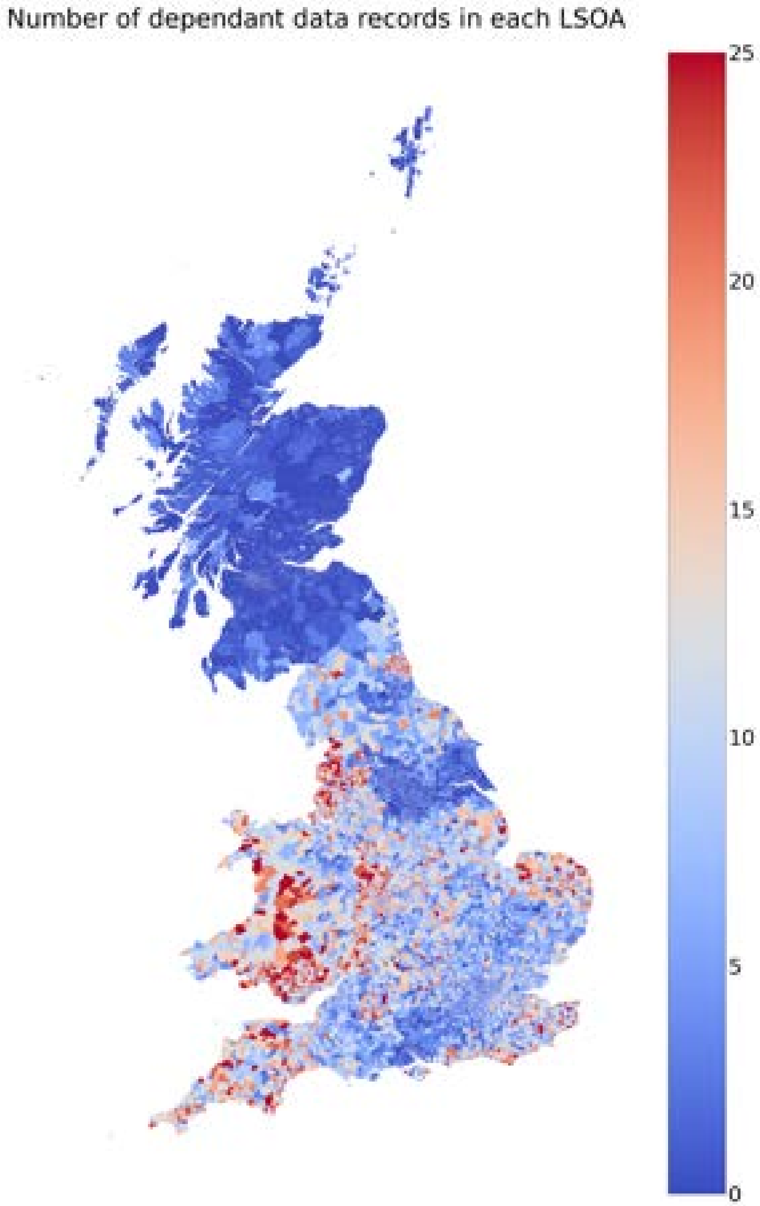


Figure 5.63: Number of dependant members, between 1st April 2013 and 31st March 2020, by Scottish Data Zone and LSOA in England and Wales.

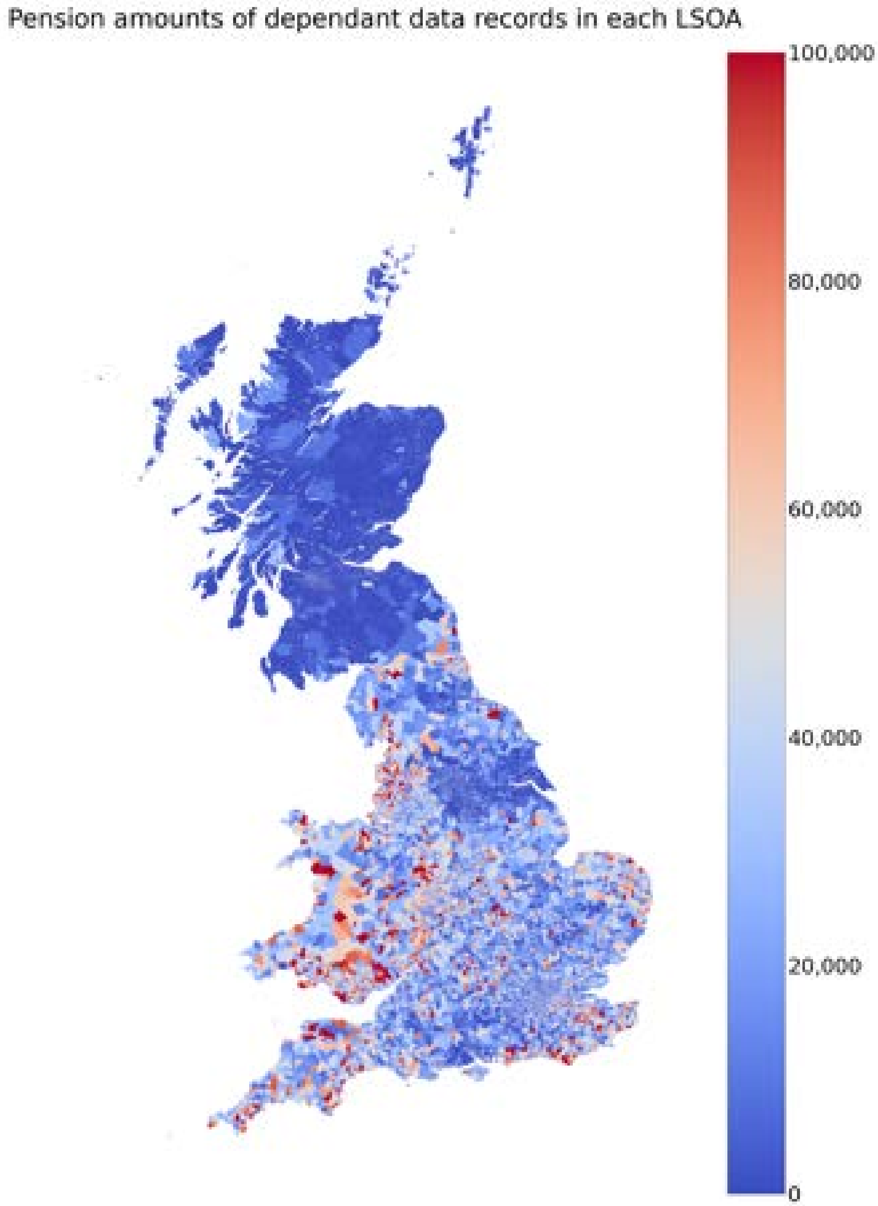


Figure 5.64: Pension amounts (in 2020 terms) of dependant members, between 1st April 2013 and 31st March 2020, by Scottish Data Zone and LSOA in England and Wales.

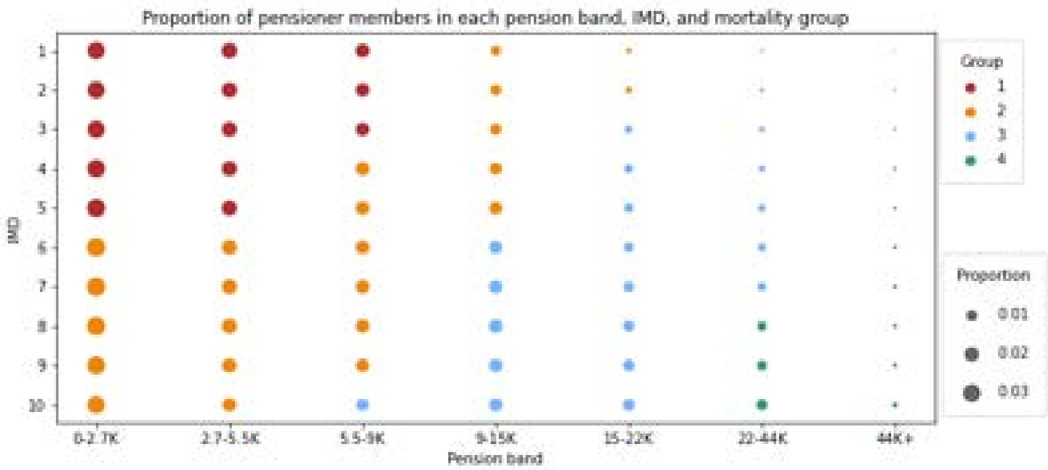


Figure 5.65: Proportion of male pensioner members, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

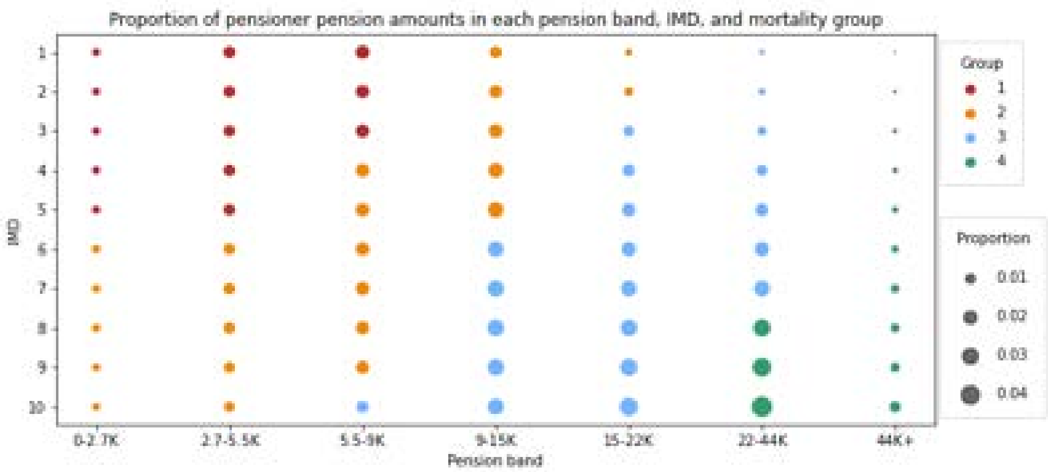


Figure 5.66: Proportion of pension amounts for male pensioners, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

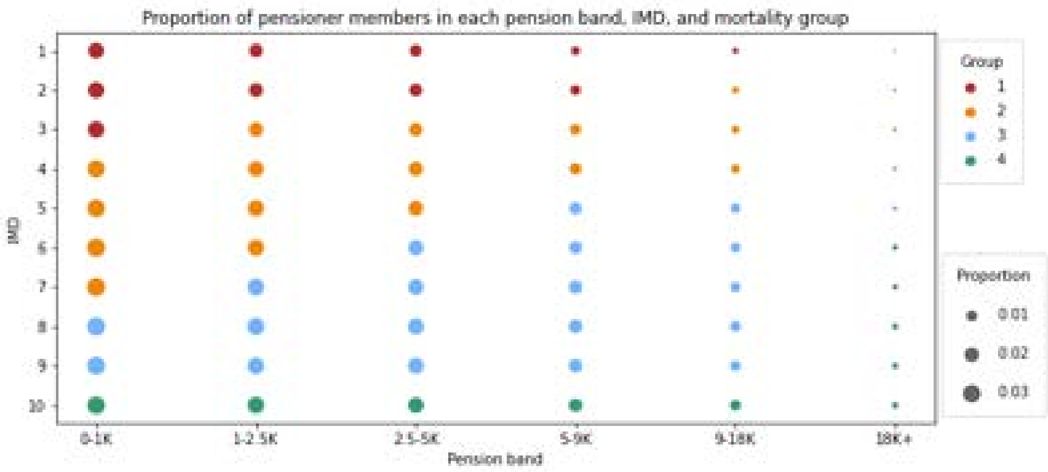


Figure 5.67: Proportion of female pensioner members, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

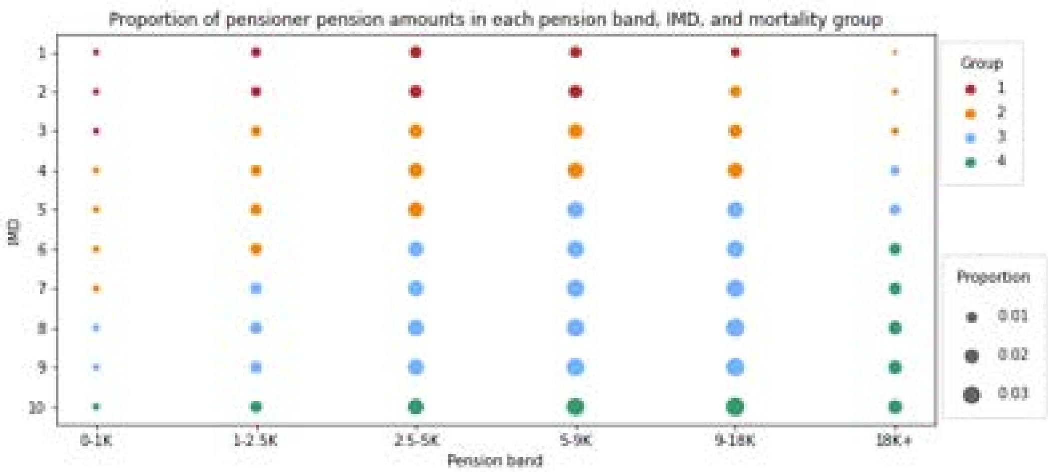


Figure 5.68: Proportion of pension amounts for female pensioners, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

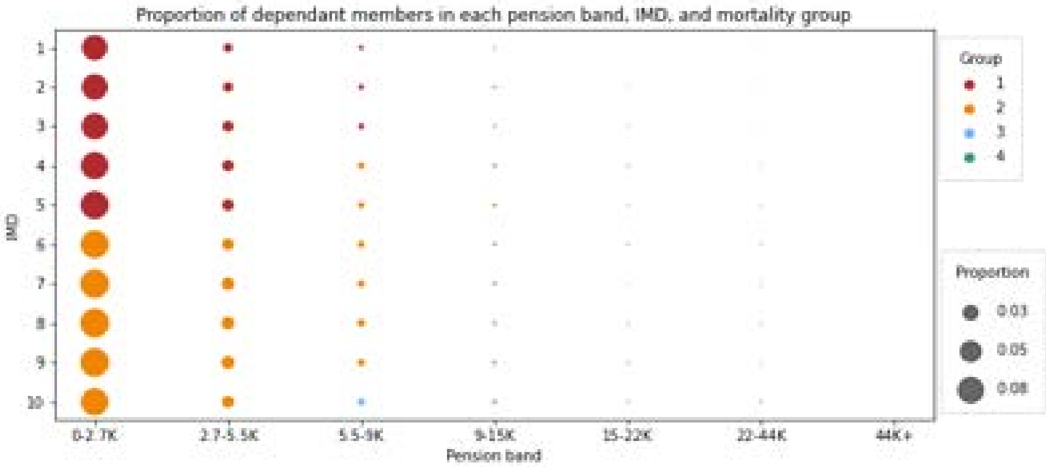


Figure 5.69: Proportion of male dependant members, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

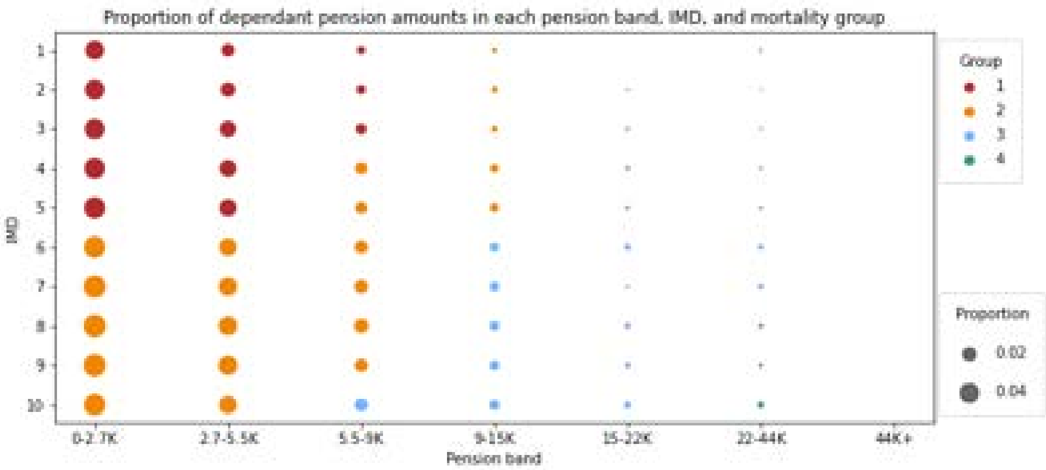


Figure 5.70: Proportion of pension amounts for male dependants, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

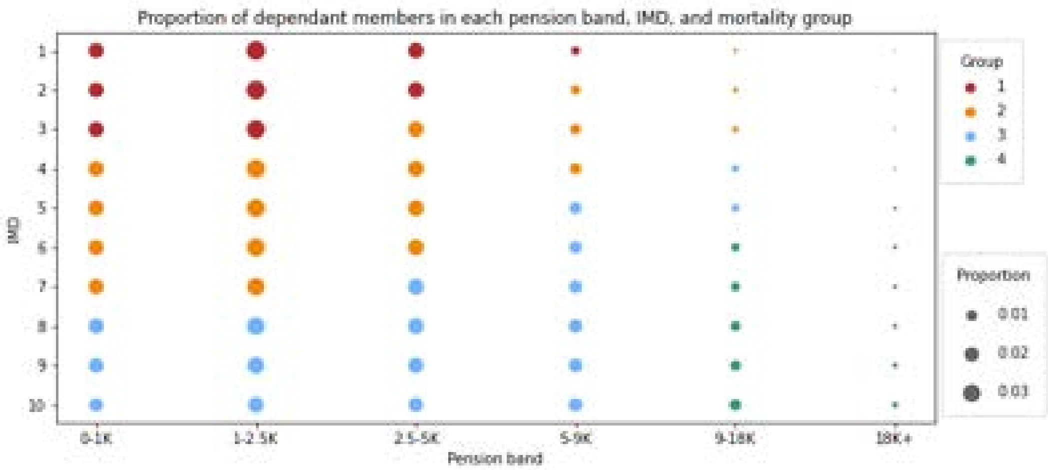


Figure 5.71: Proportion of female dependant members, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

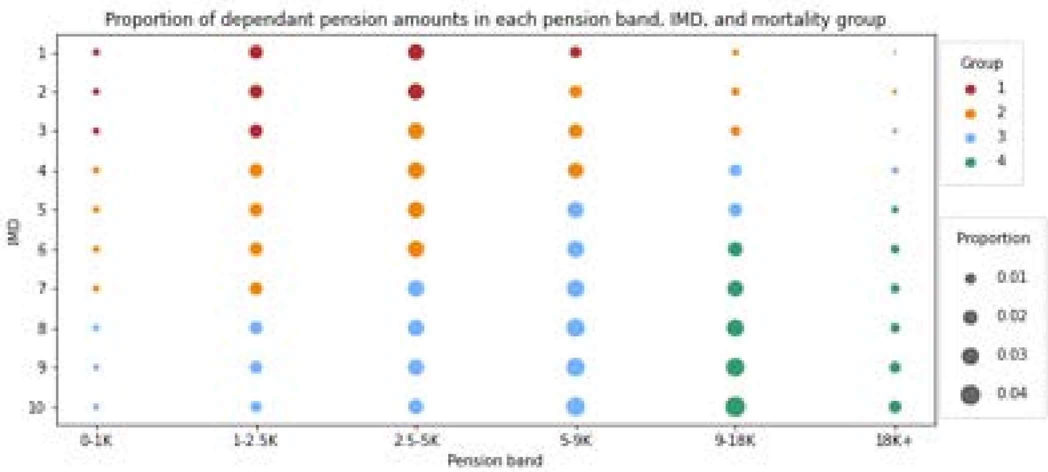


Figure 5.72: Proportion of pension amounts for female dependants, between 1st April 2013 and 31st March 2020, in each IMD, pension band and mortality group.

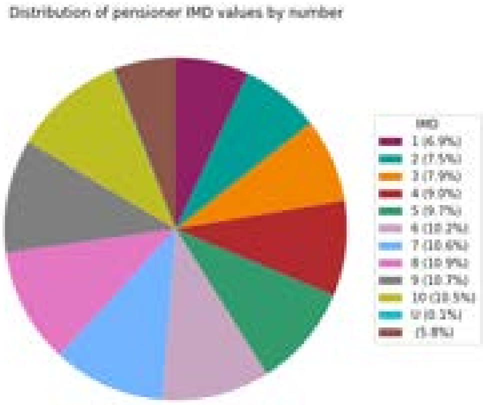


Figure 5.73: Proportion of all pensioner members in each IMD, between 1st April 2013 and 31st March 2020.

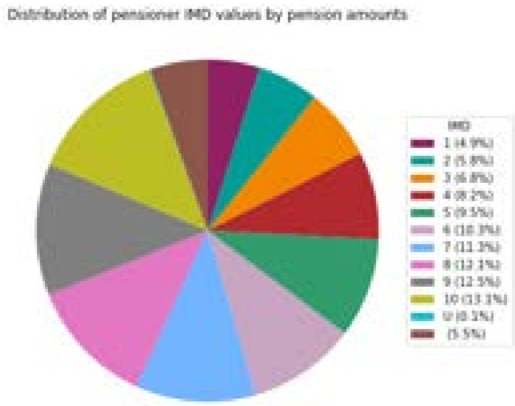


Figure 5.74: Proportion of pension amounts for all pensioners in each IMD, between 1st April 2013 and 31st March 2020.

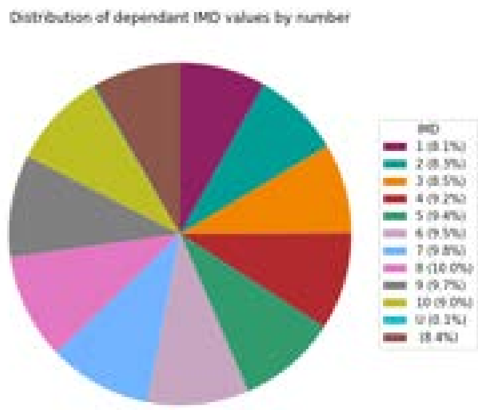


Figure 5.75: Proportion of all dependant members in each IMD, between 1st April 2013 and 31st March 2020.

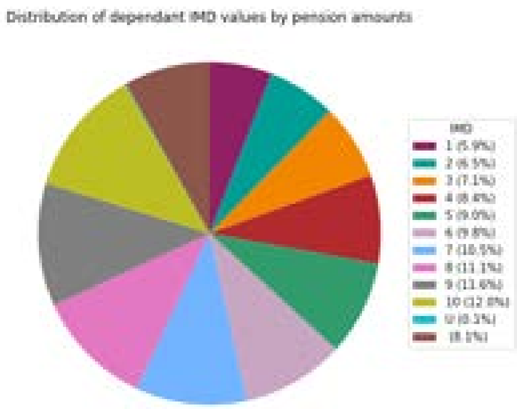


Figure 5.76: Proportion of pension amounts for all dependants in each IMD, between 1st April 2013 and 31st March 2020.

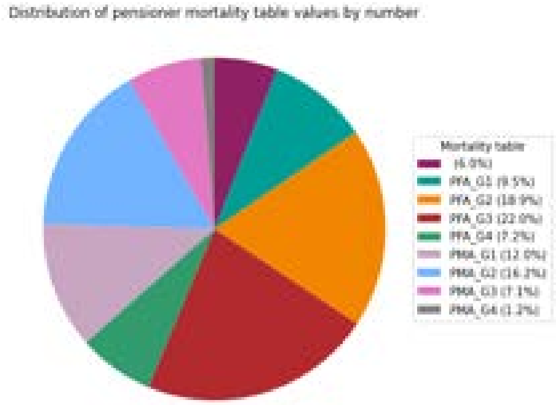


Figure 5.77: Proportion of all pensioner members in each SAPS S4 IMD mortality table, between 1st April 2013 and 31st March 2020.

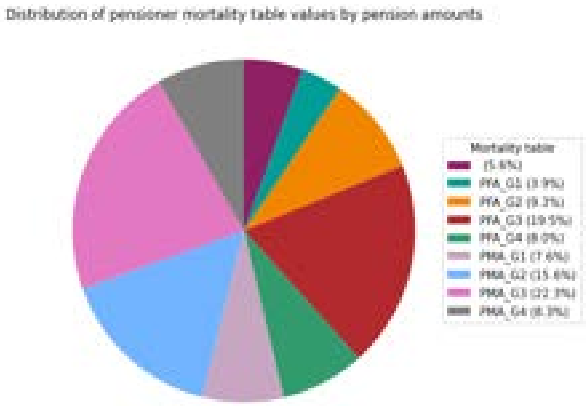


Figure 5.78: Proportion of ion amounts for all pensioners in each SAPS S4 IMD mortality table, between 1st April 2013 and 31st March 2020.

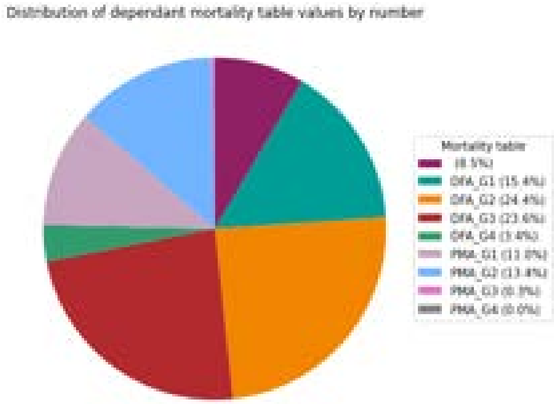


Figure 5.79: Proportion of all dependant members in each SAPS S4 IMD mortality table, between 1st April 2013 and 31st March 2020.

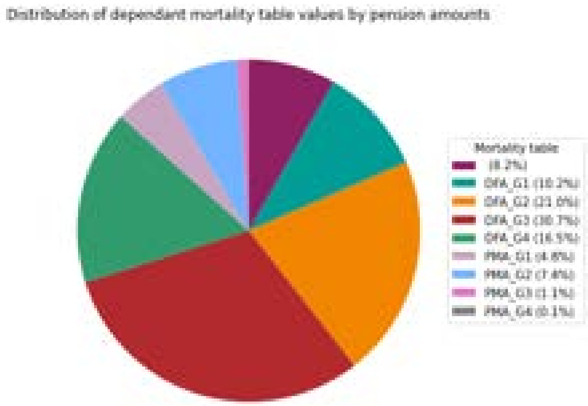


Figure 5.80: Proportion of ion amounts for all dependants in each SAPS S4 IMD mortality table, between 1st April 2013 and 31st March 2020.

5.2.3 IMD SAPS tables compared to baseline

This section further subdivides the categories analysed in Section 5.1 by mortality group. Table 5.82 illustrates optimal scaling factors required to align total expected and actual exits for each category; Table 5.83 presents the necessary optimal scaling factors for each category to minimise the least square residuals of actual and expected exit amounts across each age. The same S3 tables from Section 5.1 are applied to each group (i.e. G1-G4), whereas each category is scaled to best fit its relevant SAPS S4 IMD table in the S4 column, as depicted in Table 5.81.

Optimal scaling factors for S3 tables decrease as group number increases, supporting the proposition that higher IMD and pension combinations correlate to lower mortality rates; for normal health, optimal scaling factors are below one for the S4 table, indicating fewer deaths than suggested in the data used to derive the S4 mortality tables; on the other hand, ill health groups exhibit heavier mortality as expected, given the absence of S4 IMD-based tables that differentiate between normal and ill health. The optimal scaling factors for spouses are typically lower than one, indicating relatively light mortality compared to the underlying tables, with the exception of group 3 spouse males; this can possibly be attributed to random variation, resulting from lower volumes of data in this category. The ill health categories show a relatively large differential between the two optimisation metrics, especially for females. Other categories exhibit reasonably consistent results between the two metrics. The optimal scaling factors suggest that the males display slightly heavier mortality than the underlying tables for their female counterparts; this heaviness, relative to data used to derive the mortality tables, increases in accordance to group number.

Table 5.84 displays EtR, actual exits and expected exit amounts, based on optimally scaled SAPS S4 IMD and optimally scaled S3 mortality tables, without group specification from Table 5.22 in Section 5.1. Categories in groups 2 and 3, particularly in the latter, are the primary contributors to exposure and actual exits. Intuitively, expected exits using

Category	S4 IMD mortality table
N-health male G1	PMA_G1
N-health male G2	PMA_G2
N-health male G3	PMA_G3
N-health male G4	PMA_G4
N-health female G1	PFA_G1
N-health female G2	PFA_G2
N-health female G3	PFA_G3
N-health female G4	PFA_G4
I-health male G1	PMA_G1
I-health male G2	PMA_G2
I-health male G3	PMA_G3
I-health male G4	PMA_G4
I-health female G1	PFA_G1
I-health female G2	PFA_G2
I-health female G3	PFA_G3
I-health female G4	PFA_G4
Spouse male G1	PMA_G1
Spouse male G2	PMA_G2
Spouse male G3	PMA_G3
Spouse male G4	PMA_G4
Spouse female G1	DFA_G1
Spouse female G2	DFA_G2
Spouse female G3	DFA_G3
Spouse female G4	DFA_G4

Figure 5.81: Relevant S4 IMD table applicable to each category.

Category	S4	S3
N-health male G1	0.89	1.32
N-health male G2	0.88	1.07
N-health male G3	0.88	0.86
N-health male G4	0.90	0.73
N-health female G1	0.86	1.21
N-health female G2	0.86	1.02
N-health female G3	0.87	0.90
N-health female G4	0.89	0.82
I-health male G1	1.32	1.36
I-health male G2	1.46	1.20
I-health male G3	1.59	1.04
I-health male G4	1.87	0.98
I-health female G1	1.46	1.59
I-health female G2	1.59	1.44
I-health female G3	1.67	1.32
I-health female G4	1.94	1.34
Spouse male G1	0.96	1.08
Spouse male G2	1.00	0.90
Spouse male G3	1.17	0.73
Spouse male G4	0.00	0.00
Spouse female G1	0.96	1.21
Spouse female G2	0.94	1.03
Spouse female G3	0.94	0.93
Spouse female G4	0.88	0.79

Figure 5.82: Optimal scaling factors applicable to unscaled S4 IMD and S3 mortality base tables, with ONS 2021 and ONS 2018 improvements for each category, to equalise total expected and actual exit amounts.

optimal scaling factors from 5.22 generally underestimate actual exits at lower group numbers, overestimating them at higher group numbers; expected exits using S4 tables are scaled to their respective categories, albeit by least squares instead of total exits; unsurprisingly, they are close to the actual values. There were no actual exits for group 4 spouse males, resulting in the group-specific S4 scaling factor and subsequent expected exits being determined as zero; the S3 table scaling factors used are independent of the group, leading to a jagged and highly fluctuating expected exit line by age when applied to the limited exposure in this category.

Table 5.85 presents an F-test comparing S4 IMD table models with optimally scaled S3 tables without group specification. The DoF is calculated as the number of parameters in the model subtracted from the number of ages used. S3 tables utilise four parameters for males and five for females [7]. S4 IMD tables use four parameters, for both genders [9]. The RSS and DoF for both models are used to determine the F value and subsequent probabilities. Lower probability values indicate greater statistical difference between the two models, thereby emphasising the greater suitability of optimally scaled S4 IMD table models.

All normal health probabilities, except for group 3 females, are below the critical value of 0.05; this sufficiently rejects the null hypothesis, accepting the alternative hypothesis that the optimally scaled S4 IMD table model is a better fit; this is also true for the female spouse categories. As previously discussed, the S3 models usually underestimate actual exits in lower groups, overestimating them in higher groups; it is unsurprising that the significance level decreases in the middle groups (i.e. G2-G3) where these models more closely reflect the data. None of the ill health categories highlight a significant difference between S4 and S3 models at the 5% level; thus, the null hypothesis that both models are equally effective stands, with differences attributed to random variation. In fact, numerous probabilities for ill health categories exceed 0.5, some to a substantial degree; this proposes that S3 tables provide the better fit, as evidenced when S3 tables were specifically derived for ill health members, whereas underlying S4 tables were derived without distinguishing

Category	S4	S3
N-health male G1	0.89	1.31
N-health male G2	0.89	1.08
N-health male G3	0.89	0.87
N-health male G4	0.91	0.73
N-health female G1	0.87	1.19
N-health female G2	0.86	1.02
N-health female G3	0.87	0.90
N-health female G4	0.91	0.83
I-health male G1	1.25	1.36
I-health male G2	1.29	1.15
I-health male G3	1.32	0.94
I-health male G4	1.53	0.86
I-health female G1	1.26	1.45
I-health female G2	1.25	1.26
I-health female G3	1.25	1.12
I-health female G4	1.28	1.07
Spouse male G1	0.95	1.09
Spouse male G2	0.98	0.91
Spouse male G3	1.07	0.69
Spouse male G4	0.00	0.00
Spouse female G1	0.97	1.15
Spouse female G2	0.94	0.99
Spouse female G3	0.95	0.93
Spouse female G4	0.89	0.80

Figure 5.83: Optimal scaling factors applicable to unscaled S4 IMD and S3 mortality base tables, with ONS 2021 and ONS 2018 improvements for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

Category	EtR	Actual exits	Expected exits (S4)	Expected exits (S3)
N-health male G1	2,165,292,712	75,418,132	75,810,378	55,280,508
N-health male G2	5,129,995,801	134,791,775	136,214,304	122,265,248
N-health male G3	8,448,817,228	167,259,735	169,283,599	189,054,379
N-health male G4	3,403,340,379	50,396,682	50,926,023	67,264,793
N-health female G1	1,187,865,516	24,659,185	24,934,969	19,344,620
N-health female G2	3,092,108,778	49,861,202	50,064,643	46,533,444
N-health female G3	6,761,831,114	91,668,548	92,423,218	96,855,158
N-health female G4	2,930,782,762	30,510,877	31,168,357	35,273,071
I-health male G1	993,985,727	44,945,295	42,552,903	37,433,097
I-health male G2	1,353,162,139	53,595,584	47,591,267	50,347,247
I-health male G3	1,057,409,818	34,510,220	28,749,457	37,627,369
I-health male G4	174,108,771	4,925,708	4,032,374	5,664,197
I-health female G1	437,355,780	15,705,856	13,491,784	11,738,148
I-health female G2	730,207,411	23,315,519	18,241,393	19,221,786
I-health female G3	1,335,127,451	38,759,793	29,047,288	34,951,566
I-health female G4	363,699,089	9,785,071	6,427,322	8,716,900
Spouse male G1	127,896,461	4,448,134	4,386,547	4,022,713
Spouse male G2	199,724,974	5,429,712	5,336,499	5,908,237
Spouse male G3	29,176,955	408,361	374,434	545,230
Spouse male G4	1,572,485	0	0	24,085
Spouse female G1	313,005,875	18,334,767	18,446,822	14,298,783
Spouse female G2	632,942,047	35,037,505	34,967,909	31,913,879
Spouse female G3	930,687,157	51,243,799	51,571,644	51,636,225
Spouse female G4	493,868,929	23,961,212	24,173,646	28,498,389

Figure 5.84: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on optimally scaled SAPS S4 IMD and S3 mortality tables, without group specification.

between member type. Consequently, the improved shape of mortality rates with age, for S3 tables specific to ill health, outweighs the optimisation of scaling factors used on S4 IMD tables specific to normal health; the strong performance of S4 tables across this category illustrates how the underlying data is weighted towards normal health rather than members with ill health issues. Male spouses do not present any significant differences between the application of S4 and S3 models. However, this category has sparse data, to the extent of a complete absence of actual exits in group 4; Group 1 is close to the critical level, albeit with weaker significance in the middling groups; therefore, the lack of significance in these categories could be attributed to extenuating factors, such as poor data quantity.

On balance, S4 mortality tables and groups provide additional capability in differentiating mortality between data records. Further IMD-based tables distinguishing between normal and ill health information would plausibly improve performance for those with health issues. Normal health categories already perform well despite the absence of segregation.

Category	DoF (S4)	DoF (S3)	Scale (S4)	Scale (S3)	RSS (S4)	RSS (S3)	F	p-value
N-health male G1	53	53	0.89	0.97	895,020,897,799	13,270,951,706,196	14.83	0.00000
N-health male G2	56	56	0.89	0.97	2,990,363,890,376	6,601,563,620,853	2.21	0.00179
N-health male G3	51	51	0.89	0.97	5,755,628,333,613	19,692,391,275,970	3.42	0.00001
N-health male G4	47	47	0.91	0.97	1,145,999,475,389	10,967,243,124,514	9.57	0.00000
N-health female G1	55	54	0.87	0.95	250,961,369,144	975,464,003,077	3.89	0.00000
N-health female G2	55	54	0.86	0.95	455,304,261,460	739,012,371,100	1.62	0.03810
N-health female G3	58	57	0.87	0.95	1,268,932,176,326	1,787,301,658,094	1.41	0.09830
N-health female G4	58	57	0.91	0.95	445,620,199,989	1,303,478,719,800	2.93	0.00004
I-health male G1	74	74	1.25	1.13	1,148,436,438,540	2,612,297,921,231	2.27	0.00026
I-health male G2	76	76	1.29	1.13	3,486,685,716,242	1,069,880,219,596	0.31	1.00000
I-health male G3	73	73	1.32	1.13	2,882,504,644,741	2,271,245,035,158	0.79	0.84458
I-health male G4	61	61	1.53	1.13	202,606,473,505	212,209,258,026	1.05	0.42854
I-health female G1	75	74	1.26	1.19	413,620,576,795	540,207,806,408	1.31	0.12557
I-health female G2	84	83	1.25	1.19	1,491,384,685,670	826,120,382,671	0.55	0.99620
I-health female G3	81	80	1.25	1.19	4,938,722,978,958	2,544,798,443,536	0.52	0.99834
I-health female G4	72	71	1.28	1.19	708,171,168,136	468,495,817,969	0.66	0.95847
Spouse male G1	88	88	0.95	0.98	13,209,171,788	18,314,832,308	1.39	0.06362
Spouse male G2	84	84	0.98	0.98	17,619,753,459	17,058,264,795	0.97	0.55882
Spouse male G3	68	68	1.07	0.98	5,594,144,352	6,023,795,902	1.08	0.38059
Spouse male G4	-4	-4	0.00	0.98	0	26,947,323	0.00	NaN
Spouse female G1	97	96	0.97	0.94	48,108,876,395	603,892,206,798	12.55	0.00000
Spouse female G2	97	96	0.94	0.94	97,891,734,874	421,016,474,554	4.30	0.00000
Spouse female G3	90	89	0.95	0.94	318,638,824,037	361,058,343,552	1.13	0.27766
Spouse female G4	83	82	0.89	0.94	325,403,374,732	1,208,949,268,167	3.72	0.00000

Figure 5.85: F-test results comparing optimally scaled SAPS S4 IMD and S3 mortality tables for each category, without group specification.

Figures 5.86 to 5.89 illustrate the tendency of S4 IMD table models overestimating exits for normal health categories; this may be attributed to how underlying mortality tables are derived from an analysis of amalgamated health data, with those in ill health more frequently exiting at a young age; conversely, the model usually significantly underestimates these exits, which could again be a consequence of the greater proportion of normal health records underlying the S4 IMD-based mortality tables. Despite applying the optimal scaling factors, the fit at these lower ages appears less accurate than stated in S3 tables, which are not optimally scaled to the specific mortality group. The ill health effect diminishes at higher ages, resulting in closer resemblance between S3 and S4 tables in normal and ill health categories. The S4 tables seem to fit the spouse data well; this is particularly the case for females where the data is more plentiful. These patterns remain fairly consistent across mortality groups G1 to G4. The S3 tables underestimate actual exits in the G1 group and overestimate them in the G4 group, fitting more closely in the middle groups, particularly G2; this logical pattern of underestimation to overestimation, increasing with group number, highlights the successful differentiation of subdivision between mortality groups.

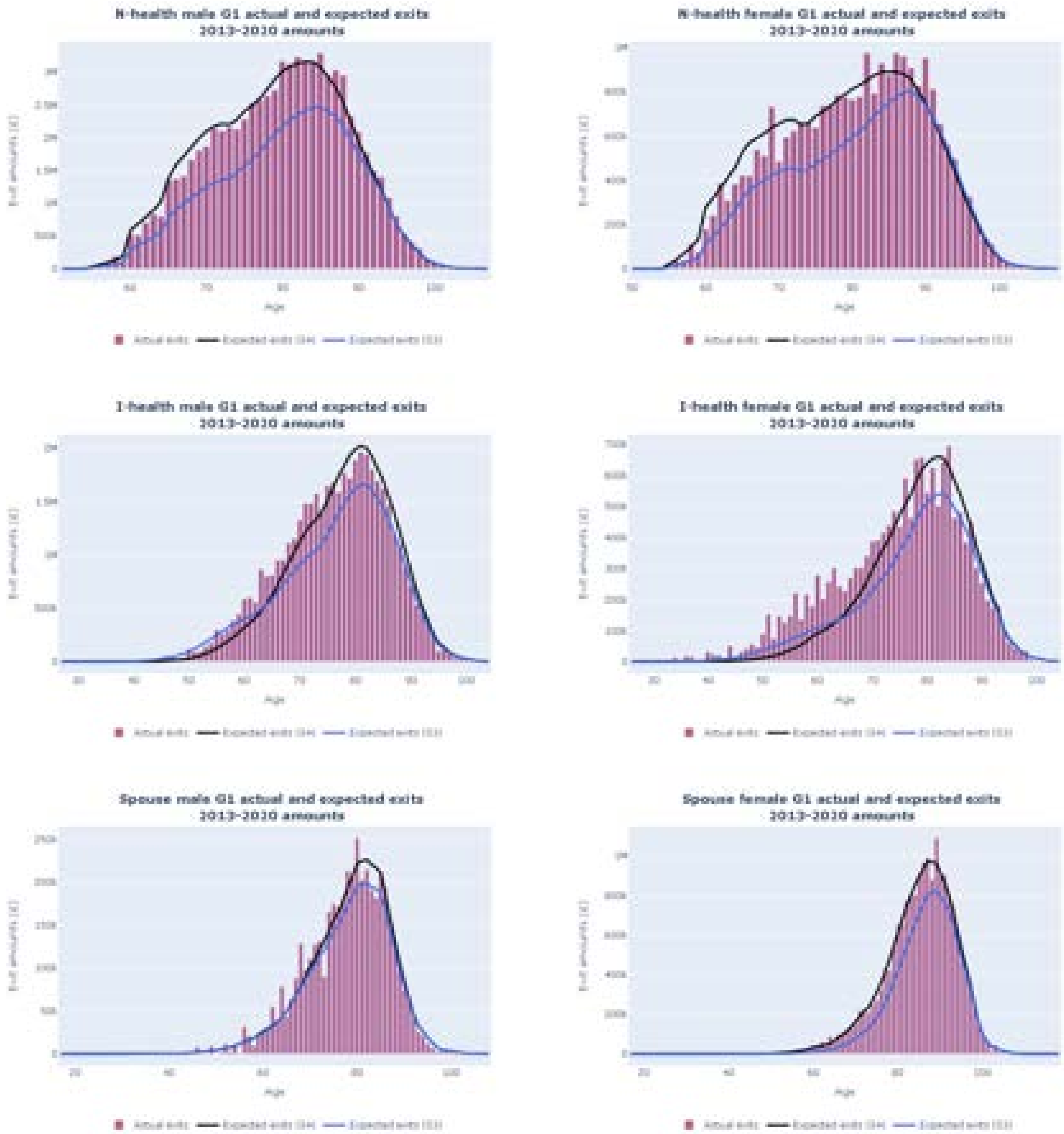


Figure 5.86: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to relevant S4 IMD and S3 CMI tables.

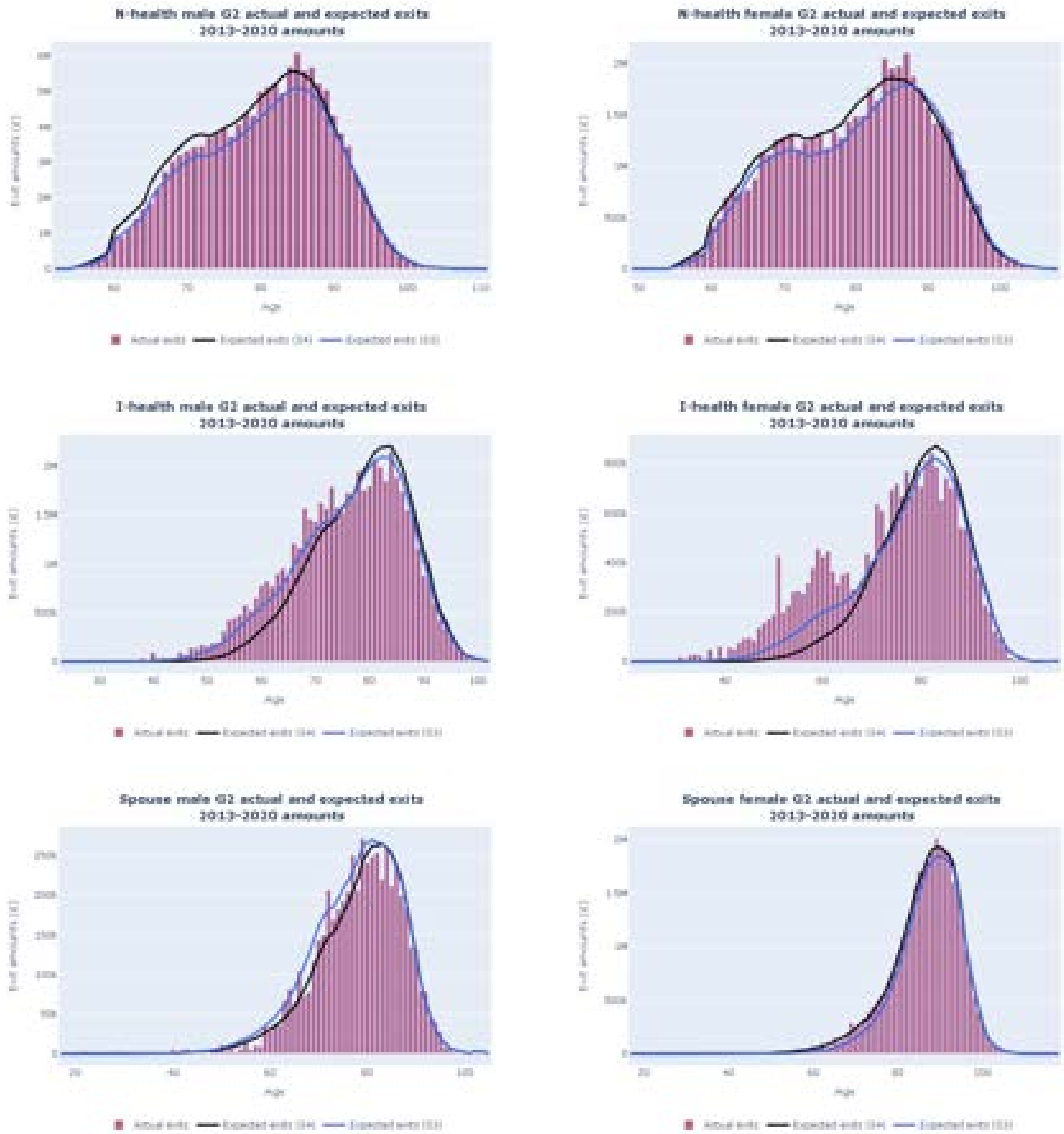


Figure 5.87: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to relevant S4 IMD and S3 CMI tables.

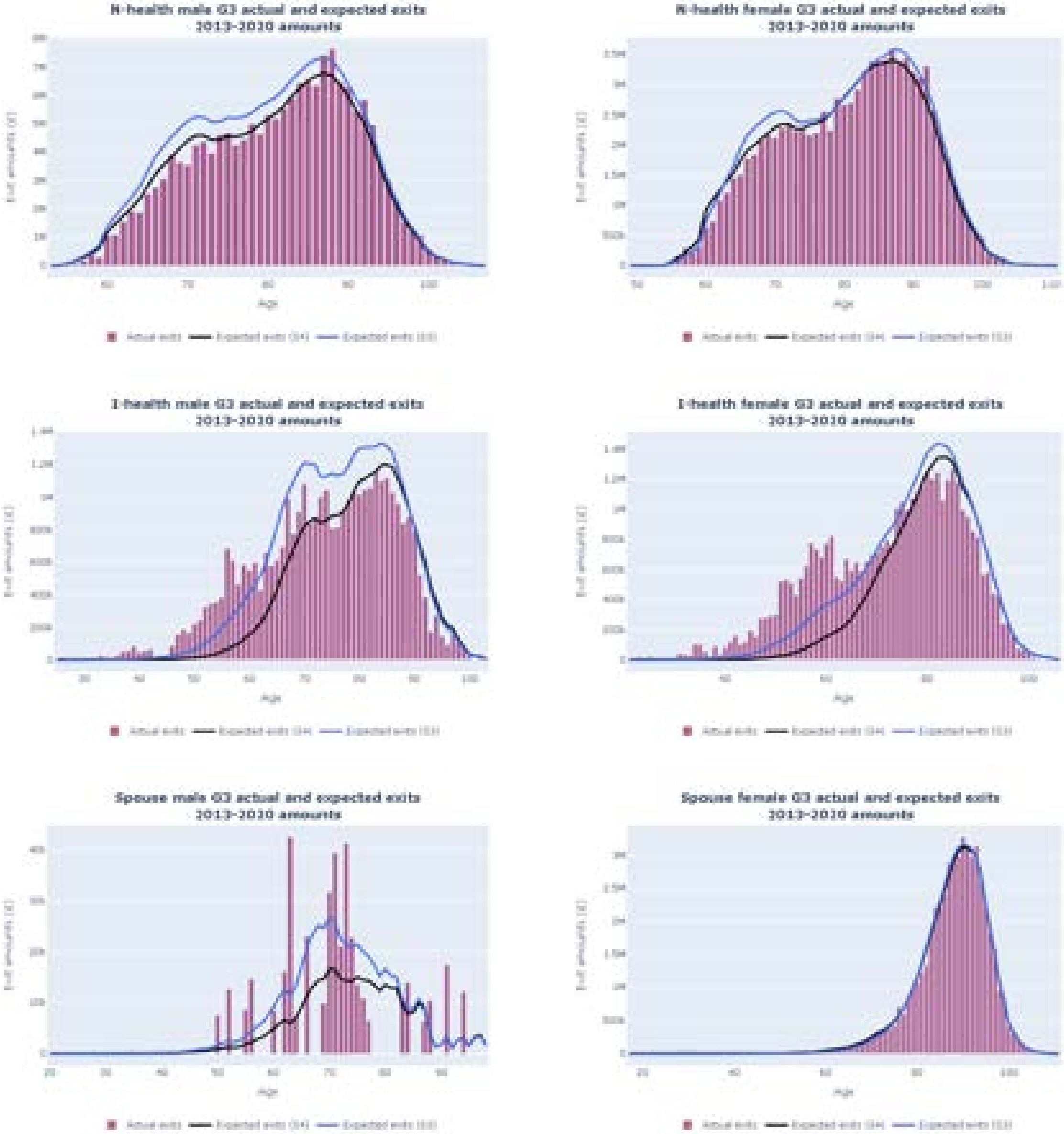


Figure 5.88: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to relevant S4 IMD and S3 CMI tables.

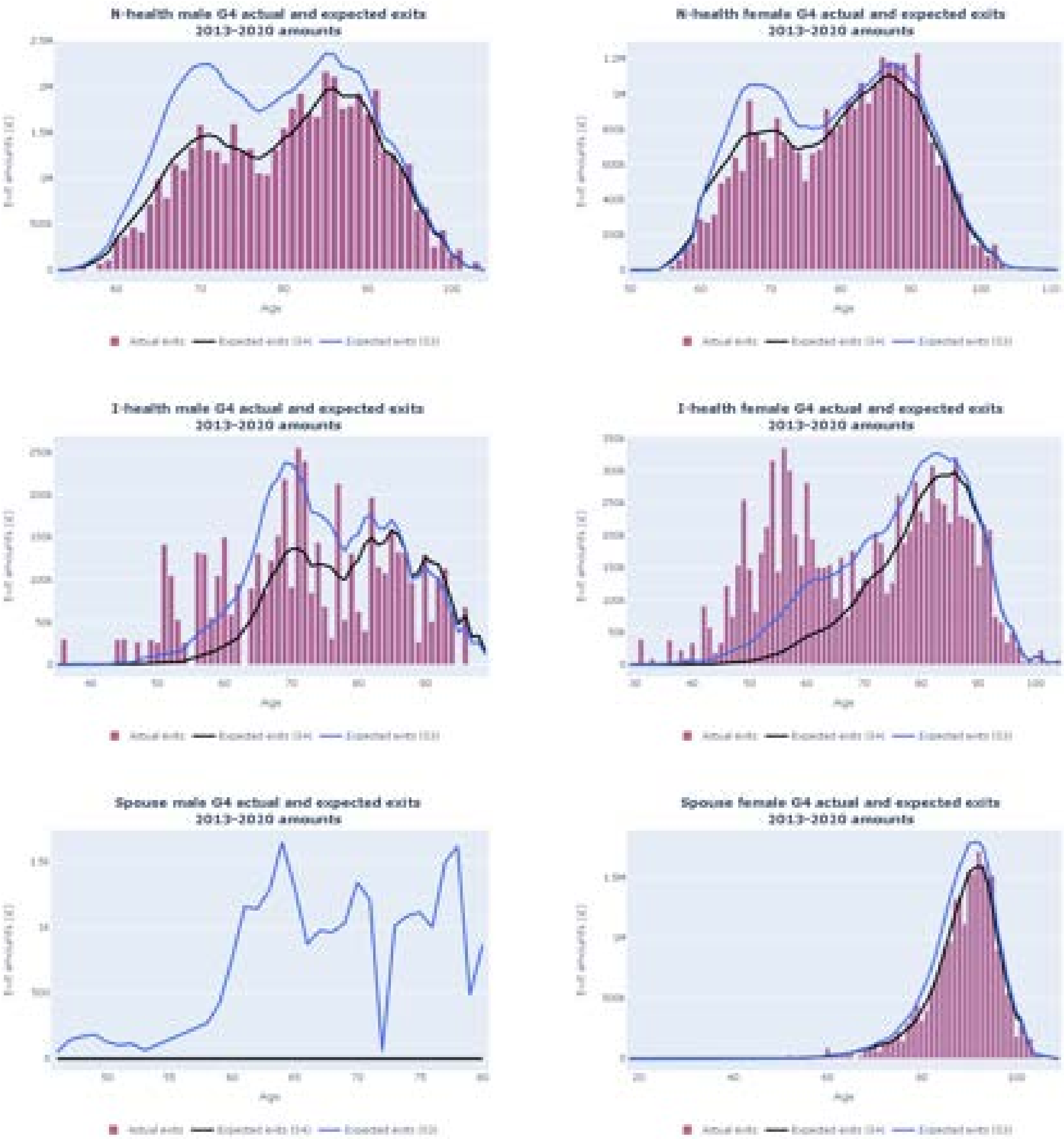


Figure 5.89: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts when optimal scaling factors are applied to relevant S4 IMD and S3 CMI tables.

5.3 Regional analysis

This section further subdivides the categories analysed in Section 5.1 by region. The CMI mapping tool associates UK postcodes with the Isle of Man, Channel Islands or NUTS 1 regions of the UK, which include Northern Ireland, Scotland, Wales and nine English regions [70]. Regions outside England and Wales, in addition to records without a corresponding postcode or regional value in the CMI mapping tool, were consolidated into a single more substantial category to offset the low volumes of data.

Table 5.90 displays each category's exits and exposure on a lives basis. Females generally have higher exposure than males across all regions; in the spouse categories, this is typically accompanied by higher deaths. Normal health categories depict greater death and exposure values than those with ill health, regardless of gender, with spouses attaining the lowest values. The most prominent death and exposure values are located in the North West (NW), South East (SE) and Yorkshire and the Humber (YH); the weakest values are observed in the North East (NE) and Wales (WL). The South East exhibits the highest exposure values for males and females of normal health, whereas Wales maintains some of the lowest. For members with ill health, the North West shows greater exposure for men and women; for spouses, the North West and South East obtain the highest death and exposure values. Although categories with unassigned regions or regions outside England and Wales have less data than those with assigned regions, they follow a similar pattern regarding their order of magnitude for exposure and death.

Table 5.91 displays the equivalent exposure and actual exits in 2020 pension terms, alongside additional expected exits, using the 2013, 2016 and 2020 valuation assumptions for each category. This data is distinctly weighted towards categories with male members, including males of normal and ill health as well as female spouses; this significantly contrasts with the dominant female exposure witnessed on a purely lives basis. Nevertheless, general trends across regions remain largely consistent with the lives table.

Expected exits for each region consistently increase with each valuation, indicating

Category	d_x	E_x
N-health male EE	6,816	237,160
N-health male EM	5,249	201,101
N-health male LN	3,917	137,906
N-health male NE	3,754	143,469
N-health male NW	9,660	334,890
N-health male SE	9,699	345,192
N-health male SW	6,939	269,539
N-health male WL	3,516	132,748
N-health male WM	7,452	271,096
N-health male YH	6,223	234,502
N-health male other	4,983	143,139
N-health female EE	6,272	402,234
N-health female EM	5,411	346,725
N-health female LN	3,728	211,176
N-health female NE	3,061	190,969
N-health female NW	9,063	528,975
N-health female SE	9,393	590,466
N-health female SW	5,969	430,892
N-health female WL	3,138	189,916
N-health female WM	7,408	455,386
N-health female YH	6,468	388,198
N-health female other	4,161	205,003
I-health male EE	1,668	38,553
I-health male EM	1,760	37,839
I-health male LN	936	23,196
I-health male NE	1,907	46,826
I-health male NW	4,504	104,051
I-health male SE	2,203	46,630
I-health male SW	1,900	42,671
I-health male WL	1,873	43,855
I-health male WM	2,337	55,205
I-health male YH	2,615	58,667
I-health male other	1,412	32,200
I-health female EE	1,508	49,310
I-health female EM	1,731	50,259
I-health female LN	985	34,713
I-health female NE	1,604	50,529
I-health female NW	4,055	121,370
I-health female SE	2,204	59,135
I-health female SW	1,523	49,820
I-health female WL	1,583	57,907
I-health female WM	2,262	69,925
I-health female YH	2,581	75,884
I-health female other	1,244	39,157
Spouse male EE	651	17,569
Spouse male EM	915	20,411
Spouse male LN	501	12,528
Spouse male NE	484	11,997
Spouse male NW	1,727	39,941
Spouse male SE	1,124	29,489
Spouse male SW	749	21,931
Spouse male WL	698	16,705
Spouse male WM	1,300	31,865
Spouse male YH	183	4,838
Spouse male other	688	16,994
Spouse female EE	3,569	58,346
Spouse female EM	3,057	53,314
Spouse female LN	2,344	44,819
Spouse female NE	2,137	37,733
Spouse female NW	7,023	117,675
Spouse female SE	5,447	92,638
Spouse female SW	4,293	75,788
Spouse female WL	3,351	57,428
Spouse female WM	5,087	90,625
Spouse female YH	924	17,090
Spouse female other	3,285	55,713

Figure 5.90: Actual number of exits (d_x) and exact initial EtR, from 1st April 2013 to 31st March 2020 for each category, after excluding regions outside England and Wales.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
N-health male EE	1,997,688,628	46,856,780	50,220,389	48,177,779	48,030,999
N-health male EM	1,552,812,031	33,762,133	33,761,324	32,293,722	32,033,380
N-health male LN	1,240,073,945	27,281,229	29,167,149	27,933,980	27,933,414
N-health male NE	1,188,811,795	24,333,602	23,498,773	22,417,405	22,376,445
N-health male NW	2,748,242,825	64,810,270	61,159,922	58,503,196	57,896,569
N-health male SE	2,984,249,914	68,493,413	76,850,288	73,734,965	73,268,459
N-health male SW	2,166,529,517	48,526,142	55,133,453	52,860,207	52,510,052
N-health male WL	1,171,403,414	25,217,790	25,852,850	24,723,193	24,781,273
N-health male WM	2,171,847,807	47,725,551	46,924,944	44,874,343	44,715,658
N-health male YH	1,818,415,770	38,122,046	37,843,358	36,168,827	35,714,710
N-health male other	1,036,206,659	33,282,694	32,174,378	30,925,167	31,335,005
N-health female EE	1,490,949,414	21,492,273	22,459,351	21,586,749	21,555,431
N-health female EM	1,187,941,633	15,839,684	15,715,906	15,172,967	15,124,726
N-health female LN	1,107,608,572	16,461,500	17,884,629	17,169,493	17,139,509
N-health female NE	734,924,488	9,578,978	9,314,976	9,060,787	9,084,408
N-health female NW	2,016,281,032	28,817,114	27,319,610	26,416,603	26,207,125
N-health female SE	2,267,860,757	32,553,754	35,254,967	33,788,273	33,594,943
N-health female SW	1,446,991,080	19,177,542	21,441,387	20,575,409	20,467,435
N-health female WL	730,139,377	10,373,815	10,304,069	9,932,128	9,964,927
N-health female WM	1,627,075,386	22,830,838	21,994,039	21,279,715	21,268,734
N-health female YH	1,277,143,040	18,260,852	17,063,453	16,495,355	16,342,819
N-health female other	690,924,524	14,685,139	13,588,818	12,956,822	13,066,002
I-health male EE	295,352,274	10,729,180	11,210,132	10,487,608	9,847,829
I-health male EM	261,811,844	10,919,406	10,497,096	9,868,676	9,199,176
I-health male LN	178,985,406	6,627,870	6,608,313	6,168,336	5,800,178
I-health male NE	303,384,275	11,221,582	10,826,294	10,081,357	9,462,957
I-health male NW	718,797,876	27,805,894	26,611,677	24,896,140	23,156,554
I-health male SE	371,881,177	14,701,782	15,715,983	14,735,526	13,743,431
I-health male SW	318,176,987	12,242,397	13,315,844	12,480,956	11,641,323
I-health male WL	308,249,496	11,618,306	11,202,757	10,480,974	9,895,725
I-health male WM	396,494,659	15,006,456	14,768,479	13,889,847	13,040,017
I-health male YH	401,633,445	16,313,809	14,132,121	13,240,508	12,342,927
I-health male other	224,221,377	8,911,126	10,048,441	9,385,432	8,891,970
I-health female EE	248,162,768	6,968,350	6,638,335	5,894,637	5,654,247
I-health female EM	226,264,146	7,526,672	6,565,180	5,816,990	5,552,227
I-health female LN	191,446,755	5,109,600	5,659,343	4,992,195	4,789,186
I-health female NE	206,828,818	6,156,015	5,962,282	5,310,817	5,154,804
I-health female NW	539,298,843	17,360,502	16,274,479	14,446,869	13,837,062
I-health female SE	315,118,013	10,664,120	10,017,900	8,794,061	8,353,081
I-health female SW	246,148,647	6,967,293	7,178,146	6,340,411	6,046,658
I-health female WL	245,798,198	6,369,419	6,781,998	6,014,944	5,801,430
I-health female WM	315,084,977	9,675,154	8,946,442	7,926,126	7,610,694
I-health female YH	309,386,727	10,280,774	8,749,125	7,786,260	7,450,818
I-health female other	183,450,163	5,977,399	6,047,153	5,352,805	5,268,007
Spouse male EE	29,909,014	802,649	873,512	820,610	757,784
Spouse male EM	33,682,652	941,886	945,780	890,164	808,692
Spouse male LN	26,865,349	703,910	887,883	862,851	796,606
Spouse male NE	19,952,052	519,630	510,376	468,660	434,646
Spouse male NW	68,522,151	2,172,683	1,944,078	1,828,247	1,667,094
Spouse male SE	51,437,461	1,462,555	1,515,055	1,441,157	1,328,187
Spouse male SW	34,258,998	873,528	943,701	883,919	816,910
Spouse male WL	31,410,798	942,731	924,118	874,533	808,616
Spouse male WM	52,762,930	1,625,418	1,489,134	1,399,147	1,287,029
Spouse male YH	7,654,897	202,328	205,394	190,201	173,098
Spouse male other	27,848,386	888,025	845,176	806,721	760,066
Spouse female EE	220,738,530	12,639,934	13,219,895	12,649,690	11,705,843
Spouse female EM	189,256,367	9,771,968	9,634,197	9,206,232	8,404,907
Spouse female LN	174,158,674	8,375,506	9,192,017	8,784,030	8,144,565
Spouse female NE	119,649,322	6,402,709	5,315,996	5,050,891	4,720,148
Spouse female NW	406,178,808	22,384,560	20,801,023	19,867,826	18,252,426
Spouse female SE	368,677,175	20,632,282	23,128,473	22,200,784	20,438,449
Spouse female SW	285,867,314	15,892,532	17,617,275	16,901,344	15,603,711
Spouse female WL	215,132,294	11,577,004	11,464,189	10,947,970	10,168,024
Spouse female WM	313,704,932	16,900,511	15,986,029	15,257,777	14,149,296
Spouse female YH	63,257,167	3,160,869	3,193,827	3,058,380	2,796,462
Spouse female other	194,486,014	10,743,398	11,026,921	10,539,998	9,980,917

Figure 5.91: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

diminishing prudence within the assumptions over time; as a whole, the assumptions are reasonably similar to actual exits, with notable exceptions in the South East and South West for those with normal health, particularly men, where actual exits substantially fall short of expectations.

Table 5.92 presents average exit ages weighted by pension amounts across all categories, highlighting comparisons with assumptions set at the 2013, 2016 and 2020 pension scheme valuations.

The actual mean exit ages tend to be higher than their assumed counterparts across the majority of regions for normal health categories, with the reverse holding true in ill health and spousal categories. The discrepancy between the actual and expected values is strikingly high in numerous regions for females with health issues. The actual mean exit age is usually higher for males than females with ill health and lower for spouses; a pattern for those of normal health cannot be established.

Table 5.93 reveals a similar trend of decline in the ratio of actual to expected exit values from the 2013 to 2020 assumptions across most categories, indicating a gradual increase to assumed exits over time; this expectation of heavier mortality is less prudent since it projects fewer pension payments and, consequently, a lower remaining liability for the pension scheme.

The East of England, South East, South West and London typically display the lowest ratio of actual to expected exits, whereas the highest ratios are located in the North East, North West, West Midlands and Yorkshire and the Humber. These results potentially impact mortality depending on the location of the region, such as the north or south of the UK.

Figures 5.94 to 5.104 juxtapose actual and expected exits under each valuation assumption; these illustrate that black 2020 assumption lines tend to be higher than blue 2016 assumption lines, which subsequently exceed red 2013 assumption lines. This contrast is more pronounced at older ages for females of normal and ill health, as well as males with health issues. For male spouses, the effect is more conspicuous at younger

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
N-health male EE	82.05	81.20	81.37	81.23
N-health male EM	80.83	79.91	80.10	79.94
N-health male LN	80.92	80.73	80.92	80.78
N-health male NE	79.70	79.08	79.31	79.16
N-health male NW	80.38	80.27	80.47	80.33
N-health male SE	82.65	81.54	81.70	81.56
N-health male SW	82.41	81.50	81.66	81.51
N-health male WL	80.12	80.06	80.25	80.11
N-health male WM	80.28	79.78	79.98	79.84
N-health male YH	79.68	79.47	79.68	79.52
N-health male other	81.07	83.03	83.16	83.06
N-health female EE	81.18	80.97	80.56	80.15
N-health female EM	80.22	79.86	79.43	79.01
N-health female LN	81.95	81.87	81.40	80.99
N-health female NE	79.94	79.85	79.28	78.90
N-health female NW	80.38	80.35	79.86	79.44
N-health female SE	82.06	81.46	81.03	80.60
N-health female SW	81.89	81.16	80.71	80.27
N-health female WL	81.24	80.72	80.23	79.82
N-health female WM	80.37	80.27	79.76	79.35
N-health female YH	80.70	80.35	79.84	79.41
N-health female other	79.82	83.46	83.01	82.64
I-health male EE	75.41	77.11	77.25	76.97
I-health male EM	75.36	77.76	77.81	77.50
I-health male LN	74.21	76.68	76.88	76.63
I-health male NE	74.69	76.07	76.36	76.14
I-health male NW	75.83	76.82	76.99	76.72
I-health male SE	76.72	78.57	78.57	78.23
I-health male SW	76.88	78.35	78.38	78.06
I-health male WL	75.30	76.55	76.70	76.44
I-health male WM	75.49	76.83	76.93	76.66
I-health male YH	74.34	75.99	76.24	75.99
I-health male other	76.97	79.09	79.09	78.81
I-health female EE	71.87	77.08	76.93	77.14
I-health female EM	72.84	78.06	77.88	77.99
I-health female LN	73.51	78.29	78.08	78.18
I-health female NE	73.99	77.79	77.68	77.86
I-health female NW	74.68	78.40	78.25	78.35
I-health female SE	73.64	79.45	79.15	79.13
I-health female SW	73.64	78.32	78.08	78.14
I-health female WL	73.63	77.47	77.28	77.46
I-health female WM	73.27	77.68	77.53	77.70
I-health female YH	72.53	77.72	77.57	77.70
I-health female other	72.79	79.53	79.24	79.24
Spouse male EE	78.53	78.06	79.77	79.60
Spouse male EM	78.35	78.07	79.92	79.71
Spouse male LN	78.28	79.51	81.12	80.95
Spouse male NE	77.03	76.88	78.74	78.56
Spouse male NW	77.74	78.01	79.77	79.58
Spouse male SE	79.37	78.57	80.41	80.23
Spouse male SW	79.52	77.85	79.73	79.55
Spouse male WL	78.17	78.31	80.06	79.90
Spouse male WM	77.59	77.91	79.68	79.48
Spouse male YH	75.86	77.43	79.24	79.07
Spouse male other	77.21	79.05	81.06	80.89
Spouse female EE	88.35	88.65	88.83	88.52
Spouse female EM	87.30	87.87	88.05	87.66
Spouse female LN	87.20	88.07	88.24	87.85
Spouse female NE	86.00	86.88	87.04	86.56
Spouse female NW	86.94	87.89	88.07	87.67
Spouse female SE	88.77	89.12	89.30	89.01
Spouse female SW	88.76	89.04	89.23	88.94
Spouse female WL	87.46	88.03	88.20	87.82
Spouse female WM	86.98	87.90	88.07	87.67
Spouse female YH	87.69	88.16	88.34	87.94
Spouse female other	87.71	88.69	88.89	88.55

Figure 5.92: Actual mean exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected mean exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
N-health male EE	0.93	0.97	0.98
N-health male EM	1.00	1.05	1.05
N-health male LN	0.94	0.98	0.98
N-health male NE	1.04	1.09	1.09
N-health male NW	1.06	1.11	1.12
N-health male SE	0.89	0.93	0.93
N-health male SW	0.88	0.92	0.92
N-health male WL	0.98	1.02	1.02
N-health male WM	1.02	1.06	1.07
N-health male YH	1.01	1.05	1.07
N-health male other	1.03	1.08	1.06
N-health female EE	0.96	1.00	1.00
N-health female EM	1.01	1.04	1.05
N-health female LN	0.92	0.96	0.96
N-health female NE	1.03	1.06	1.05
N-health female NW	1.05	1.09	1.10
N-health female SE	0.92	0.96	0.97
N-health female SW	0.89	0.93	0.94
N-health female WL	1.01	1.04	1.04
N-health female WM	1.04	1.07	1.07
N-health female YH	1.07	1.11	1.12
N-health female other	1.08	1.13	1.12
I-health male EE	0.96	1.02	1.09
I-health male EM	1.04	1.11	1.19
I-health male LN	1.00	1.07	1.14
I-health male NE	1.04	1.11	1.19
I-health male NW	1.04	1.12	1.20
I-health male SE	0.94	1.00	1.07
I-health male SW	0.92	0.98	1.05
I-health male WL	1.04	1.11	1.17
I-health male WM	1.02	1.08	1.15
I-health male YH	1.15	1.23	1.32
I-health male other	0.89	0.95	1.00
I-health female EE	1.05	1.18	1.23
I-health female EM	1.15	1.29	1.36
I-health female LN	0.90	1.02	1.07
I-health female NE	1.03	1.16	1.19
I-health female NW	1.07	1.20	1.25
I-health female SE	1.06	1.21	1.28
I-health female SW	0.97	1.10	1.15
I-health female WL	0.94	1.06	1.10
I-health female WM	1.08	1.22	1.27
I-health female YH	1.18	1.32	1.38
I-health female other	0.99	1.12	1.13
Spouse male EE	0.92	0.98	1.06
Spouse male EM	1.00	1.06	1.16
Spouse male LN	0.79	0.82	0.88
Spouse male NE	1.02	1.11	1.20
Spouse male NW	1.12	1.19	1.30
Spouse male SE	0.97	1.01	1.10
Spouse male SW	0.93	0.99	1.07
Spouse male WL	1.02	1.08	1.17
Spouse male WM	1.09	1.16	1.26
Spouse male YH	0.99	1.06	1.17
Spouse male other	1.05	1.10	1.17
Spouse female EE	0.96	1.00	1.08
Spouse female EM	1.01	1.06	1.16
Spouse female LN	0.91	0.95	1.03
Spouse female NE	1.20	1.27	1.36
Spouse female NW	1.08	1.13	1.23
Spouse female SE	0.89	0.93	1.01
Spouse female SW	0.90	0.94	1.02
Spouse female WL	1.01	1.06	1.14
Spouse female WM	1.06	1.11	1.19
Spouse female YH	0.99	1.03	1.13
Spouse female other	0.97	1.02	1.08

Figure 5.93: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

ages.

Normal health tables overestimate actual exits at younger ages; conversely, ill health tables routinely underestimate actual exits at younger ages, overstating them at older ages, especially for females. The spouse tables appear to visually fit the data well, notably for females using the 2020 assumptions. Males demonstrate greater variability from age to age due to the data's increased sparseness. Although this appears reasonably common across the majority of regions, there are a few exceptions. For example, the North West (NW) and West Midlands (WM) expected exits complement actual exits even at lower ages for males of ill health in addition to those of normal health, regardless of gender. On the contrary, regions outside of England and Wales underestimate normal health exits at younger ages.

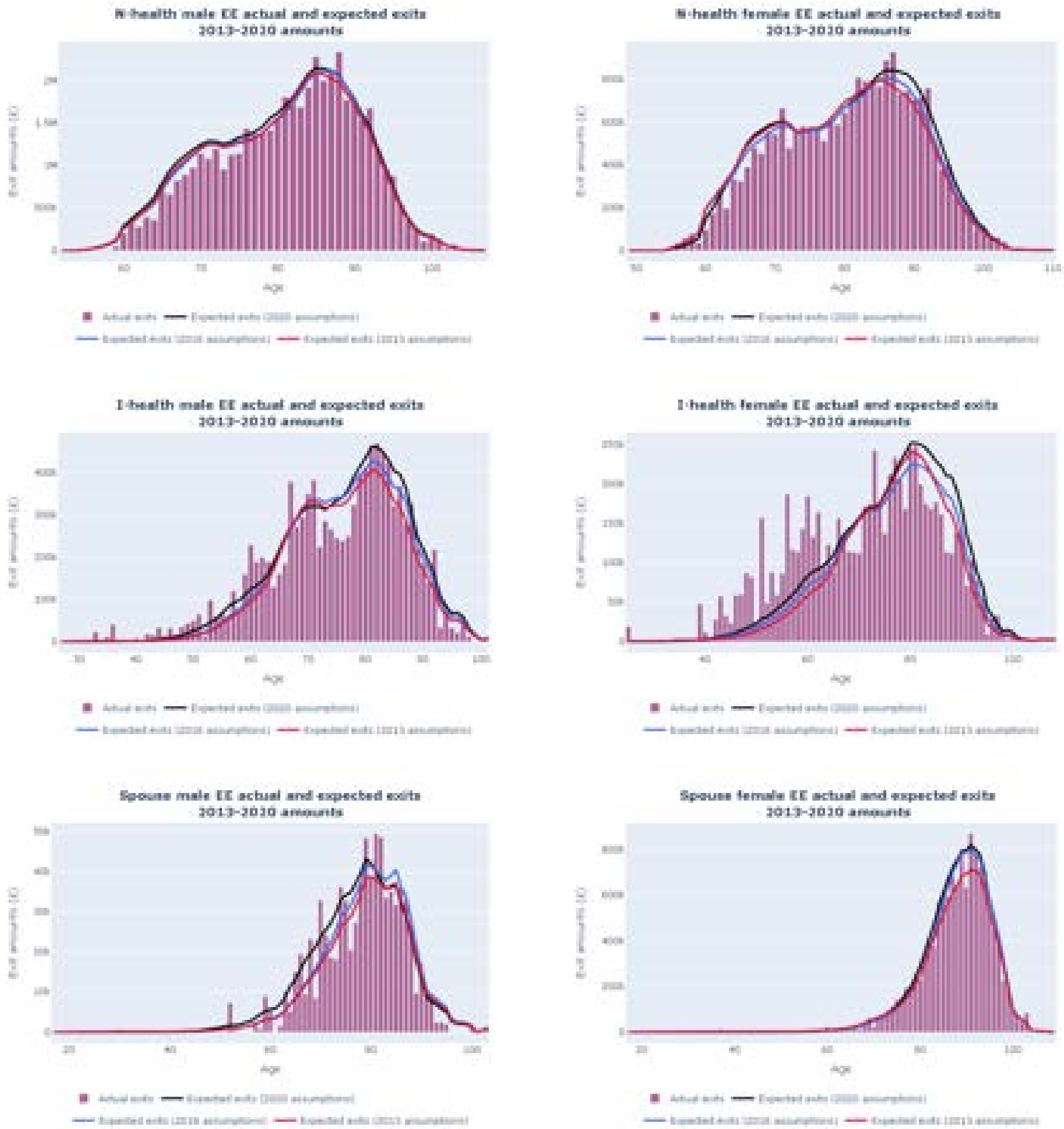


Figure 5.94: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

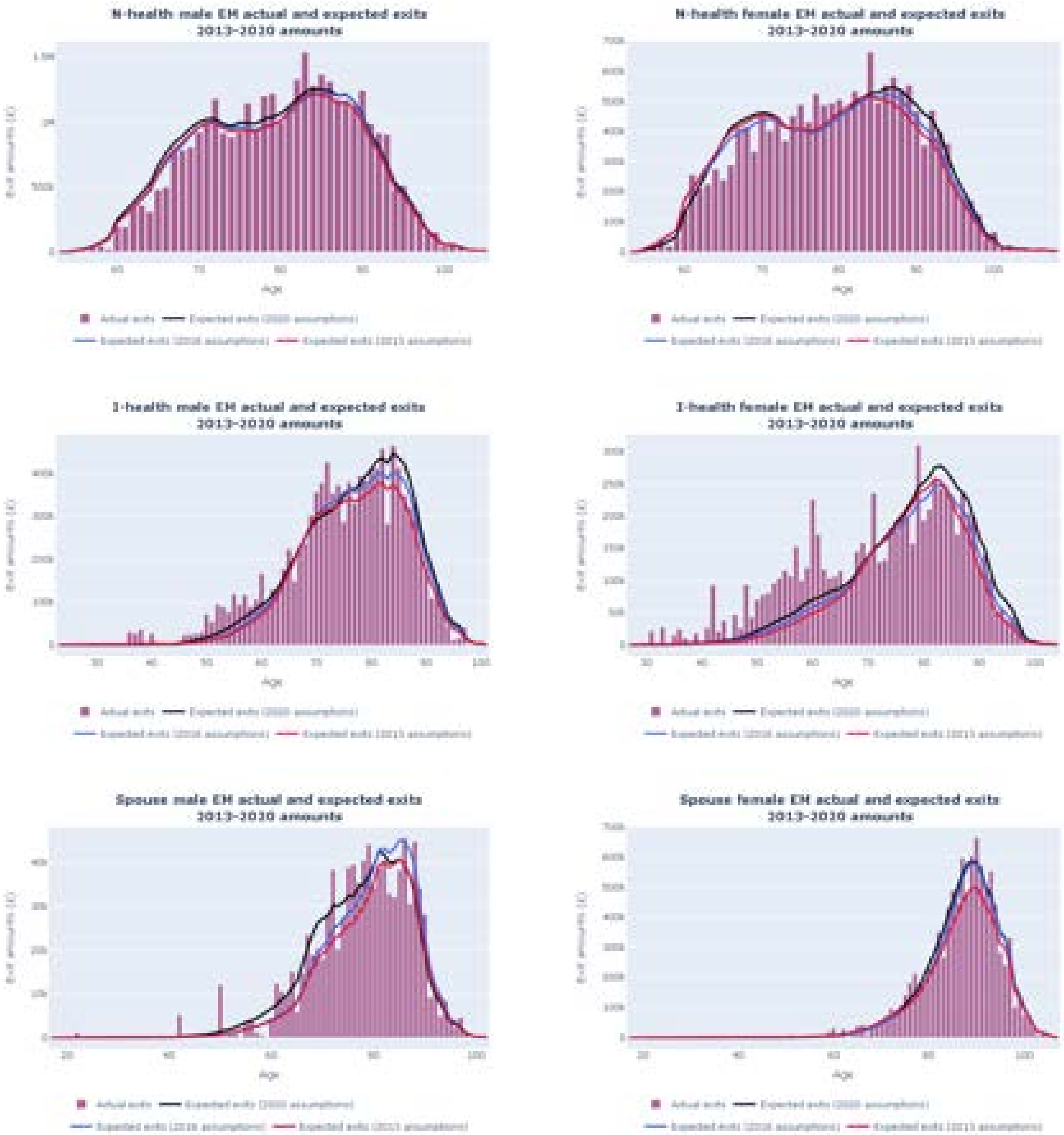


Figure 5.95: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

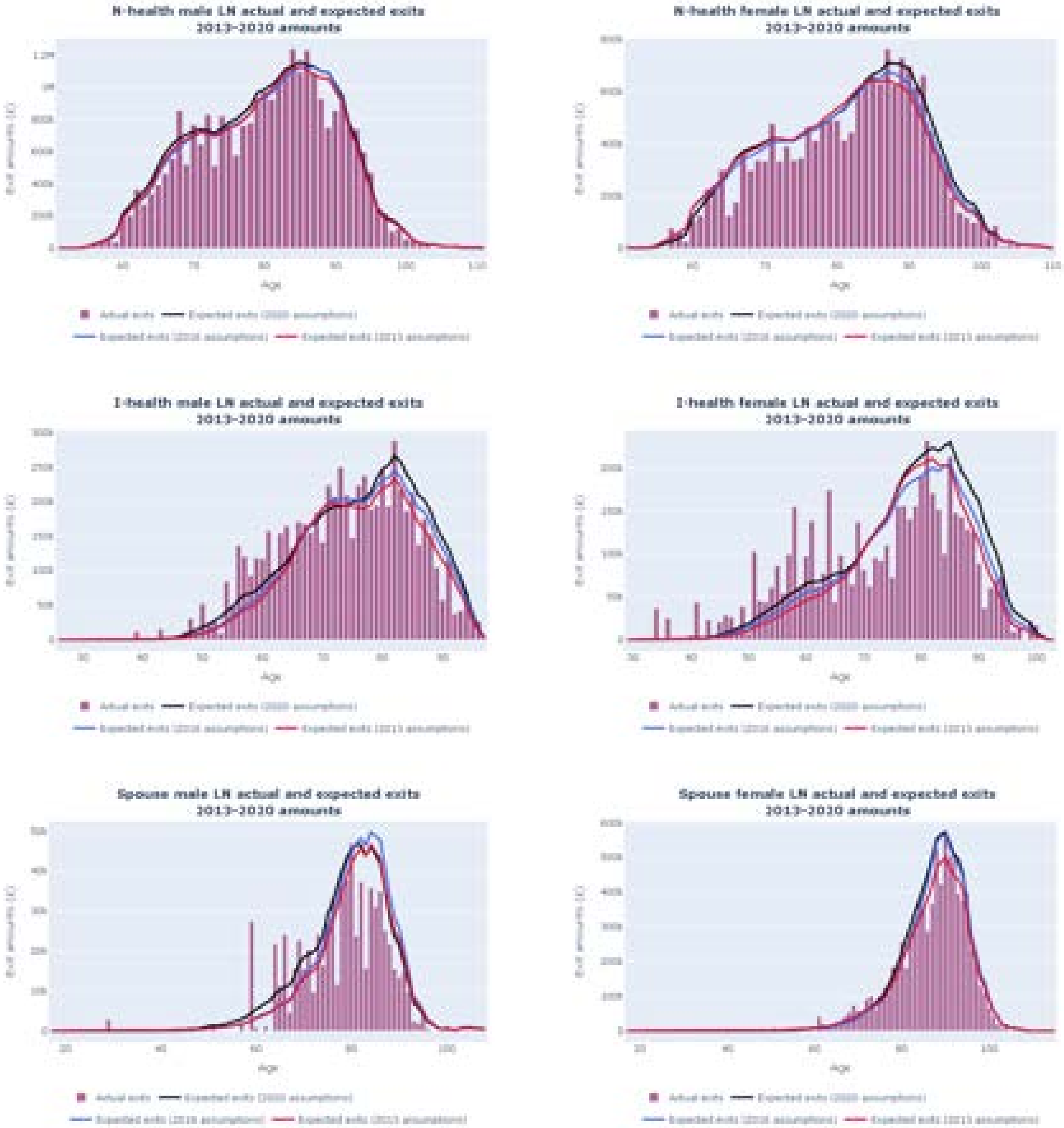


Figure 5.96: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

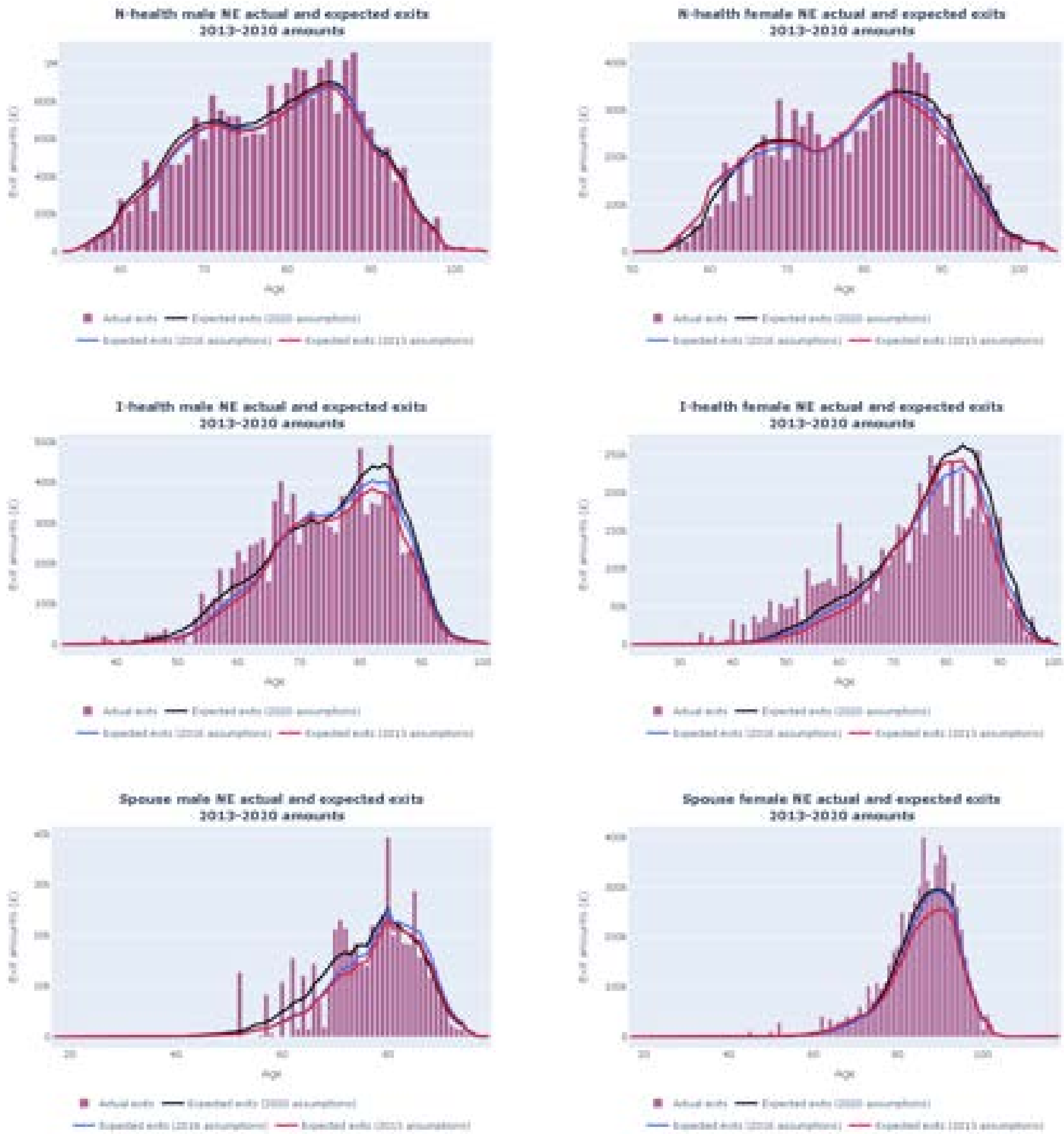


Figure 5.97: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

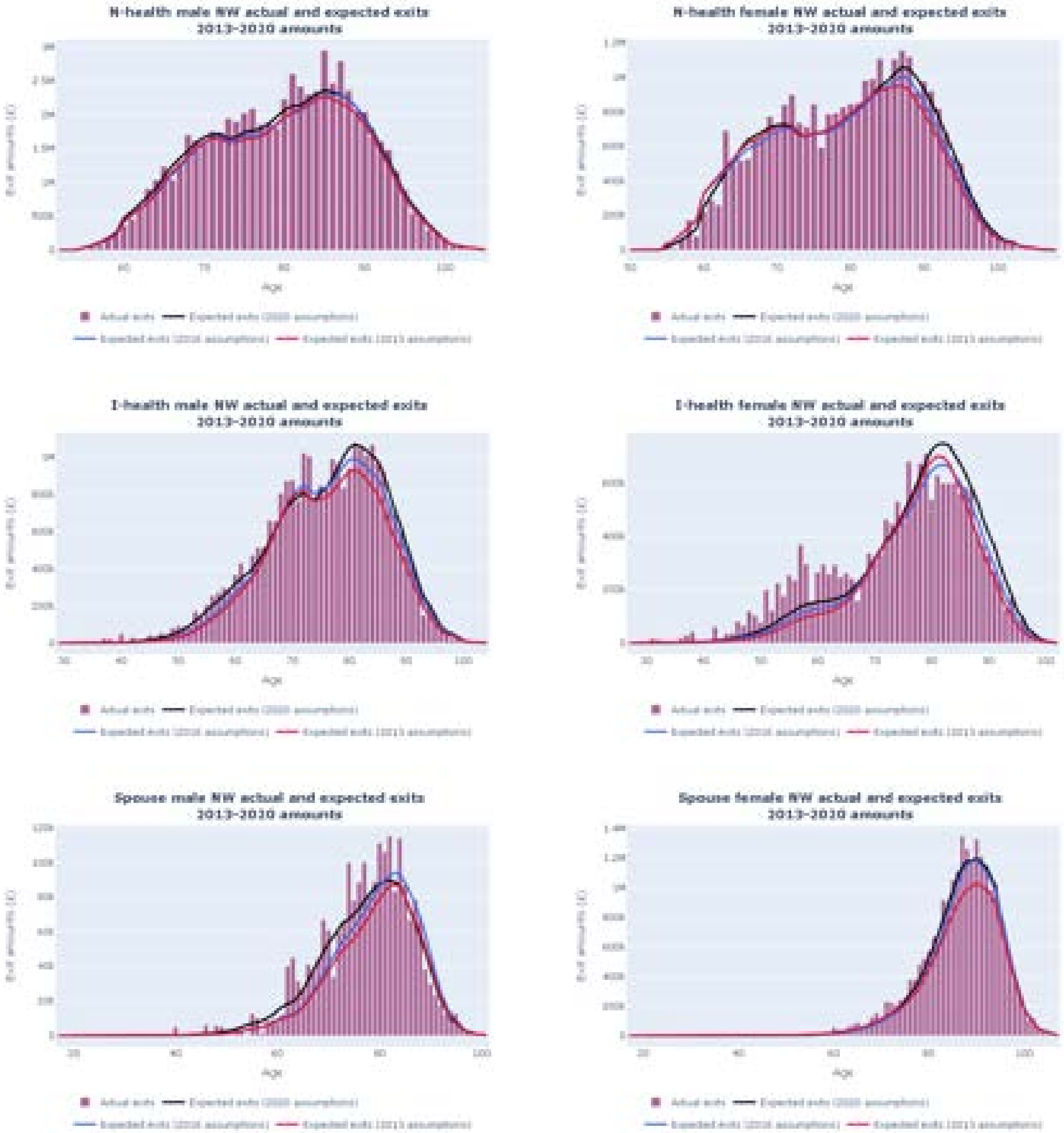


Figure 5.98: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

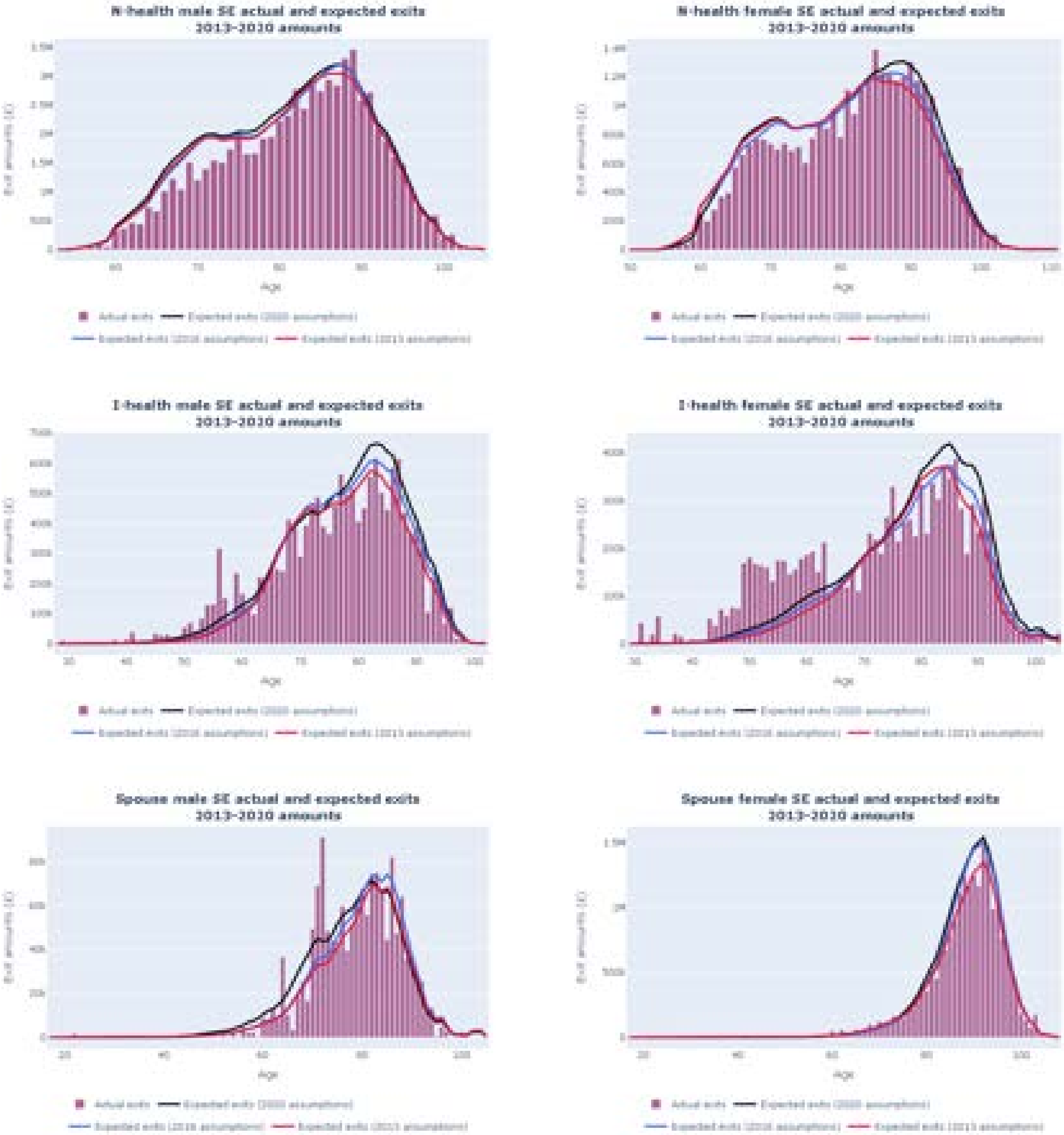


Figure 5.99: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

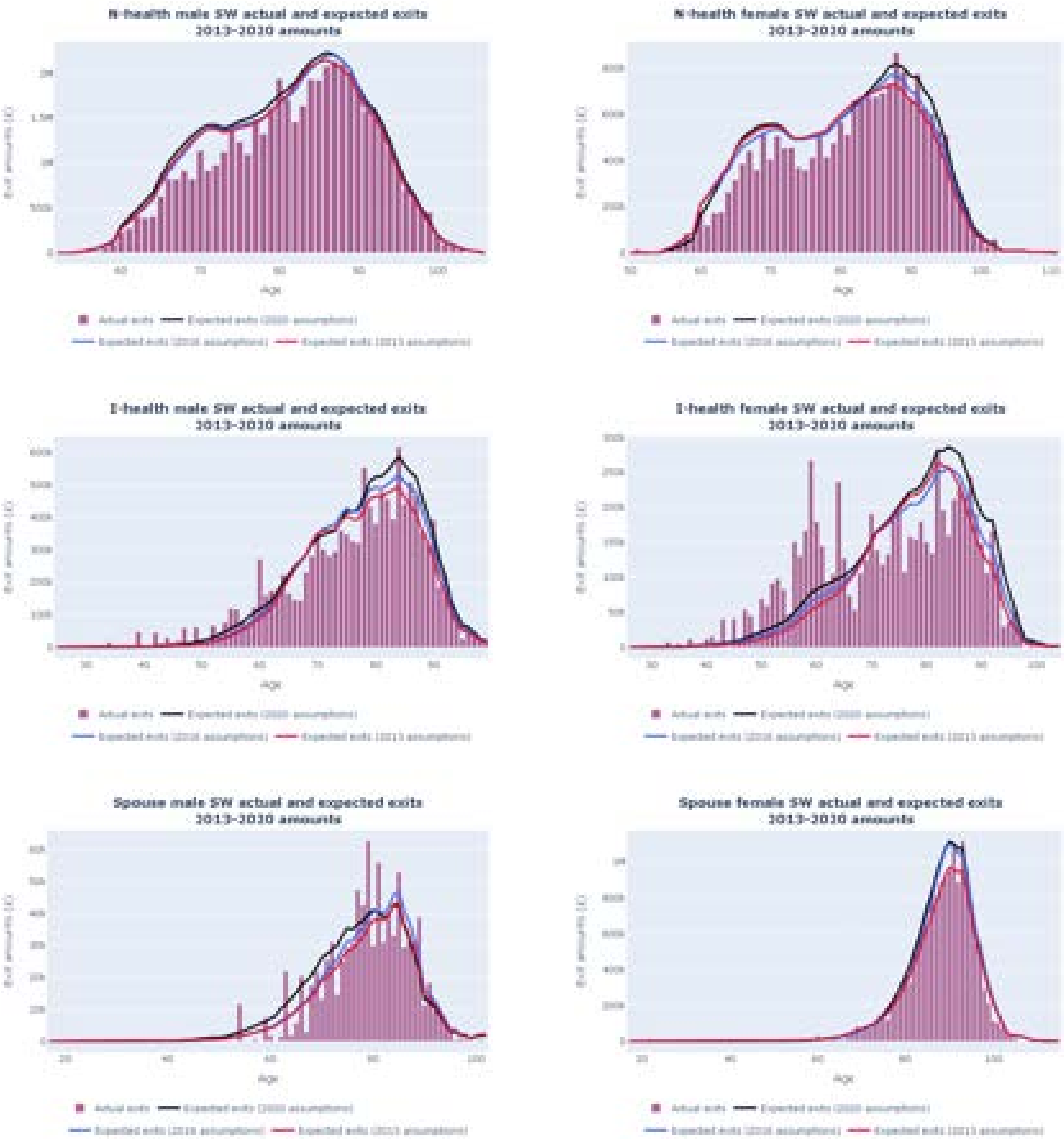


Figure 5.100: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

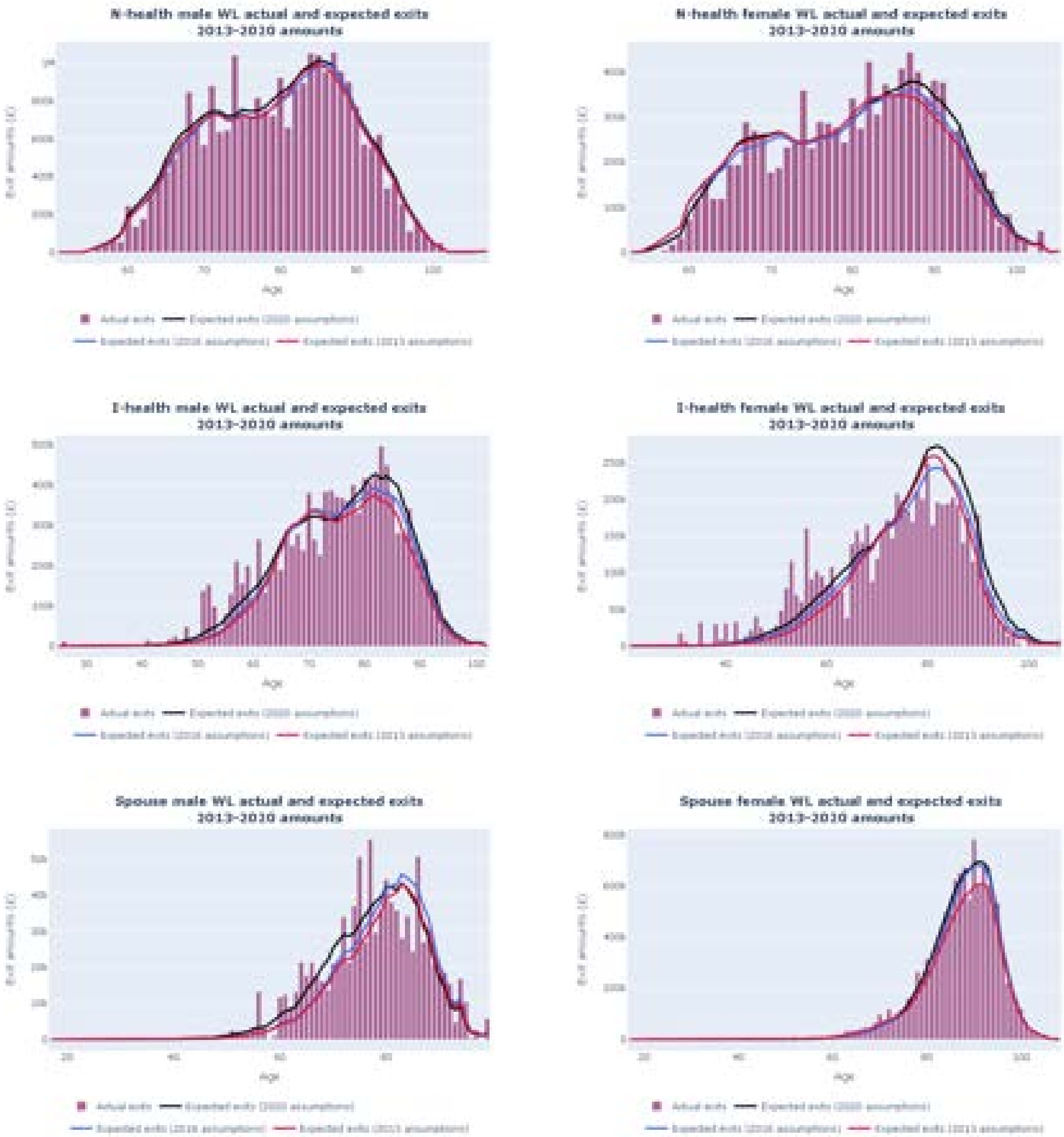


Figure 5.101: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

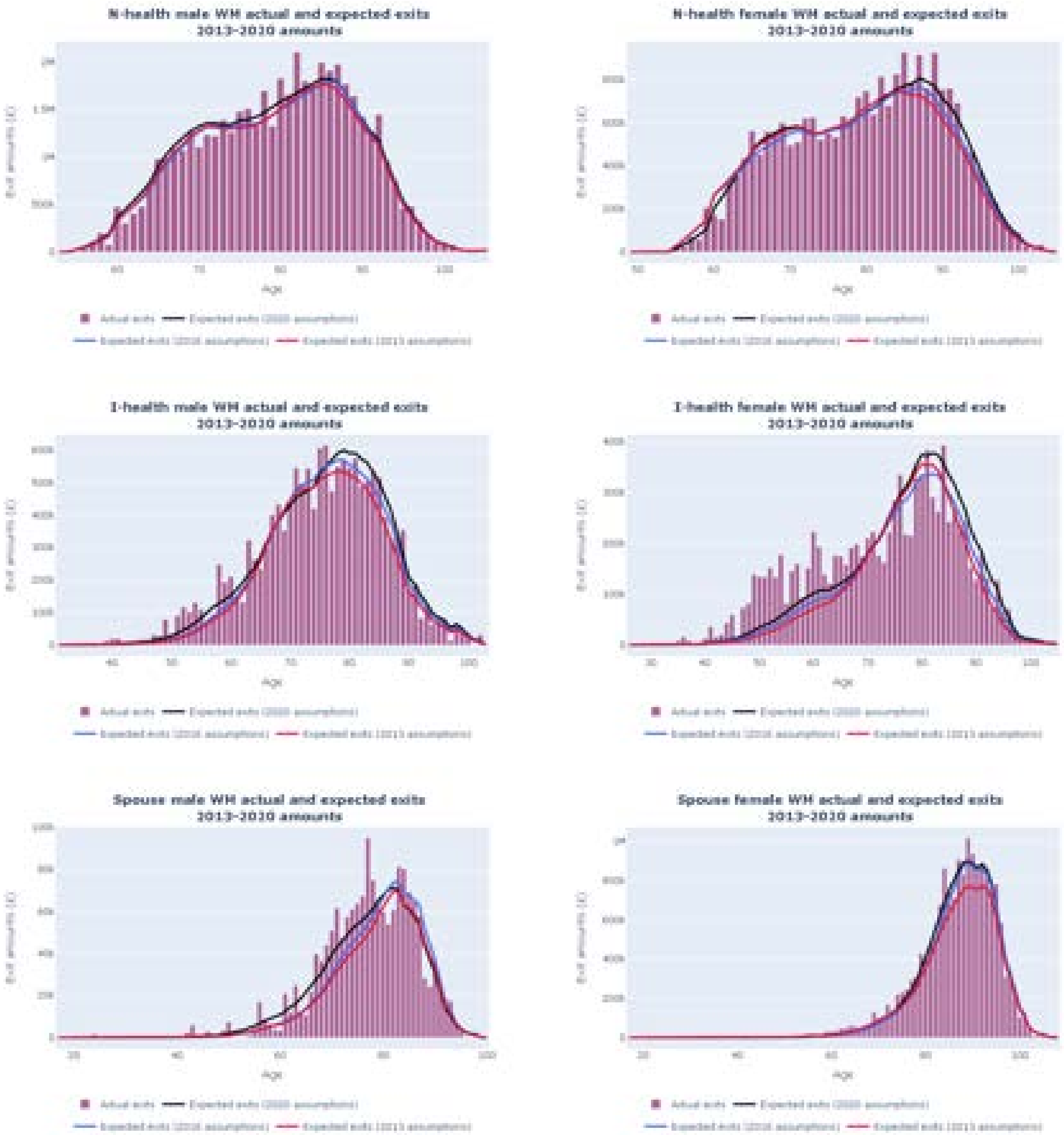


Figure 5.102: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

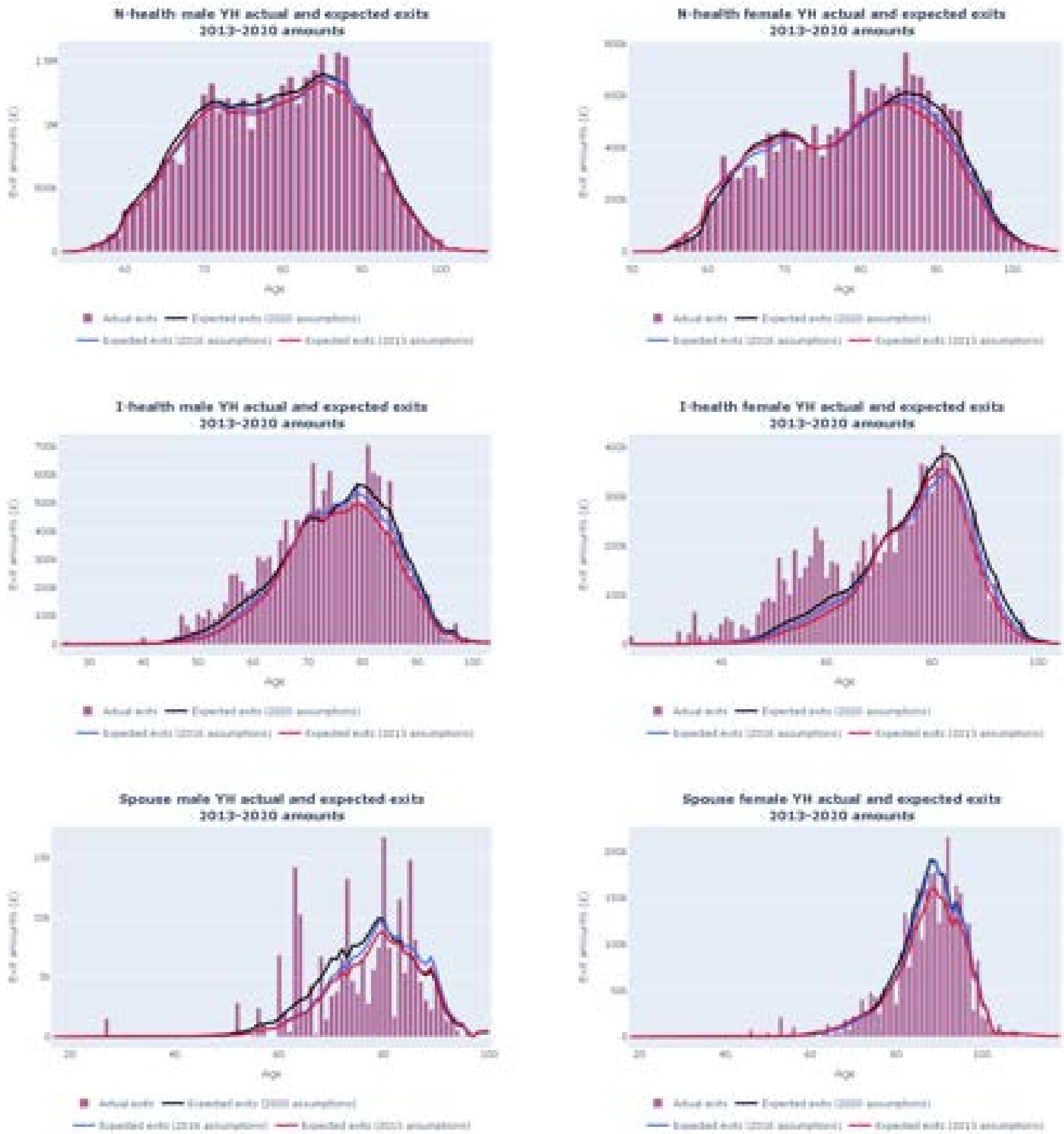


Figure 5.103: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

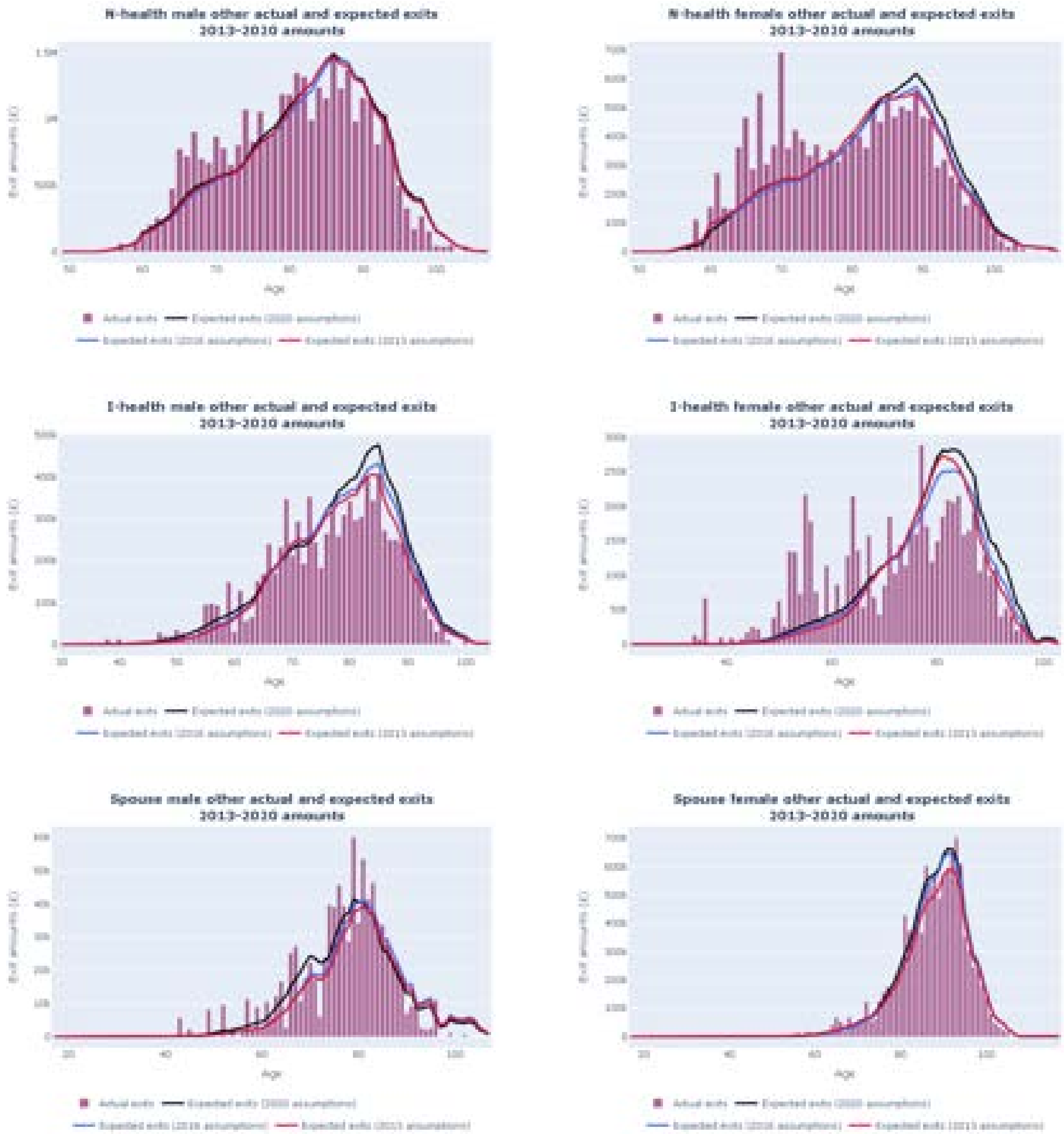


Figure 5.104: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

5.3.1 Regional subdivision compared to baseline

By utilising postcodes to create more refined regional groupings, optimal scaling factors for SAPS mortality tables can be identified; these should provide a better fit to the data within their respective categories. For each region, the fit of models is assessed with and without region-specific scaling to determine any statistically significant improvements in model performance, resulting from these scaling factors.

Table 5.105 illustrates the necessary optimal scaling factors to ensure that total expected exits equal total actual exits for each category; Table 5.106 presents applicable optimal scaling factors for each category, minimising the least square residuals of actual and expected exit amounts across each age.

The optimal scaling factors are similar for both genders across many regions, with a tendency to be slightly higher for males. Ill health categories exhibit a relatively large difference between the two metrics, particularly for females. Other categories have reasonably consistent results for the two metrics. Northern regions often have higher optimal factors, which are indicative of heavier mortality rates, as evidenced in Figures 5.119 to 5.124; these usually depict southern regions as darker blue in normal health and spouse categories, whereas northern regions are darker red in ill health categories.

Table 5.107 portray F-test results juxtaposing optimally scaled models with and without region-specific scaling; the DoF is always four for males and five for females [7]. Most regions do not provide sufficient evidence to validate the hypothesis that region-specific scaling factors offer a better fit than non-region-specific factors, as indicated by p-values greater than 0.05; it is reasonable to attribute differences between expected exits from both models due to random variation. Exceptions where region-specific models perform significantly better are evident in the South West, for both genders of normal health and in the North West, for males of normal health; other exceptions include London for female and male spouses, males with health issues in Yorkshire and Humber and female spouses in the North East, North West and South East. In conclusion, there is insufficient evi-

Category	2020 assumption	2016 assumption	2013 assumption
N-health male EE	0.92	0.98	0.97
N-health male EM	0.99	1.06	1.04
N-health male LN	0.93	0.99	0.97
N-health male NE	1.03	1.10	1.08
N-health male NW	1.05	1.12	1.11
N-health male SE	0.88	0.94	0.93
N-health male SW	0.87	0.93	0.91
N-health male WL	0.97	1.03	1.01
N-health male WM	1.01	1.07	1.06
N-health male YH	1.00	1.06	1.06
N-health male other	1.02	1.09	1.05
N-health female EE	0.92	0.92	0.93
N-health female EM	0.97	0.96	0.97
N-health female LN	0.88	0.88	0.89
N-health female NE	0.99	0.97	0.98
N-health female NW	1.01	1.00	1.02
N-health female SE	0.89	0.89	0.90
N-health female SW	0.86	0.86	0.87
N-health female WL	0.97	0.96	0.97
N-health female WM	1.00	0.99	1.00
N-health female YH	1.03	1.02	1.04
N-health female other	1.04	1.04	1.05
I-health male EE	1.12	1.09	1.13
I-health male EM	1.22	1.18	1.23
I-health male LN	1.17	1.15	1.19
I-health male NE	1.21	1.19	1.23
I-health male NW	1.22	1.20	1.25
I-health male SE	1.09	1.07	1.11
I-health male SW	1.08	1.05	1.09
I-health male WL	1.21	1.19	1.22
I-health male WM	1.19	1.16	1.20
I-health male YH	1.35	1.32	1.37
I-health male other	1.04	1.02	1.04
I-health female EE	1.40	1.25	1.31
I-health female EM	1.52	1.37	1.44
I-health female LN	1.20	1.08	1.13
I-health female NE	1.37	1.23	1.27
I-health female NW	1.42	1.27	1.33
I-health female SE	1.42	1.29	1.35
I-health female SW	1.29	1.16	1.22
I-health female WL	1.25	1.12	1.16
I-health female WM	1.44	1.29	1.35
I-health female YH	1.56	1.40	1.46
I-health female other	1.31	1.18	1.20
Spouse male EE	0.88	1.29	1.27
Spouse male EM	0.96	1.40	1.40
Spouse male LN	0.76	1.08	1.06
Spouse male NE	0.98	1.46	1.43
Spouse male NW	1.07	1.57	1.56
Spouse male SE	0.93	1.34	1.32
Spouse male SW	0.89	1.30	1.28
Spouse male WL	0.98	1.42	1.40
Spouse male WM	1.05	1.53	1.52
Spouse male YH	0.95	1.40	1.40
Spouse male other	1.01	1.45	1.40
Spouse female EE	0.93	1.06	1.09
Spouse female EM	0.98	1.13	1.17
Spouse female LN	0.88	1.01	1.04
Spouse female NE	1.17	1.34	1.37
Spouse female NW	1.04	1.19	1.24
Spouse female SE	0.87	0.99	1.02
Spouse female SW	0.88	1.00	1.03
Spouse female WL	0.98	1.12	1.15
Spouse female WM	1.03	1.17	1.21
Spouse female YH	0.96	1.10	1.14
Spouse female other	0.95	1.08	1.09

Figure 5.105: Optimal scaling factors applied to unscaled mortality base and improvement tables, used for the 2013, 2016 and 2020 valuation assumptions for each category, to ensure that total expected exit amounts equal actual exit amounts.

Category	2020 assumption	2016 assumption	2013 assumption
N-health male EE	0.94	1.00	0.98
N-health male EM	1.01	1.07	1.06
N-health male LN	0.93	0.98	0.97
N-health male NE	1.04	1.10	1.08
N-health male NW	1.06	1.13	1.12
N-health male SE	0.90	0.95	0.94
N-health male SW	0.88	0.93	0.92
N-health male WL	0.98	1.04	1.02
N-health male WM	1.02	1.08	1.06
N-health male YH	1.00	1.07	1.06
N-health male other	0.98	1.03	1.00
N-health female EE	0.93	0.93	0.94
N-health female EM	0.97	0.97	0.97
N-health female LN	0.90	0.90	0.91
N-health female NE	1.00	0.99	0.99
N-health female NW	1.02	1.01	1.03
N-health female SE	0.90	0.90	0.91
N-health female SW	0.87	0.87	0.88
N-health female WL	0.98	0.98	0.98
N-health female WM	1.00	1.00	1.01
N-health female YH	1.03	1.03	1.05
N-health female other	0.94	0.95	0.96
I-health male EE	1.08	1.04	1.06
I-health male EM	1.15	1.11	1.15
I-health male LN	1.12	1.07	1.10
I-health male NE	1.16	1.12	1.15
I-health male NW	1.21	1.16	1.20
I-health male SE	1.03	0.99	1.03
I-health male SW	1.02	0.98	1.02
I-health male WL	1.16	1.11	1.13
I-health male WM	1.15	1.10	1.12
I-health male YH	1.30	1.23	1.27
I-health male other	0.98	0.96	0.98
I-health female EE	1.19	1.05	1.05
I-health female EM	1.29	1.14	1.14
I-health female LN	1.05	0.93	0.93
I-health female NE	1.20	1.06	1.04
I-health female NW	1.26	1.11	1.11
I-health female SE	1.17	1.05	1.06
I-health female SW	1.10	0.98	0.97
I-health female WL	1.13	1.00	0.99
I-health female WM	1.23	1.09	1.08
I-health female YH	1.34	1.17	1.17
I-health female other	1.03	0.92	0.89
Spouse male EE	0.92	1.30	1.29
Spouse male EM	0.95	1.32	1.33
Spouse male LN	0.73	0.97	0.96
Spouse male NE	1.00	1.42	1.40
Spouse male NW	1.08	1.49	1.50
Spouse male SE	0.96	1.32	1.30
Spouse male SW	0.94	1.33	1.32
Spouse male WL	0.93	1.28	1.26
Spouse male WM	1.05	1.43	1.42
Spouse male YH	0.89	1.26	1.27
Spouse male other	1.05	1.47	1.42
Spouse female EE	0.92	1.04	1.09
Spouse female EM	0.98	1.11	1.19
Spouse female LN	0.85	0.97	1.02
Spouse female NE	1.14	1.30	1.36
Spouse female NW	1.01	1.15	1.23
Spouse female SE	0.85	0.96	1.01
Spouse female SW	0.89	1.00	1.06
Spouse female WL	0.95	1.08	1.14
Spouse female WM	1.00	1.13	1.19
Spouse female YH	0.94	1.06	1.14
Spouse female other	0.93	1.05	1.09

Figure 5.106: Optimal scaling factors applied to unscaled mortality base and improvement tables, used for the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

Category	DoF	Scale (region)	Scale (no region)	RSS (region)	RSS (no region)	F	p-value
N-health male EE	52	0.94	0.97	763,589,864,529	816,068,672,680	1.07	0.40575
N-health male EM	49	1.01	0.97	607,014,683,868	654,407,670,122	1.08	0.39676
N-health male LN	57	0.93	0.97	413,051,313,667	451,247,251,464	1.09	0.36981
N-health male NE	48	1.04	0.97	380,176,132,721	447,810,772,987	1.18	0.28640
N-health male NW	49	1.06	0.97	1,032,953,829,344	1,963,980,943,109	1.90	0.01325
N-health male SE	49	0.90	0.97	2,499,276,430,575	3,319,882,075,267	1.33	0.16181
N-health male SW	49	0.88	0.97	1,058,724,227,767	1,717,610,544,088	1.62	0.04679
N-health male WL	51	0.98	0.97	377,815,205,649	379,171,604,293	1.00	0.49492
N-health male WM	49	1.02	0.97	711,090,167,656	858,142,680,937	1.21	0.25653
N-health male YH	51	1.00	0.97	389,778,544,473	438,283,657,978	1.12	0.33847
N-health male other	54	0.98	0.97	1,539,250,950,036	1,545,190,631,902	1.00	0.49438
N-health female EE	55	0.93	0.95	149,571,599,369	158,030,507,531	1.06	0.41955
N-health female EM	51	0.97	0.95	146,621,736,598	148,587,582,974	1.01	0.48113
N-health female LN	54	0.90	0.95	160,983,539,992	184,196,140,931	1.14	0.31120
N-health female NE	50	1.00	0.95	70,913,130,460	77,129,125,673	1.09	0.38379
N-health female NW	54	1.02	0.95	250,369,194,850	349,989,880,190	1.40	0.11081
N-health female SE	57	0.90	0.95	321,424,419,825	429,580,124,379	1.34	0.13825
N-health female SW	57	0.87	0.95	146,173,378,616	239,109,618,766	1.64	0.03285
N-health female WL	48	0.98	0.95	72,256,697,708	74,386,568,814	1.03	0.46013
N-health female WM	51	1.00	0.95	154,606,885,129	196,145,143,315	1.27	0.19917
N-health female YH	52	1.03	0.95	171,015,199,161	225,133,440,044	1.32	0.16231
N-health female other	55	0.94	0.95	745,977,582,349	747,280,141,538	1.00	0.49743
I-health male EE	71	1.08	1.13	124,106,703,778	129,250,672,266	1.04	0.43231
I-health male EM	75	1.15	1.13	118,189,908,632	119,277,264,224	1.01	0.48424
I-health male LN	64	1.12	1.13	74,530,320,984	74,718,345,760	1.00	0.49600
I-health male NE	67	1.16	1.13	157,165,673,926	159,499,546,539	1.01	0.47604
I-health male NW	72	1.21	1.13	266,643,555,969	347,289,729,296	1.30	0.13230
I-health male SE	69	1.03	1.13	256,963,670,081	307,614,618,609	1.20	0.22843
I-health male SW	71	1.02	1.13	150,750,461,109	197,727,972,768	1.31	0.12774
I-health male WL	73	1.16	1.13	143,454,135,558	145,973,280,958	1.02	0.47046
I-health male WM	69	1.15	1.13	147,474,484,349	149,996,388,872	1.02	0.47203
I-health male YH	73	1.30	1.13	210,344,613,825	320,619,605,270	1.52	0.03689
I-health male other	71	0.98	1.13	139,066,613,429	186,040,928,208	1.34	0.11131
I-health female EE	77	1.19	1.19	141,444,435,276	141,456,475,137	1.00	0.49985
I-health female EM	72	1.29	1.19	135,730,643,124	142,625,063,508	1.05	0.41704
I-health female LN	69	1.05	1.19	89,235,064,269	99,316,601,374	1.11	0.32894
I-health female NE	72	1.20	1.19	75,528,310,441	75,557,101,976	1.00	0.49936
I-health female NW	71	1.26	1.19	455,363,391,746	474,412,110,909	1.04	0.43170
I-health female SE	71	1.17	1.19	288,618,212,547	289,423,586,684	1.00	0.49533
I-health female SW	74	1.10	1.19	184,377,626,916	190,171,482,302	1.03	0.44724
I-health female WL	81	1.13	1.19	74,838,828,143	77,407,789,795	1.03	0.43983
I-health female WM	75	1.23	1.19	201,125,011,536	203,577,625,144	1.01	0.47914
I-health female YH	75	1.34	1.19	207,401,406,699	233,078,821,329	1.12	0.30727
I-health female other	77	1.03	1.19	215,065,075,673	230,745,815,632	1.07	0.37914
Spouse male EE	81	0.92	0.98	1,500,860,987	1,592,788,330	1.06	0.39486
Spouse male EM	79	0.95	0.98	1,390,147,621	1,410,688,497	1.01	0.47410
Spouse male LN	86	0.73	0.98	2,505,265,186	4,318,447,923	1.72	0.00617
Spouse male NE	71	1.00	0.98	1,080,083,592	1,084,097,814	1.00	0.49379
Spouse male NW	77	1.08	0.98	5,454,040,733	6,812,461,064	1.25	0.16562
Spouse male SE	82	0.96	0.98	6,810,355,253	6,845,057,026	1.01	0.49085
Spouse male SW	79	0.94	0.98	3,138,959,769	3,181,747,740	1.01	0.47609
Spouse male WL	76	0.93	0.98	2,407,982,613	2,470,727,815	1.03	0.45551
Spouse male WM	78	1.05	0.98	4,242,496,834	4,576,850,084	1.08	0.36924
Spouse male YH	76	0.89	0.98	600,075,945	612,630,801	1.02	0.46416
Spouse male other	83	1.05	0.98	2,488,749,184	2,612,263,630	1.05	0.41295
Spouse female EE	87	0.92	0.94	60,925,311,680	65,364,876,678	1.07	0.37181
Spouse female EM	85	0.98	0.94	76,033,085,188	81,391,619,611	1.07	0.37714
Spouse female LN	88	0.85	0.94	59,030,746,504	88,162,769,967	1.49	0.03075
Spouse female NE	86	1.14	0.94	25,029,853,305	70,756,982,491	2.83	0.00000
Spouse female NW	85	1.01	0.94	151,560,824,991	249,879,385,074	1.65	0.01112
Spouse female SE	86	0.85	0.94	170,611,186,940	381,680,518,694	2.24	0.00012
Spouse female SW	92	0.89	0.94	113,620,631,128	151,204,954,474	1.33	0.08621
Spouse female WL	85	0.95	0.94	63,711,416,983	64,735,174,837	1.02	0.47080
Spouse female WM	86	1.00	0.94	122,729,216,558	156,632,133,285	1.28	0.13001
Spouse female YH	89	0.94	0.94	19,698,074,758	19,698,198,210	1.00	0.49999
Spouse female other	91	0.93	0.94	92,914,329,938	93,615,587,322	1.01	0.48573

Figure 5.107: F-test comparing models with and without region-specific scaling factors for each category.

dence of regional discrepancies in mortality significantly impacting the results; therefore, non-region-specific optimal scaling factors should be similarly effective. Supplementary research could explore the aggregation of regional categories into northern and southern territories to reveal any significant differences in comparison to non-regional scaling.

Figures 5.108 to 5.118 further demonstrate the trend where the application of region-specific scaling (black lines) typically produce lower expected exits than those without such scaling (blue lines) for southern regions; the black lines are generally greater in northern regions. The category depicting records outside of England and Wales produces more obscure results; there are lower region-specific expected exits for ill health categories, higher exits for male spouses and similar exits for those of normal health and female spouses.

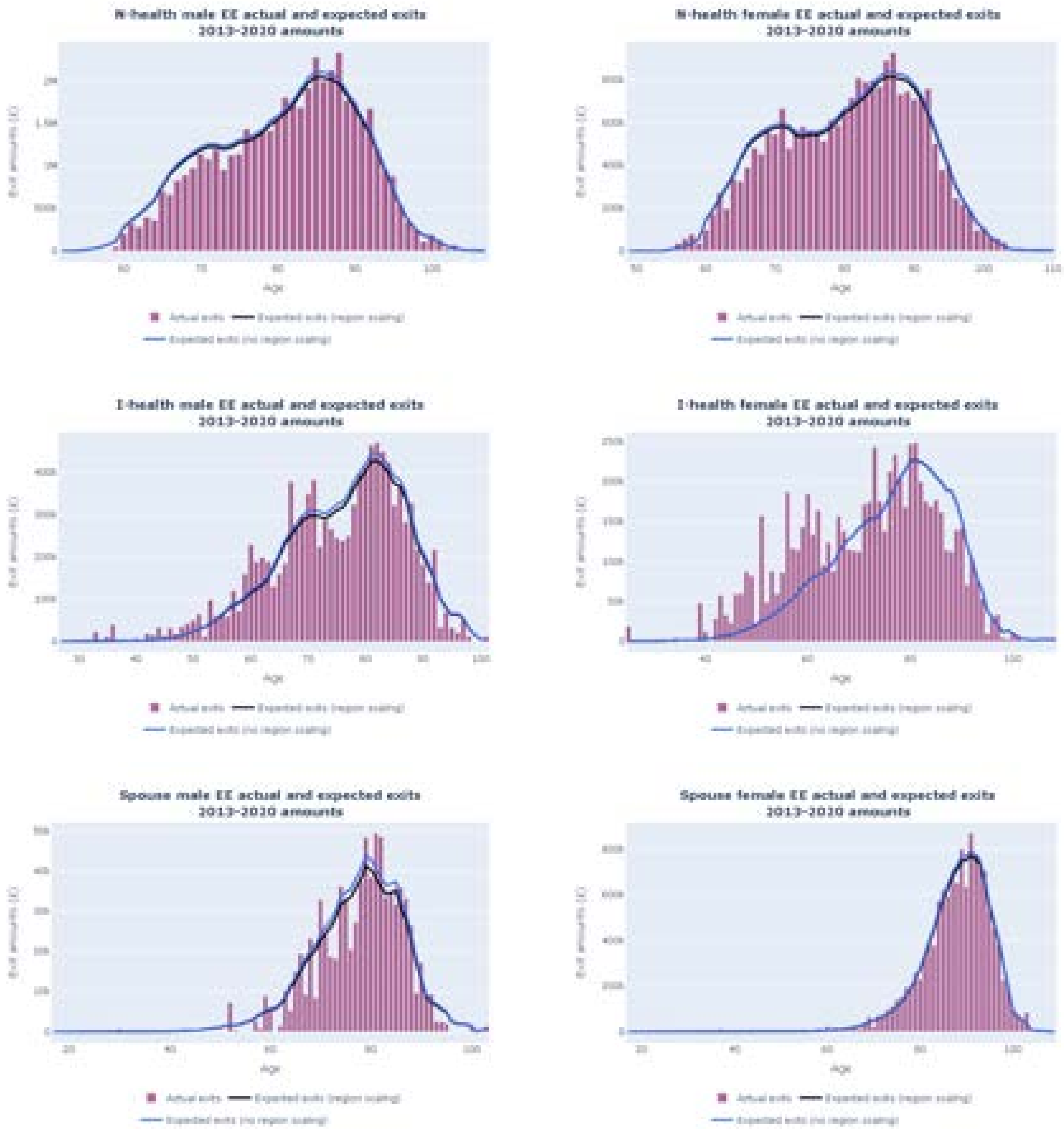


Figure 5.108: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

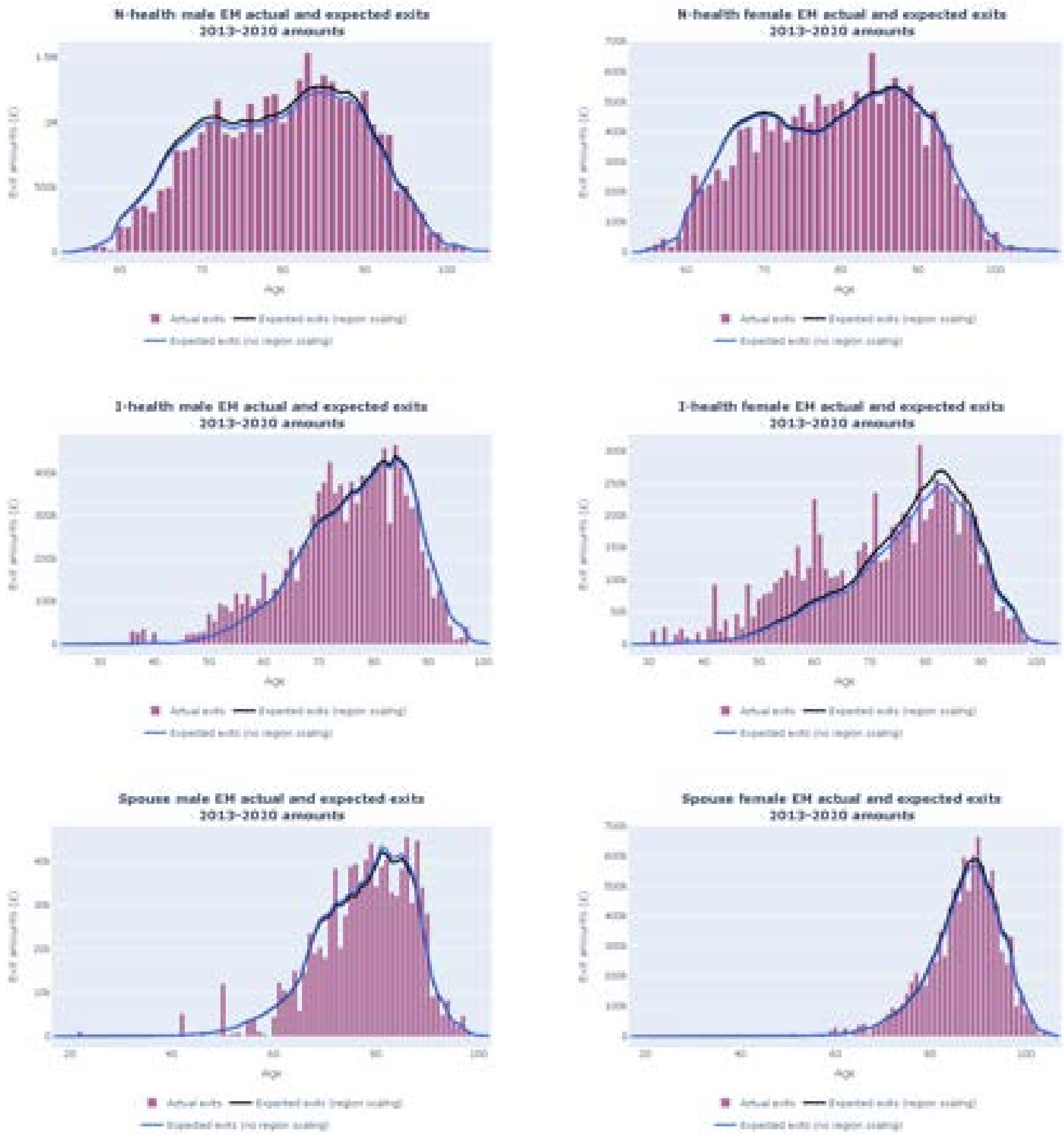


Figure 5.109: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

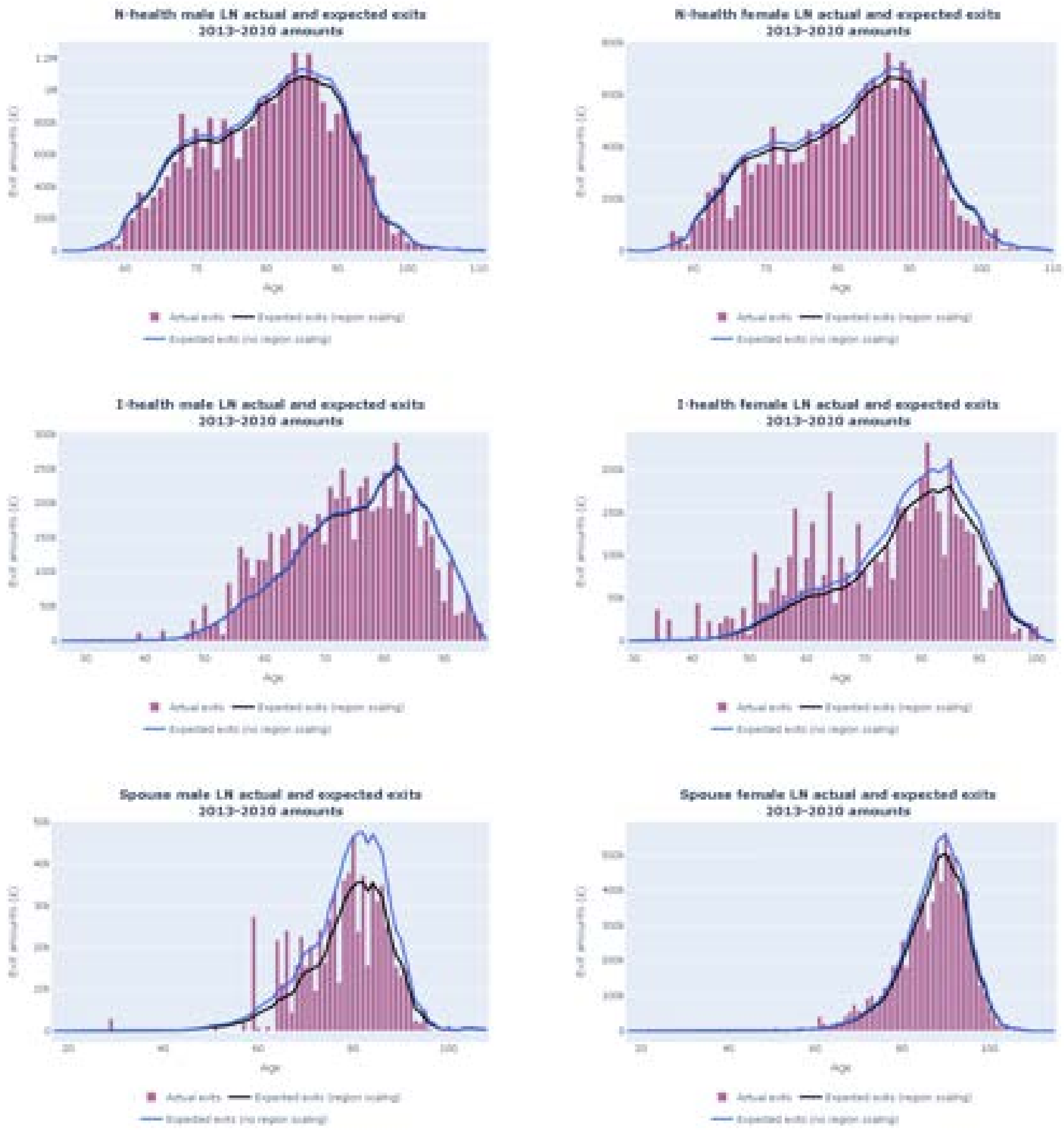


Figure 5.110: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

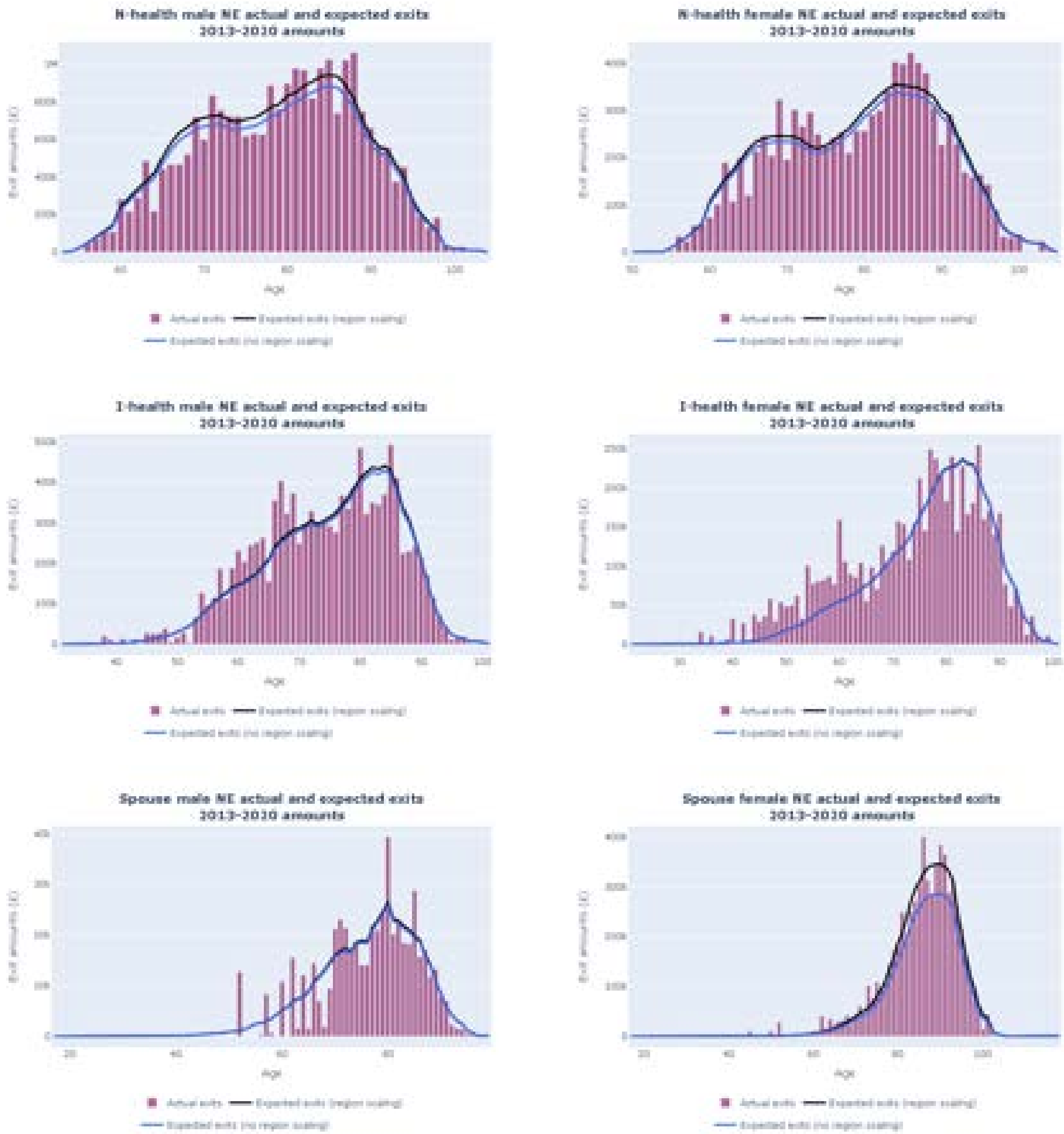


Figure 5.111: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

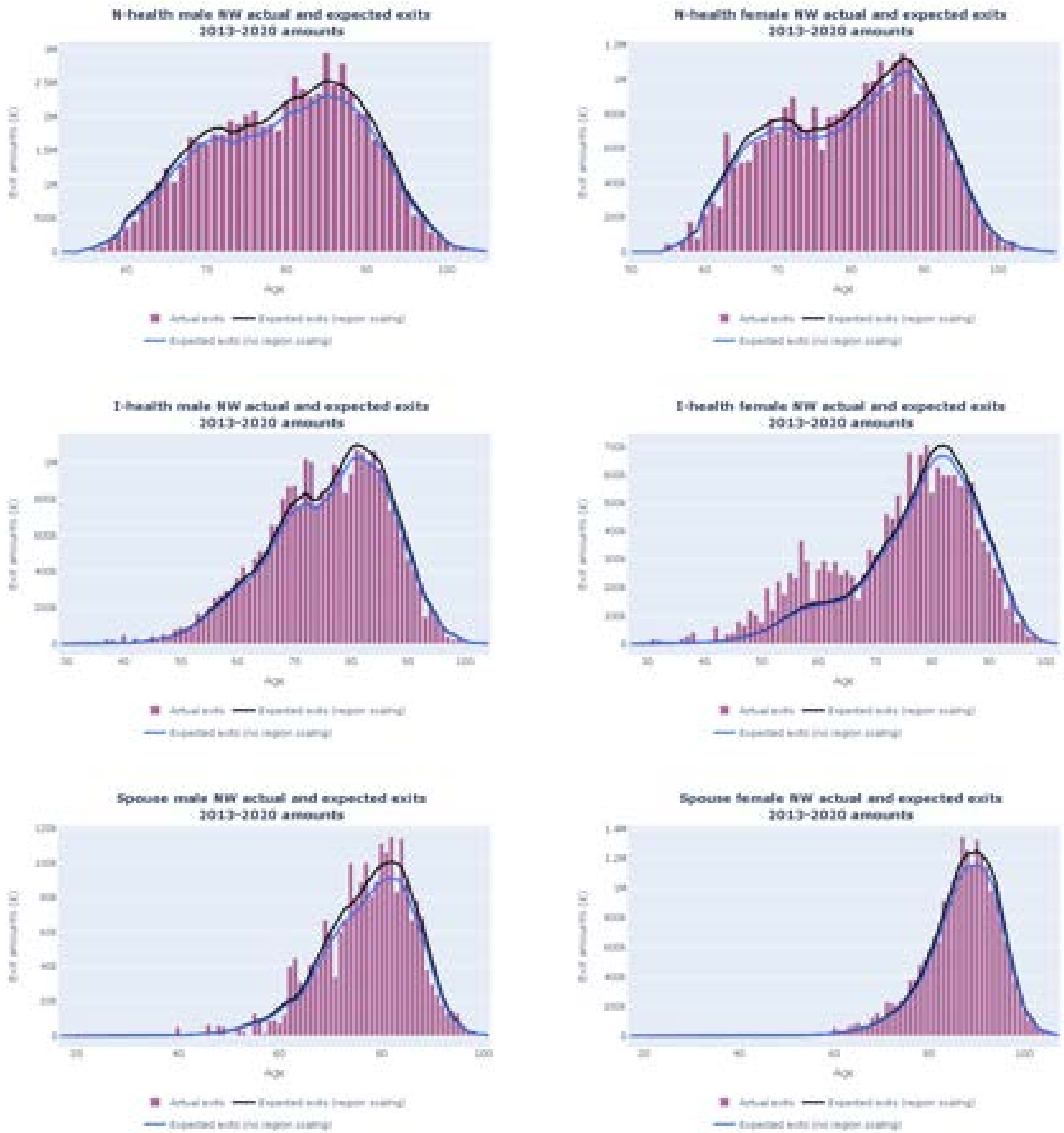


Figure 5.112: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

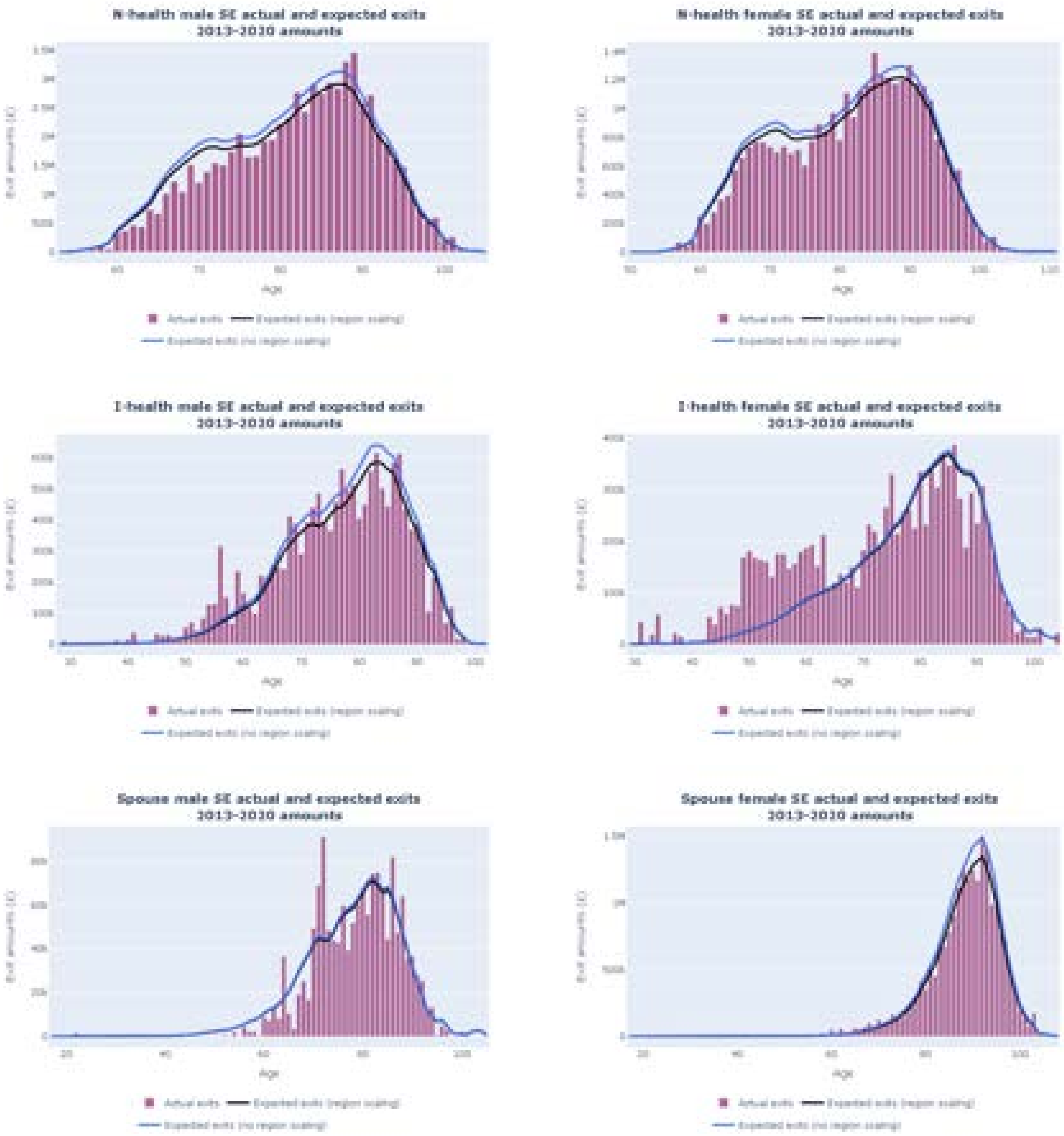


Figure 5.113: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

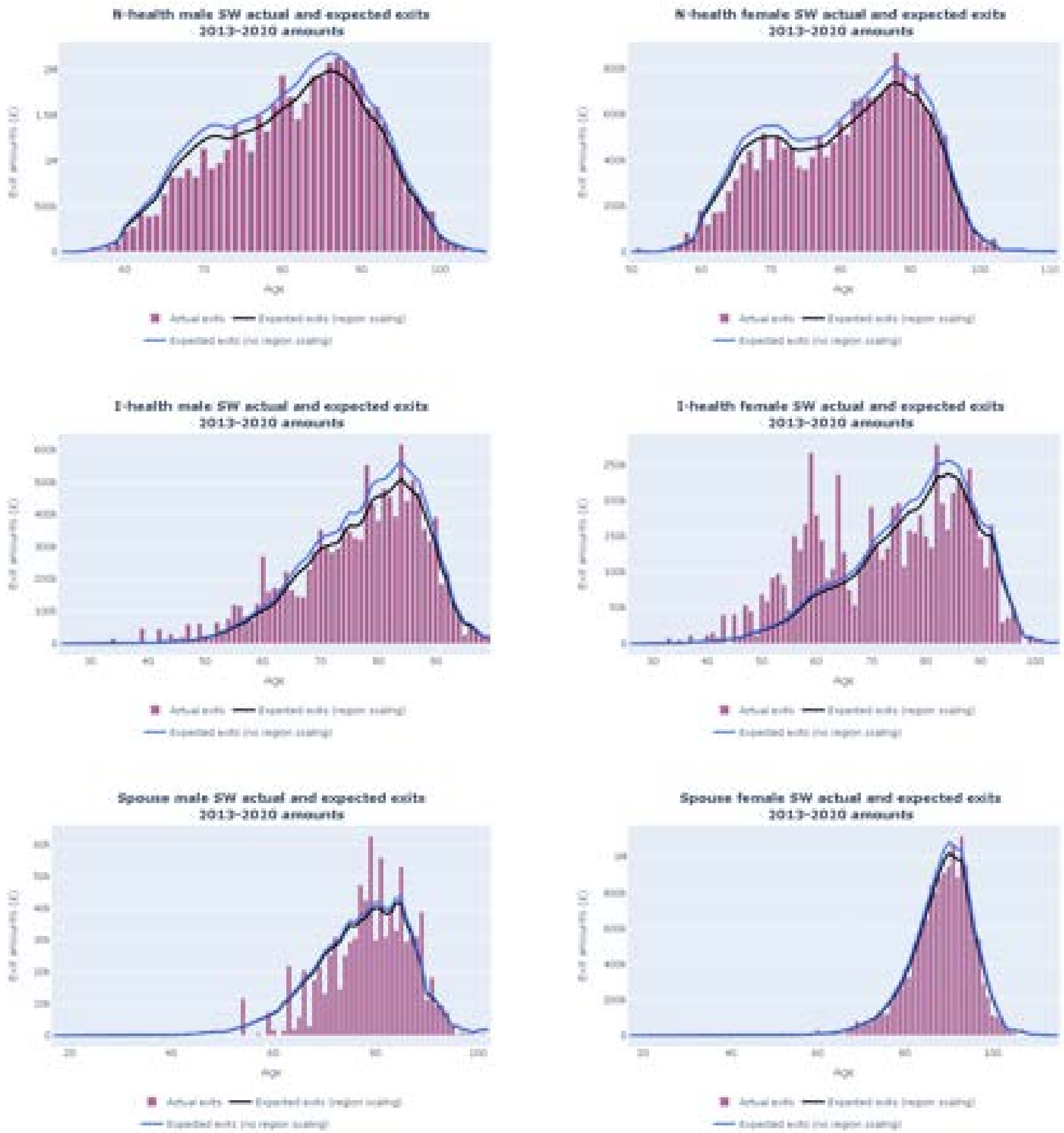


Figure 5.114: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

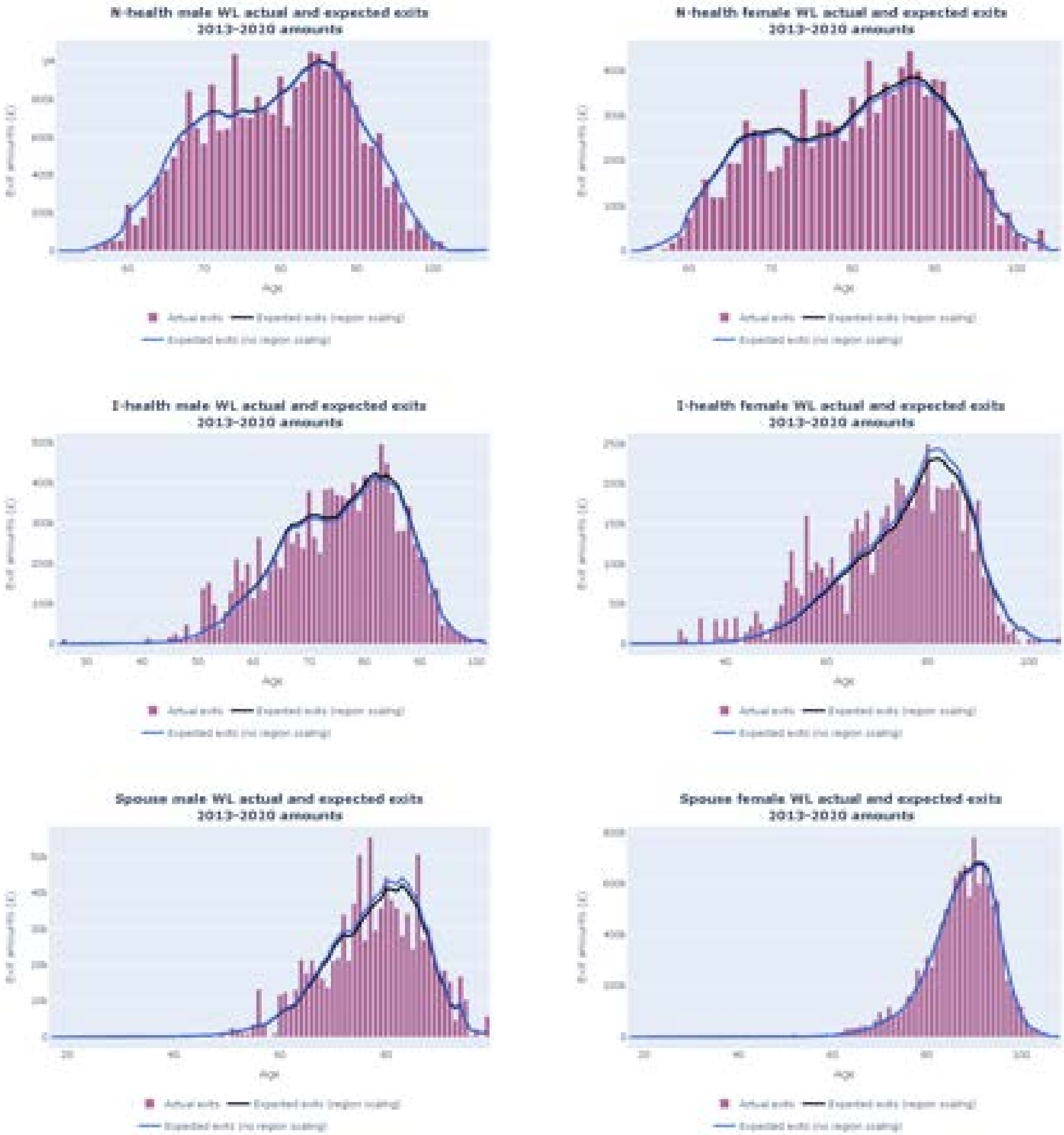


Figure 5.115: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

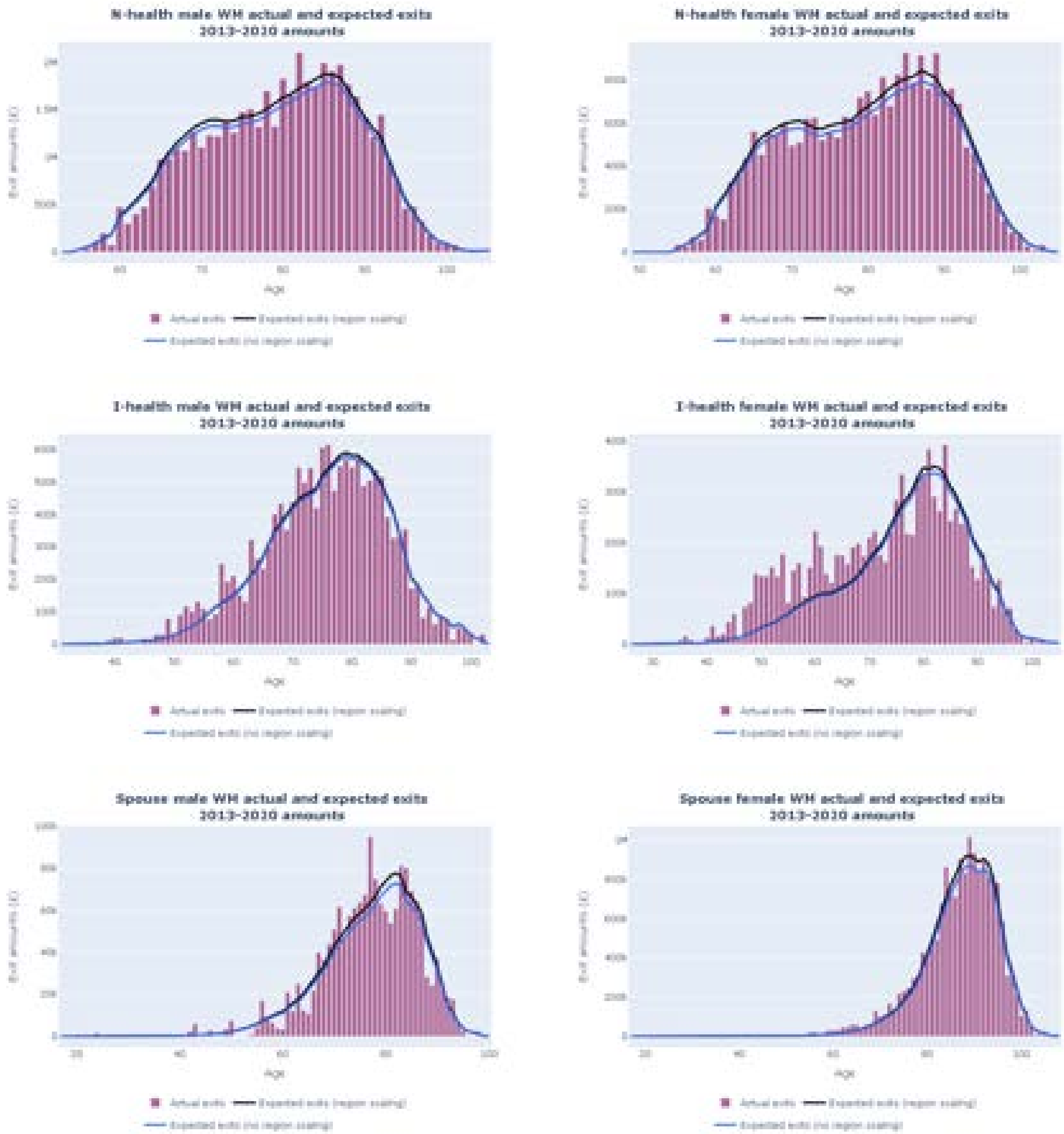


Figure 5.116: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

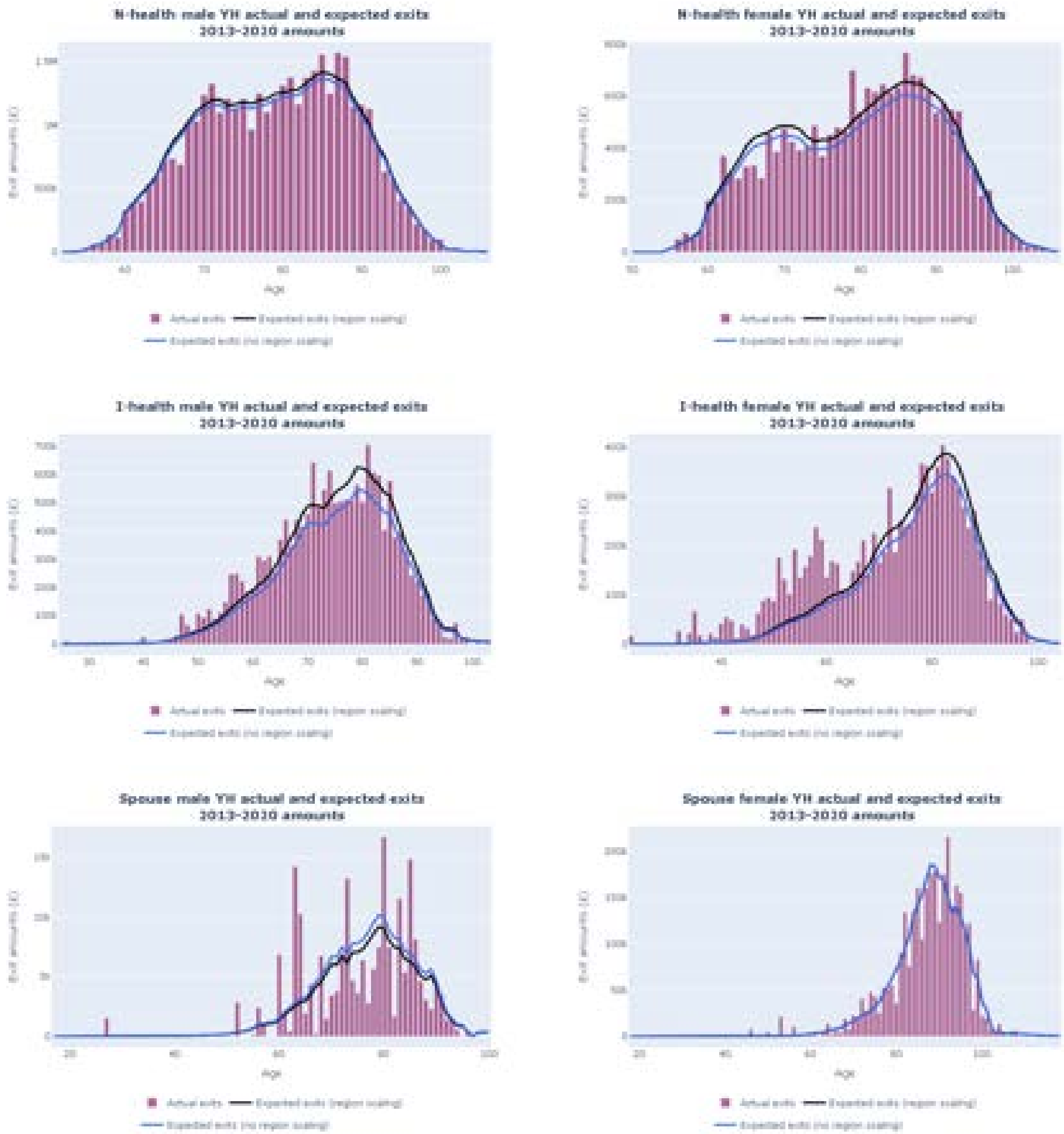


Figure 5.117: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.

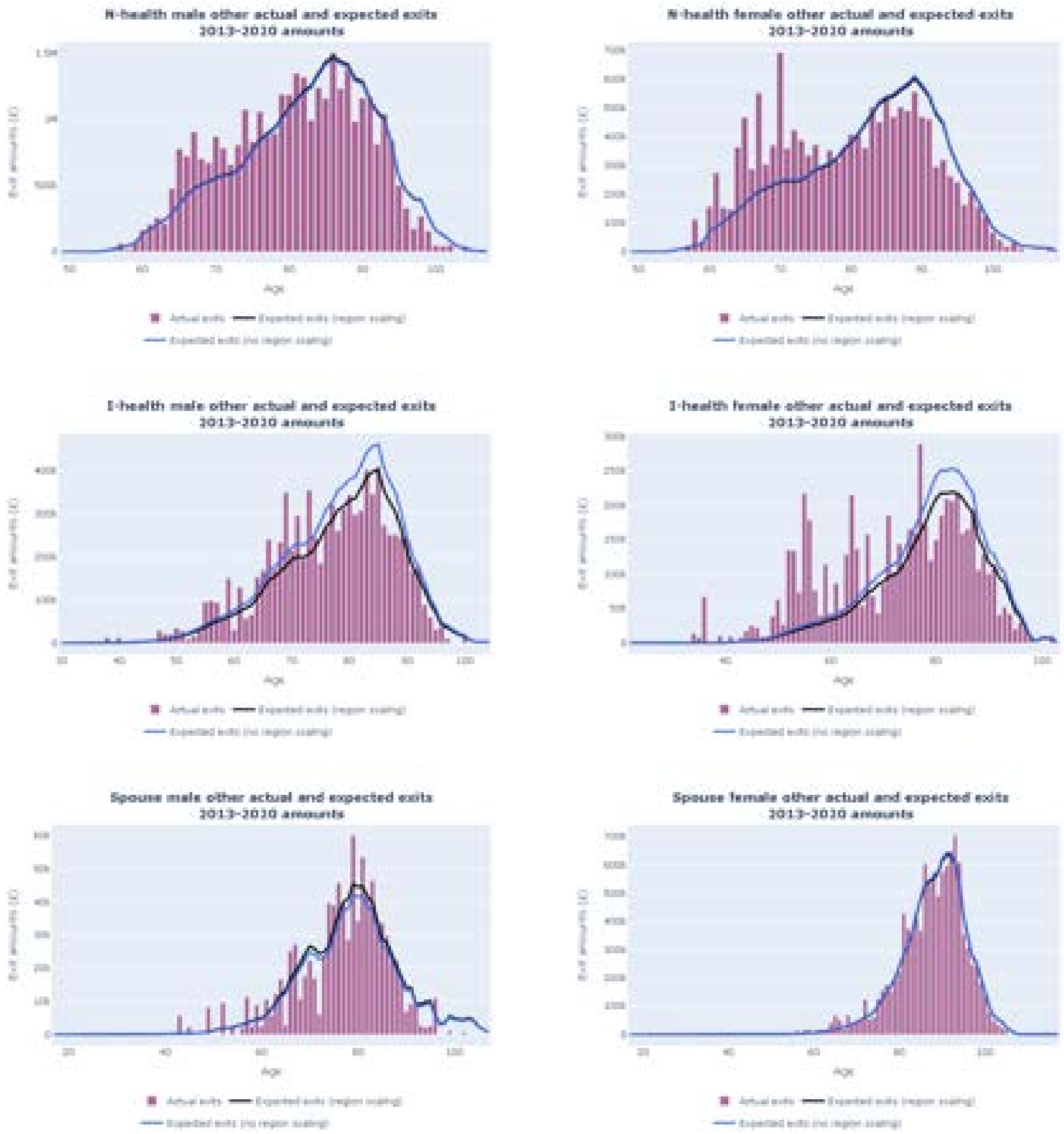
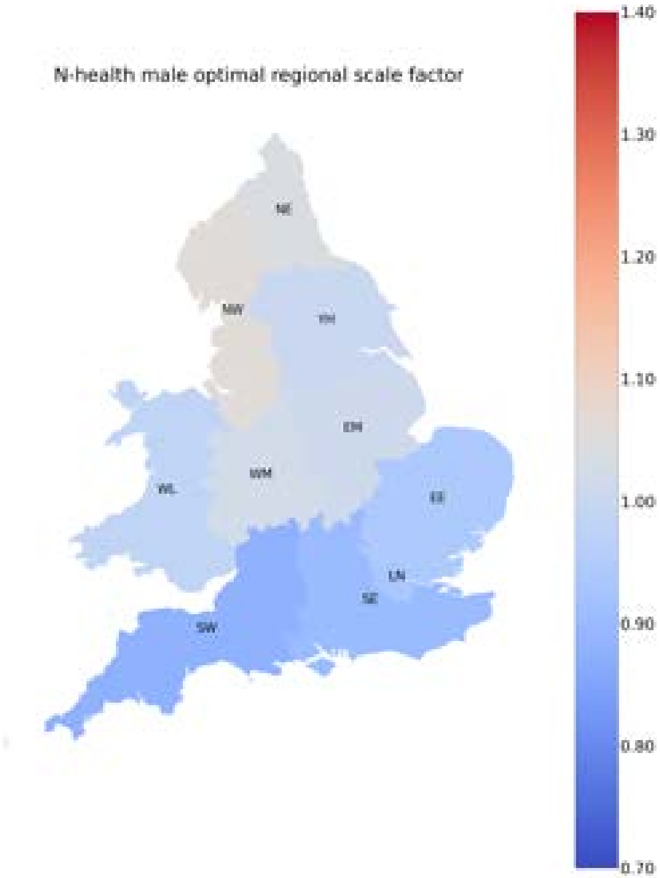
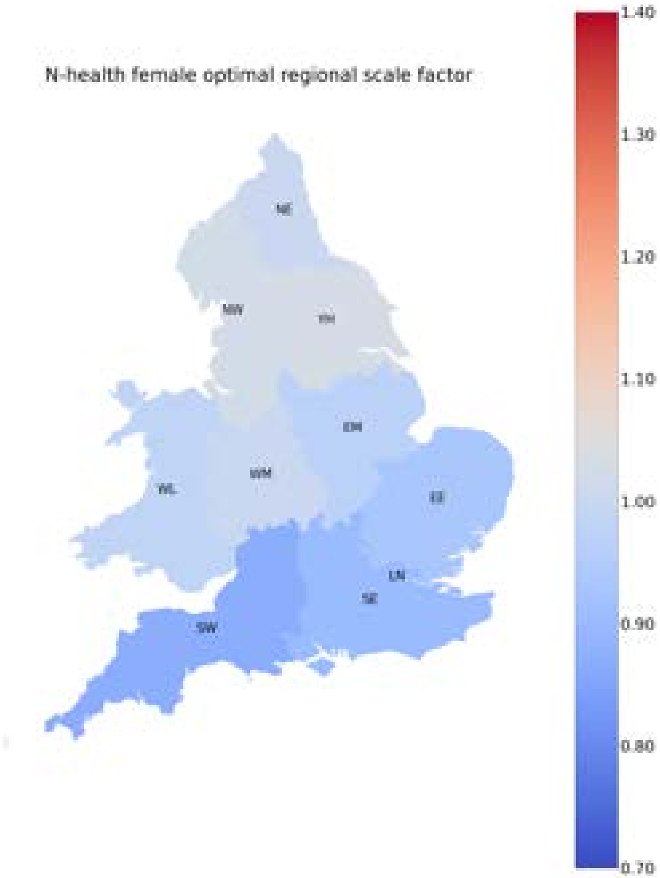


Figure 5.118: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on the application of optimal scaling factors to the relevant S3 CMI table used by GAD in the 2020 valuation, with and without region specification.



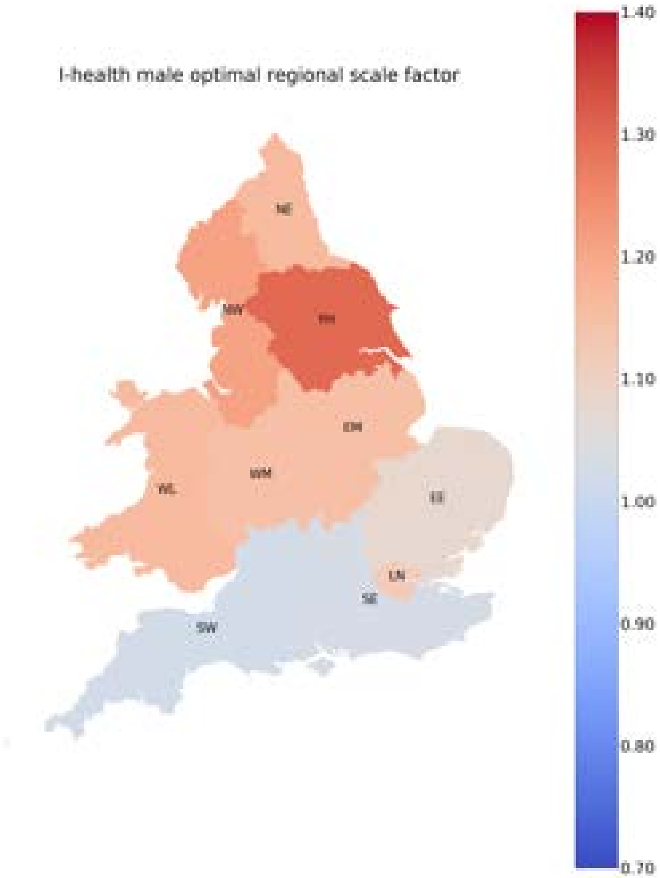
Region label	Region	N-health male optimal regional scale factor
EE	East of England	0.94
EM	East Midlands (England)	1.01
LN	London	0.93
NE	North East (England)	1.04
NW	North West (England)	1.06
SE	South East (England)	0.90
SW	South West (England)	0.88
WL	Wales	0.98
WM	West Midlands (England)	1.02
YH	Yorkshire and The Humber	1.00

Figure 5.119: Optimal scaling factor by region for normal health male pensioners, between 1st April 2013 and 31st March 2020.



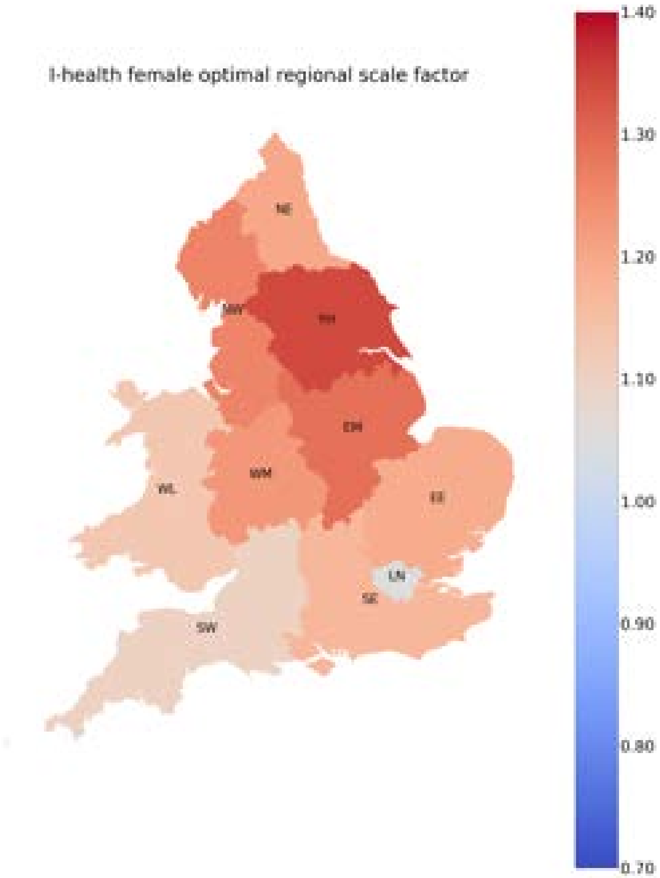
Region label	Region	N-health female optimal regional scale factor
EE	East of England	0.93
EM	East Midlands (England)	0.97
LN	London	0.90
NE	North East (England)	1.00
NW	North West (England)	1.02
SE	South East (England)	0.90
SW	South West (England)	0.87
WL	Wales	0.98
WM	West Midlands (England)	1.00
YH	Yorkshire and The Humber	1.03

Figure 5.120: Optimal scaling factor by region for normal health female pensioners, between 1st April 2013 and 31st March 2020.



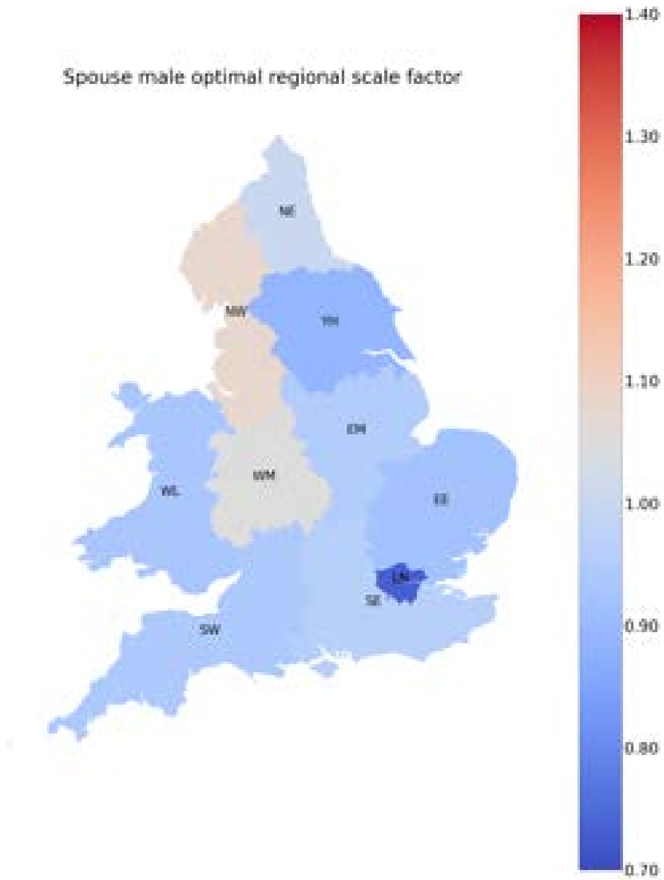
Region label	Region	I-health male optimal regional scale factor
EE	East of England	1.08
EM	East Midlands (England)	1.15
LN	London	1.12
NE	North East (England)	1.16
NW	North West (England)	1.21
SE	South East (England)	1.03
SW	South West (England)	1.02
WL	Wales	1.16
WM	West Midlands (England)	1.15
YH	Yorkshire and The Humber	1.30

Figure 5.121: Optimal scaling factor by region for ill health male pensioners, between 1st April 2013 and 31st March 2020.



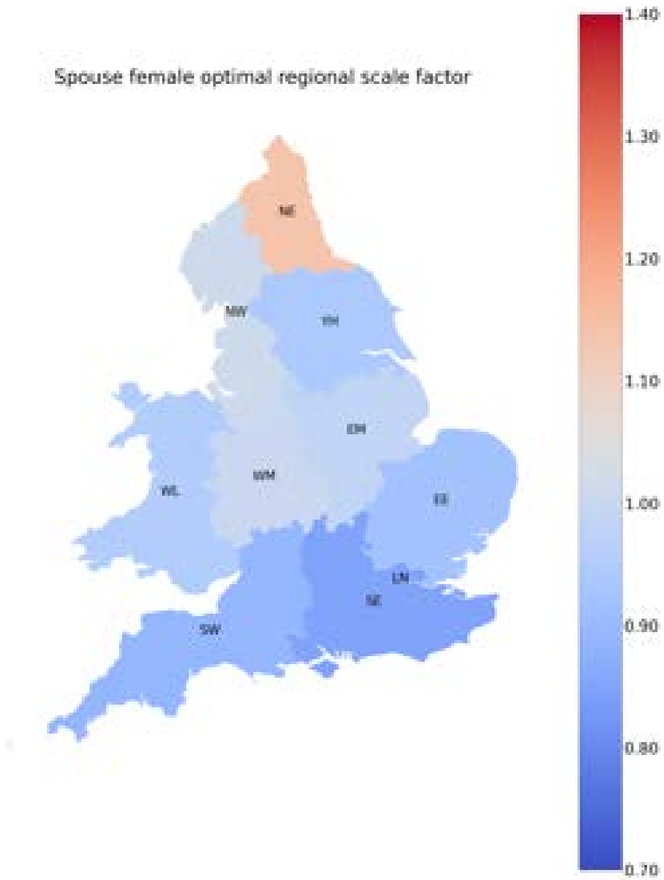
Region label	Region	I-health female optimal regional scale factor
EE	East of England	1.19
EM	East Midlands (England)	1.29
LN	London	1.05
NE	North East (England)	1.20
NW	North West (England)	1.26
SE	South East (England)	1.17
SW	South West (England)	1.10
WL	Wales	1.13
WM	West Midlands (England)	1.23
YH	Yorkshire and The Humber	1.34

Figure 5.122: Optimal scaling factor by region for ill health female pensioners, between 1st April 2013 and 31st March 2020.



Region label	Region	Spouse male optimal regional scale factor
EE	East of England	0.92
EM	East Midlands (England)	0.95
LN	London	0.73
NE	North East (England)	1.00
NW	North West (England)	1.08
SE	South East (England)	0.96
SW	South West (England)	0.94
WL	Wales	0.93
WM	West Midlands (England)	1.05
YH	Yorkshire and The Humber	0.89

Figure 5.123: Optimal scaling factor by region for male dependants, between 1st April 2013 and 31st March 2020.



Region label	Region	Spouse female optimal regional scale factor
EE	East of England	0.92
EM	East Midlands (England)	0.98
LN	London	0.85
NE	North East (England)	1.14
NW	North West (England)	1.01
SE	South East (England)	0.85
SW	South West (England)	0.89
WL	Wales	0.95
WM	West Midlands (England)	1.00
YH	Yorkshire and The Humber	0.94

Figure 5.124: Optimal scaling factor by region for female dependants, between 1st April 2013 and 31st March 2020.

5.4 Graduated mortality table analysis

A graduated mortality table is a statistical tool used in actuarial science and demography to represent a population's mortality rates, amended in order to smooth out irregularities and anomalies in the data. Graduation techniques adjust the CMR, derived from actual data, which can occasionally be erratic due to small sample sizes and other factors. Smoothing out the data stabilises the increments to mortality rates with age, improving the clarity of mortality trends. A formulaic approach may enhance accuracy and consistency of mortality rates, ensuring the criteria of smooth progression is achieved; the two types of formula applied are from the Gompertz and Gompertz-Makeham families.

The traditional Gompertz law of mortality posits that the mortality rate increases exponentially with age, a concept aligned with the biological observation where an individual's risk of death gradually grows with age. The Gompertz model is mathematically simple, utilising parameters that are relatively easy to interpret; proven to fit empirical mortality data particularly well for adults, it effectively captures this exponential increase in mortality rates. Due to its long established historical standing in demography and actuarial science, the Gompertz model's effectiveness is validated across numerous populations and time periods, bolstering its credibility. Its parameters can be estimated using widely available and recognised statistical techniques, such as maximum likelihood estimation or least squares fitting; moreover, the Gompertz model has a strong theoretical foundation, rooted in principles of probability and statistics, which compliments its rigorous application in mortality studies [75].

The Gompertz-Makeham model enhances the original Gompertz model with the addition of a constant term, accounting for age-independent mortality risks such as accidents or environmental factors; this incorporation refines the formula by making it more comprehensive and realistic. With the introduction of both age-dependent and age-independent components, this model often provides a more suitable fit to empirical mortality data across a broader age range; this is markedly useful for populations with significant nonage-related mortality risks. Its capacity to separately account for various sources of mortality risk enriches versatility, allowing the model to be employed to an assortment of populations and time periods. The parameters in the Gompertz-Makeham formula are interpretable, enabling clear communication of underlying mortality dynamics; the model retains the theoretical foundation of the original Gompertz model, whilst enhancing robustness and application with its realistic assessment of spontaneous risks that can occur, regardless of age. The Gompertz-Makeham formula is effective when fitting observed data, emphasising its credibility [76].

The graduation model applied to this analysis utilises the central EtR, assuming the number of deaths adheres to a Poisson distribution; this assumption may be misplaced for amounts-weighted data since the deviance of amounts-weighted mortality rate graduations is typically much greater than expected under the Poisson model; this often causes standard information criteria such as AIC, BIC and AICc to be inconsistent metrics when comparing model performance [77]; therefore, the amounts-weighted data has been scaled correspondingly to the S3 SAPS tables, enhancing the reliability of the applied statistical tests [7]. This process defines the average pension amount P_x at age x as:

$$P_x = \frac{E_x^A}{E_x^L} \tag{5.6}$$

E_x^A represents the exposure, weighted by amounts, at age x; E_x^L is the exposure, weighted by lives, at age x. The scaled death amounts D_x^S and scaled exposure amounts as E_x^S can be defined as follows:

$$\begin{aligned} D_x^S &= \frac{D_x^A}{P_x} \\ E_x^S &= \frac{E_x^A}{P_x} = E_x^L. \end{aligned} \tag{5.7}$$

As a result, both scaled and unscaled data, weighted by amounts, will have the same

CMR where D_x^S is broadly similar in size to D_x^L, as shown by:

$$\frac{D_x^S}{E_x^S} = \frac{D_x^A}{E_x^A}. \tag{5.8}$$

Graduations were generated for Gompertz formulae, with two to eight parameters, and Gompertz-Makeham formulae, with two to six parameters, noting that the former is a subset and simplification of the latter. For normal and ill health categories, rates were graduated for ages 60 to 95, whereas the starting age was set at 50 for spouses. Although methods for rate extensions to further ages are available, they were beyond the scope of the current analysis.

A generalised Gompertz-Makeham formula, allowing for polynomials of age and exponentials of polynomials of age, performed the graduations; it can be expressed as follows:

$$\mu_{x+0.5} = \sum_{i=1}^{r} a_i x^{i-1} + exp(\sum_{j=1}^{s} b_j x^{j-1}) \tag{5.9}$$

where:

- x is age;
- μ corresponds to the force of mortality;
- a_i and b_j are parameters to be estimated by the model.

In the event where r = 0, the GM(r,s) family formulaically reduces to Gompertz G(s), which is expressed as follows:

$$\mu_{x+0.5} = exp(\sum_{j=1}^{s} b_j x^{j-1}). \tag{5.10}$$

The formulae where $s = 0$ are also unnecessary since there is an equivalent GM(r,0) formula for any GM(r,1); therefore, the resultant list of formulae considered includes these Gompertz models:

- G(2), G(3), G(4), G(5), G(6), G(7) and G(8) and these Gompertz-Makeham formulae:

- GM(2,0), GM(1,2), GM(3,0), GM(1,3), GM(2,2), GM(4,0), GM(1,4), GM(2,3), GM(3,2), GM(5,0), GM(1,5), GM(2,4), GM(3,3), GM(4,2) and GM(6,0).

Generalising the Gompertz-Makeham formula allows for more intricate relationships between age and mortality rates; this is valuable when fitting more detailed or irregular mortality data.

The fitted $\mu_{x+0.5}$ rates can be converted to q_x rates for age x using the below conversion:

$$q_x \approx 1 - exp(-\mu_{x+0.5}). \tag{5.11}$$

Comparisons of actual q_x values against those generated by graduations, whereby various Gompertz formulae are utilised in each category, are displayed in Figure 5.125. Upon visual inspection, all models seemingly have reasonable and similar fit for normal health and spouse categories; for those with ill health, the lower order models slightly diverge at younger ages, leading to the underestimation of actual mortality rates and a somewhat flatter curve trajectory.

Comparisons of actual q_x values against those generated by graduations, whereby different order Gompertz-Makeham formulae are applied to each category, are displayed in Figure 5.126. Unsurprisingly, lower order models with exclusive polynomial age dependence do not have a close fit to actual mortality rate data; however, the higher order models, starting with GM(4,0), share a closer resemblance to the mortality rates by age. Consequently, this indicates that although powers of age can be applied to fit mortality rates, exponential terms more effectively account for most discrepancies between ages.

The charts lack clear visual distinction between many of the fitted models; in accordance with the production of SAPS S3 tables [7], the signs test, runs test, serial correlation test and QBIC were determined for each graduation. This combination of statistical tests allows for the quantitative assessment of model performance regarding the suitability of fit against actual mortality rates and in comparison to each other.

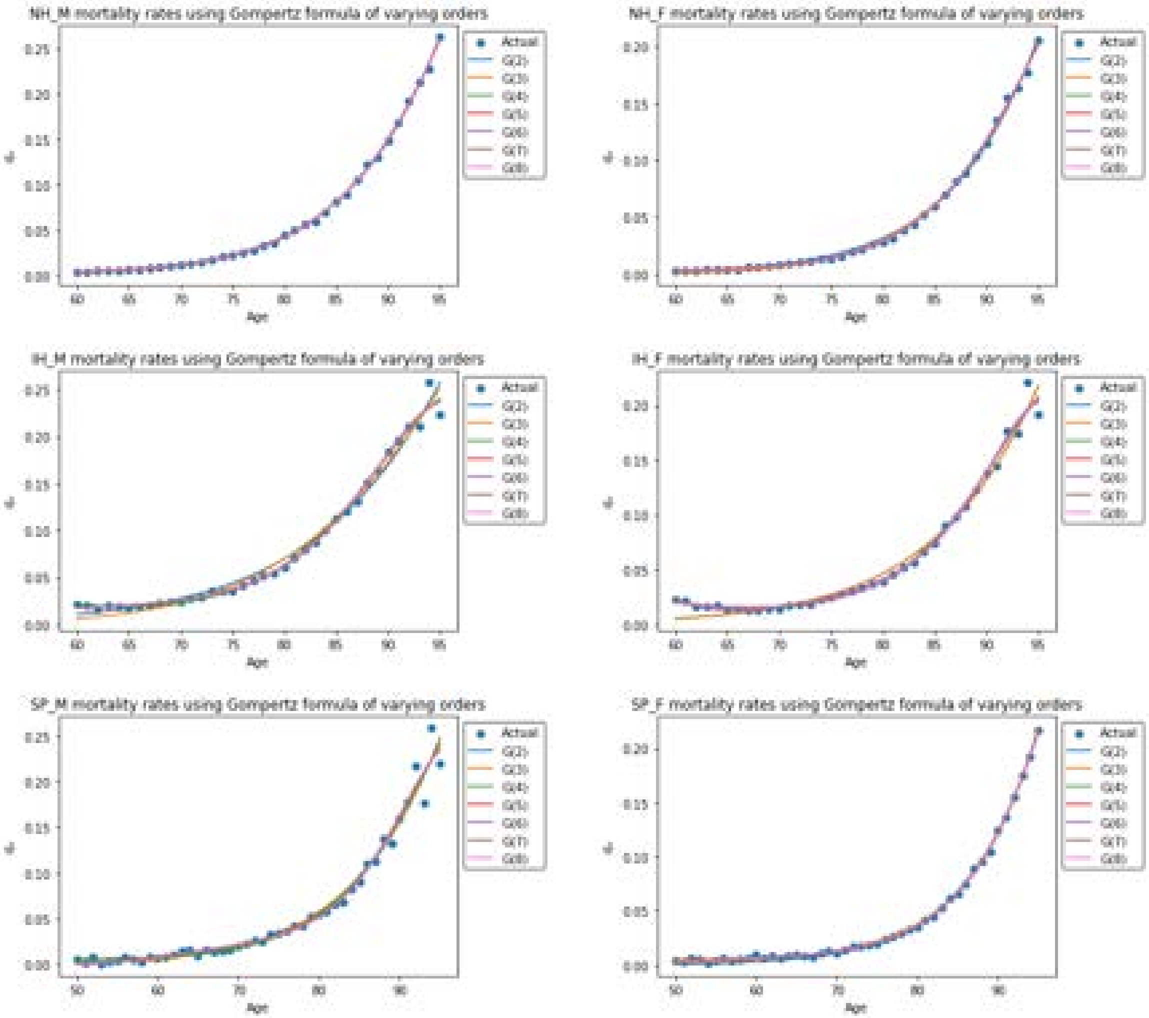


Figure 5.125: Graduated q_x mortality rates, using different Gompertz formulae against actual q_x mortality rates by age, for each category.

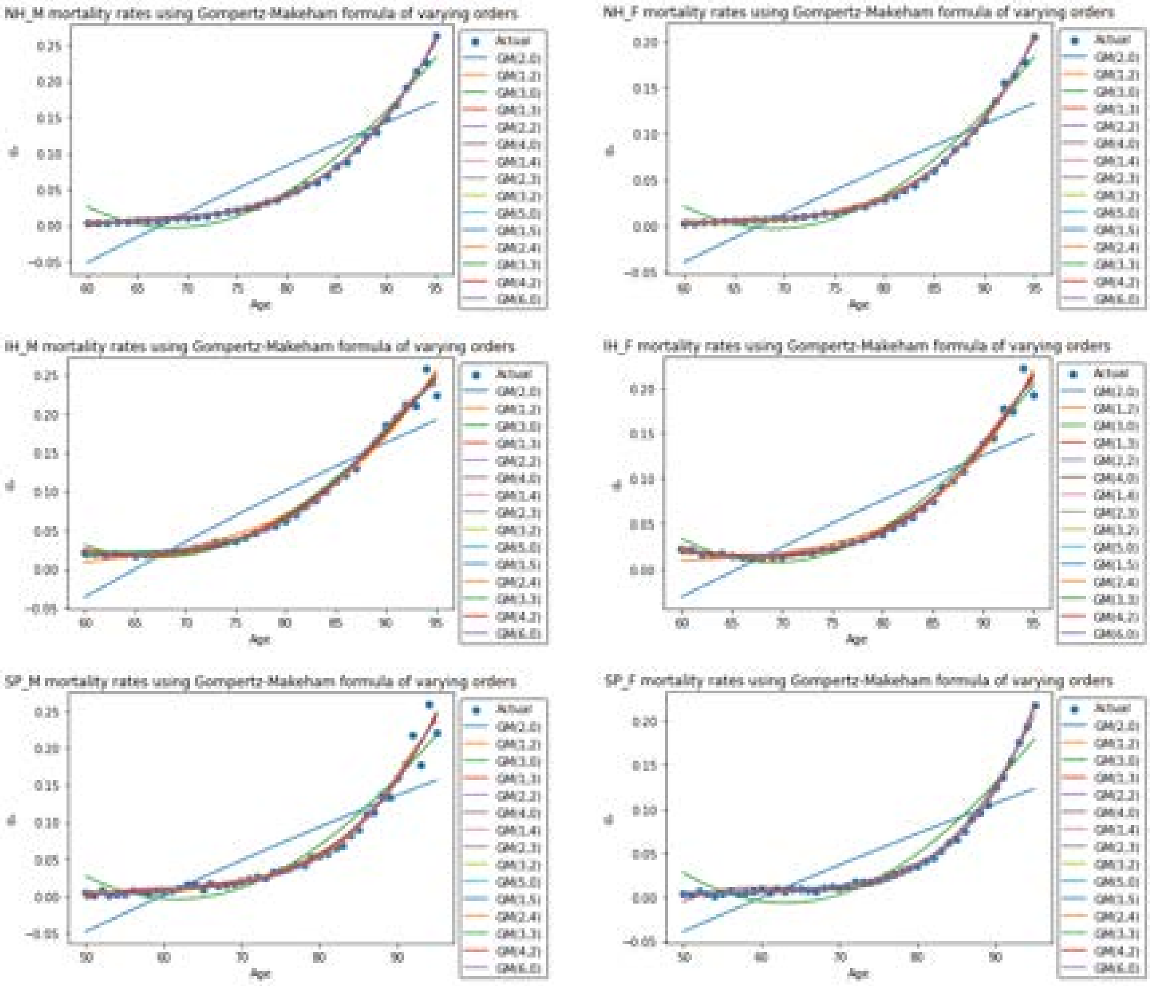


Figure 5.126: Graduated q_x mortality rates, using different Gompertz-Makeham formulae against actual q_x mortality rates by age, for each category.

For the signs, runs and serial correlation tests, the deviance residual at each age is employed; this indicates the likelihood of actual deaths as opposed to expected values derived from graduated mortality rates. The calculation is as follows:

$$DevRes_x = Sign(A_x - E_x)\sqrt{2(A_x ln(\frac{A_x}{E_x}) - A_x + E_x)} \quad (5.12)$$

where A_x and E_x are actual and expected deaths, respectively; $Sign(a)$ is +1 if $a > 0$, -1 if $a < 0$ or 0 otherwise.

If the graduated mortality rates are a suitable fit for actual data, the deviance residuals should be independent and normally distributed at each age [7]. The signs, runs and serial correlation tests assess whether the graduated models sufficiently satisfy this assumption.

The signs test aggregates the number of deviance residuals of both positive and negative signs; in other words, it calculates the amount of times the model over or underestimates the actual number of deaths. A favourable model would have an approximately equal number of each sign, therefore over or underestimating the number of deaths almost equally.

The runs test counts the amount of consecutive age groups where the deviance residual has the same sign; these are consecutive values where the model stays on a single side of over or underestimating the actual number of deaths. A suitable model would have a number of runs that is nearly half of the amount of considered ages, alternating regularly between over and underestimation.

The serial correlation test assesses the degree of correlation between sets of deviance residuals offset by an isolated age; the correlation expected to be roughly zero.

Each signs, runs and serial correlation test generates a test statistic with an associated probability value. the test is successful when the probability value is higher than a selected critical value of 5%, attributing any deviations from expected test behaviour to random variation.

Any models that failed the signs, runs or serial correlation test were excluded; out of the shortlisted models, the formula resulting in the lowest QBIC value was designated as the optimal model. However, a preferred model could be chosen in favour of the optimal model if certain conditions are satisfied. Firstly, the QBIC must be within five of the optimal formula; secondly, the preferred model is from the Gompertz family or it is the same type as the optimal model, with fewer parameters.

Standard information criterion metrics are often used to differentiate the performance of models, aiding the selection process; these attempt to determine which model fits the data best. Although these measures reward suitability of fit, they penalise adding parameters to the model and overfitting; preferred models have lower metric values. These metrics can be calculated by:

$$\begin{aligned} AIC &= Deviance + 2k \\ BIC &= Deviance + k \times ln(n) \\ AICc &= Deviance + 2k\left(\frac{n}{n-k-1}\right) \end{aligned} \tag{5.13}$$

where k is the number of parameters in the model and (n) refers to the amount of considered ages. The BIC has a harsher penalty than the AIC for the inclusion of additional parameters within the model, further discouraging overfitting. The AICc aims to correct the AIC's tendency to disadvantageously select models with excessive parameters when sample sizes are small.

The QBIC explicitly allows for overdispersion to overcome this problem of overfitting, whilst also typically leading to smoother graduations; it is calculated as:

$$QBIC = \frac{Deviance}{VIF} + k \times ln(n) \tag{5.14}$$

where VIF measures overdispersion, k signifies the number of parameters in the formula and n is the amount of considered ages. The VIF is estimated using the following formula:

$$VIF \approx \frac{Deviance_{Sat}}{n-k} \tag{5.15}$$

where $Deviance_{Sat}$ is the deviance resulting from a deliberately overfitted or saturated formula [7]. The recommended number of parameters for the saturated formula is ten [77] and is, thus, applied to this analysis.

The optimal and preferred models are shown in Figure 5.127. The only instance where the preferred model was different from the optimal model was for female spouses; similar to the preferred model for normal health males, a fourth order Gompertz model was preferred over a lower-order optimal Gompertz-Makeham formula. The simplest preferred relationship produced was for the male spouse category where a second-order Gompertz model was chosen. The ill health categories required the most complex formulae since there was a unique decrease in mortality rates with age at the younger ages, prior to a dramatic increase and hints of plateauing at the oldest ages.

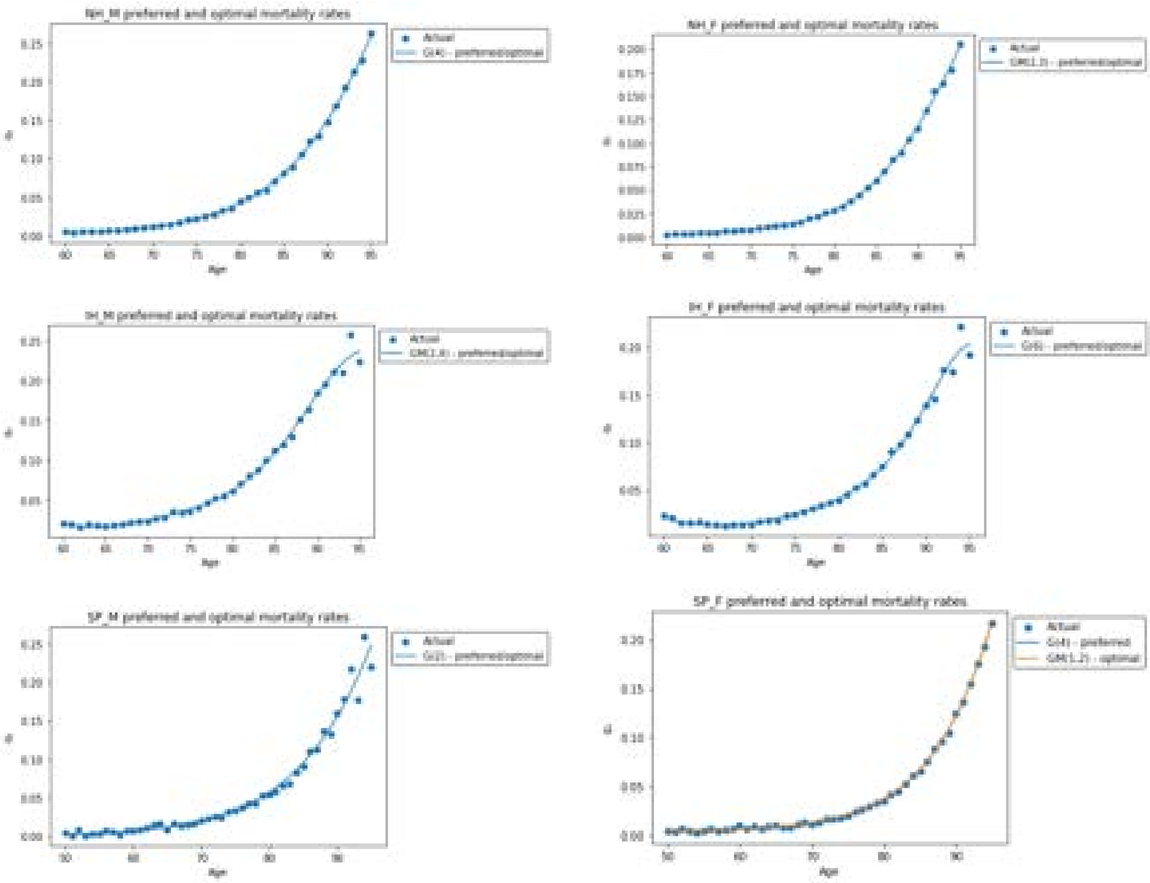


Figure 5.127: Graduated q_x mortality rates using optimal and preferred (if different) formulae against actual q_x mortality rates by age for each category.

Tables 7.79 to 7.84 in the Appendix (Chapter 7) display the fitted parameters for each

Gompertz model, along with a selection of metrics on goodness of fit. Similarly, Tables 7.85 to 7.96 present the equivalent metrics for each Gompertz-Makeham model.

The metrics for models depicting males of normal health designate the G(4) model as the simplest model to successfully pass every signs, runs and correlation test. With a probability greater than 5%, there is insufficient evidence to suggest that it systematically over or underestimates mortality rates at each age, produces sequences of over or under-estimation or results in correlation between adjacent residuals. Therefore, the occurrence of such patterns could reasonably be attributed to random variation, concluding, as the null hypothesis states, an absence of systematic bias in the model due to these elements. In addition to being the lowest order model to pass the validity tests, the G(4) model also has the lowest QBIC value, cementing its unequivocal selection as the optimal and preferred model. Although excluded from the model selection criteria, other statistics further support this conclusion, such as AIC, AICc and BIC.

For normal health females, all of the Gompertz models failed the correlation test, though the metrics suggest that G(5) would have been the best fit if successful. The optimal GM(2,3) model had the lowest QBIC, with other statistics, such as AIC, AICc, BIC, deviance and R^2 concurring with this selection. The situation is similar for ill health males, except that the optimal valid formula was GM(2,4); the best-fitting unsuccessful Gompertz model was G(4).

For females of ill health, the sole Gompertz model to pass the tests was the G(6) formula; despite this, the G(4) model has a lower QBIC and would have been preferred if the validity criteria had been met. Other metrics, such as AIC, AICc, BIC, deviance and R^2 support the G(6) formula over the G(4) model. None of the Gompertz-Makeham models satisfactorily passed the validity tests for this category.

For male spouses, all of the Gompertz models passed by the validity tests; G(2), the optimal model, was determined based on it achieving the lowest QBIC value; other statistics such as AIC, AICc, BIC and deviance agree with this selection. Although numerous of the Gompertz-Makeham models also passed the tests, they underperformed in their QBIC value in comparison to the G(2) model.

For female spouses, the G(4) model is the lowest order Gompertz model that passed the validity tests, also maintaining the lowest QBIC score within this set; the GM(1,2) model passes the signs, runs and correlation tests with a lower QBIC than the G(4) model, concluding it as the optimal choice. However, since the QBIC of the G(4) model is within five of the GM(1,2) QBIC, this model is designated as the preferred option; this is attributed to the fact that a Gompertz family model is simpler to represent mathematically than a Gompertz-Makeham.

5.4.1 Graduated mortality tables compared to baseline

After developing graduated models to fit the data, it is important to determine whether they are significantly more accurate as compared to SAPS S3 mortality tables.

Table 5.128 juxtapose F-test results for optimal graduated models with SAPS S3 tables, which have incorporated optimal scaling adjustments. For males of normal health, the optimal graduated model is as complex as the CMI model used to derive SAPS S3 tables, providing a superior fit with a probability significantly below the 5% critical threshold. Similarly, the graduated model for females of normal health contains an equal order of parameters as the formula used for SAPS S3 tables, although it is composed using the Gompertz-Makeham family; however, since probability exceeds the 5% critical value, the graduated model is not considered statistically better, owing the performance difference to random variation. This category uniquely fails to yield a significant improvement when utilising the graduated formula.

For males with health issues, a closer fit can be observed with the optimal Gompertz-Makeham graduated model, which includes additional parameters in contrast to SAPS S3 formula; with a p-value significantly below 5%, the increased model complexity is justified, as it offers a markedly closer fit to the data. The same is true for this category's female counterpart, with the exception that the optimal formula belongs to a higher order within the Gompertz family.

In the male spouse category, the optimal graduated model contains a lower order of parameters, thus supplying a weaker fit than the SAPS S3 model; nevertheless, the F-test is indicative of a significantly better performance, relative to the number of parameters, when measured against the higher order SAPS S3 model. For female spouses, the graduated model is strongly preferred as it is simpler, with fewer parameters than its SAPS counterpart, maintaining a superior fit.

Table 5.129 displays the F-test results, comparing preferred graduated models with SAPS S3 tables, which have incorporated optimal scaling adjustments. The female spouse category exclusively employs a preferred graduated formula that differs from the optimal model; the former yields identical F-test results as the latter, whilst advantageously being simpler as well as maintaining a closer fit than the SAPS S3 model. All these factors validate the adoption of the simpler, preferred model.

Category	DoF (F)	DoF (S)	Formula (F)	Formula (S)	RSS (F)	RSS (S)	F	P-value
N-health male	32	32	G(4)	G(4)	6,207,078,701,838	14,875,820,368,710	2.40	0.00785
N-health female	31	31	GM(2,3)	G(5)	1,342,108,215,863	2,335,077,169,613	1.74	0.06428
I-health male	30	32	GM(2,4)	G(4)	898,599,810,244	3,749,978,142,687	47.60	0.00000
I-health female	30	31	G(6)	G(5)	506,045,087,973	4,367,302,159,027	228.91	0.00000
Spouse male	44	42	G(2)	G(4)	31,830,912,813	24,000,521,255	5.41	0.00000
Spouse female	43	41	GM(1,2)	G(5)	427,180,475,778	1,053,486,673,444	N/A	0.00000

Figure 5.128: F-test comparing optimal graduated models (F) against SAPS S3 tables, which have optimal scaling adjustments applied (S).

Category	DoF (F)	DoF (S)	Formula (F)	Formula (S)	RSS (F)	RSS (S)	F	P-value
N-health male	32	32	G(4)	G(4)	6,207,078,701,838	14,875,820,368,710	2.40	0.00785
N-health female	31	31	GM(2,3)	G(5)	1,342,108,215,863	2,335,077,169,613	1.74	0.06428
I-health male	30	32	GM(2,4)	G(4)	898,599,810,244	3,749,978,142,687	47.60	0.00000
I-health female	30	31	G(6)	G(5)	506,045,087,973	4,367,302,159,027	228.91	0.00000
Spouse male	44	42	G(2)	G(4)	31,830,912,813	24,000,521,255	5.41	0.00000
Spouse female	42	41	G(4)	G(5)	402,111,309,699	1,053,486,673,444	N/A	0.00000

Figure 5.129: F-test comparing preferred graduated models (F) against SAPS S3 tables, which have optimal scaling adjustments applied (S).

Figure 5.130 illustrates actual exit amounts from 1st April 2013 to 31st March 2020 by age for each category, alongside expected exit amounts, derived from the preferred graduated formula in addition to relevant SAPS S3 rates with optimal scaling factors. The SAPS S3 model for normal health categories overestimates exits for younger ages and underestimates them at older ages; the graduated rates appear suitably correct this issue. The opposite is true for those of ill health, with underestimation occurring at younger ages and overestimation at older ages; this effect is more pronounced for ill health than for normal health. Although the graduated rates also seem to correct this, exits in the middle age range could potentially be slightly overstated. Due to higher volatility present in the male spouse data, there is visual obscurity regarding which model best fits the data; however, graduated rates appear to provide lower estimates at younger ages and higher estimates at older ages. The female spouse model illustrates the smoothest and simplest shape, with graduated and SAPS S3 models sharing a close resemblance; an exception can be observed where graduated estimates are slightly higher, drawing closer to actual exit amounts at younger ages.

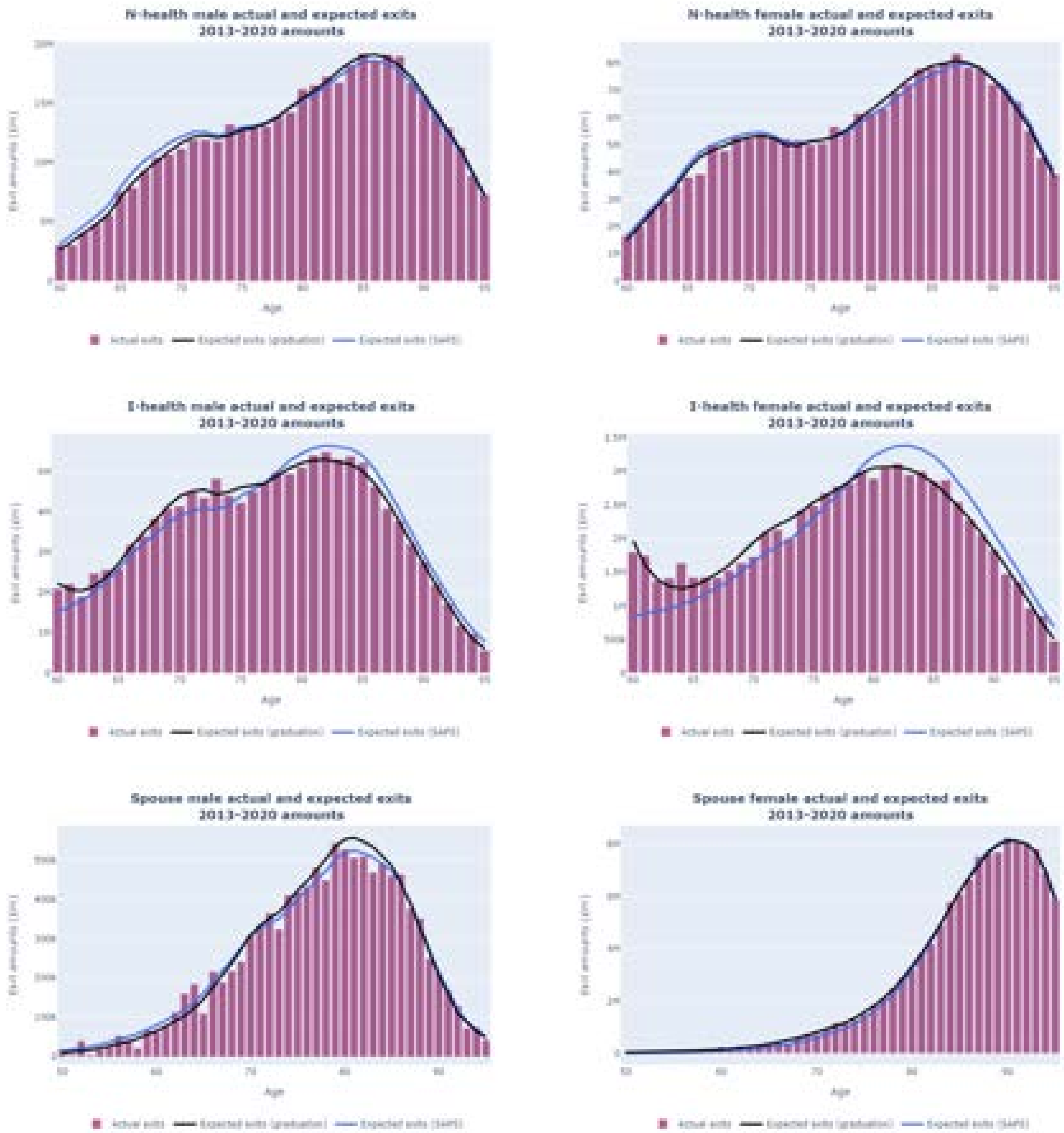


Figure 5.130: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts, based on applying graduated rates from the preferred formula, in addition to expected exit amounts, based on applying relevant SAPS rates with optimal scaling factors.

5.5 ML regression analysis

This section explores the potential employment of a ML regression approach to model mortality experience data. A decision tree algorithm is initially evaluated, with the expectation that performance enhancements could be achieved with the subsequent adoption of advanced techniques.

Decision trees have the capacity to manage both numerical and categorical variables, making them versatile tools for regression tasks. Whilst their simplicity may not always yield the most accurate models, they serve as an excellent baseline for comparison with more sophisticated methods. A decision tree regression model is a ML algorithm focused on the prediction of continuous values; it structurally consists of root nodes, internal nodes and leaf nodes. The root node is the uppermost node, which is representative of the entire dataset; internal nodes categorise data into subsets using specific features; leaf nodes, located at the ends of branches, present the predicted values [78].

Decision trees are highly intuitive and can be easily visualised, lending itself to clear interpretation and logical deductions; their hierarchical structure resembles human decision-making, increasing its accessibility even to non-professionals. This high comprehensibility is crucial in situations where understanding the reasoning behind conclusions and judgements are as essential as the prediction itself. Decision trees are versatile, handling datasets with a mix of continuous and categorical features in addition to target variables of either type; its adaptability permits their application to a vast range of problems. Since they do not require feature scaling or normalisation, they can accommodate data that has been processed less than other methods; their capability in handling missing values and outliers renders them suitable for raw and noisy data. Decision trees innately implement feature selection, splitting the data using the most informative features; a feature's importance is determined based on its frequency of appearance and its position within the tree; the greater the significance, the more recurrent and earlier the feature will appear [79].

Decision trees can become too convoluted when they reach an excessive depth, ultimately overfitting the training data; this manifests as a deficiency in smoothness of the fitted regression line or classification boundaries, resulting in poor generalisation to unseen data. Techniques such as pruning and specifying a maximum tree depth help to mitigate this issue. A small change in the training data can culminate in a completely different tree structure, making decision trees unstable and sensitive; sensitivity can be reduced by implementing ensemble methods, such as boosting or random forests, which focus on building multiple trees and the aggregation of their results [80]. Decision tree models are often less accurate than other ML methods; they can be biased towards the majority class in unbalanced datasets, inducing suboptimal splits and the misclassification of minority instances. Techniques such as oversampling, undersampling or using cost-sensitive learning help address these issues. Overall, decision trees remain a popular choice in many applications due to their simplicity, versatility and ease of interpretation.

Building a decision tree uses recursive partitioning to split the data into smaller subsets; at each step, the algorithm selects the feature and its corresponding threshold, which creates the greatest reduction in variance of the target variable. This division continues until a stopping criterion is fulfilled, such as maximum tree depth or minimum amount of samples per leaf node. Predictions within a decision tree are guided by traversing from the root to a leaf node, following the conditions defined at each internal node; the value at the leaf node is used as the predicted value for the input data provided.

Residuals represent differences between actual observed values and predicted values, which have been estimated by the regression model. Mathematically, the residual for each data point is calculated as:

$$\text{Residual} = \text{Actual Value} - \text{Predicted Value} \tag{5.16}$$

A positive residual indicates that the actual value is greater than the predicted value, whilst a negative residual indicates the reverse. Ideally, the residuals should be approaching zero, exemplifying the model's predictions close alignment to the actual data.

Residuals serve several purposes, such as evaluating how successfully a regression model fits the data; smaller residuals reflect a better fit. Upon examination, patterns of residuals can be identified, such as systematic bias, heteroscedasticity and outliers; these can signify opportunities for refinements or additional features to be explored, in order to improve the model. Residual plots help validate assumptions such as homoscedasticity and normality of regression models; scatter plots of residuals, contrasting against predicted values, help assess homoscedasticity; Q-Q (quantile-quantile) plots compare the distribution of residuals to a normal distribution, which helps confirm normality assumptions; density plots simply display the distribution of residuals. Therefore, understanding residuals is critical for model evaluation and improvement, in addition to revealing how effectively the model captures underlying relationships within the data.

Interpreting a plot of residuals against predicted values is beneficial for the performance evaluation of a ML algorithm. Ideally, residuals should be randomly distributed around zero, illustrating how the model successfully captures underlying patterns well without systematic bias; the spread of residuals should be constant across all predictions since deviations are indicative of heteroscedasticity, where error variance is inconsistent. Clear patterns, such as curves, denote the inaccurate specification of the model, whereby key features or interactions could be absent. Outlier points of a considerable distance from the bulk of residuals may indicate mistakes in measurement or data entry; high leverage points with extreme predictor values can disproportionately influence the model, especially in the presence of large residuals. Non-linear patterns in residuals imply the model fails to capture these significant relationships in the data, concluding that a more complex model or predictor transformation is necessary [81].

Deciphering a normal Q-Q plot involves the comparison of sample quantiles of the data and theoretical quantiles of a normal distribution; if the data points approximately follow a straight line, the sample data more or less follows a normal distribution. Systematic deviations from the line, such as an S-shaped curve, denotes heavier leptokurtic tails, whilst an inverted S-shaped curve suggests lighter platykurtic tails; these tails are vital elements of the distribution since ascending or descending curving tails signal unexpectedly divergent outliers in comparison to a normal distribution [82].

When optimising decision tree regression models, the chosen performance metric should conform to the objectives; two common measures for regression ML models are RMSE and R^2. RMSE calculates the average deviation between predicted values from the regression model and actual observed values in the data; lower RMSE values indicate that the model is a better fit to the data. RMSE can be calculated using the following formula:

$$\text{RMSE} = \sqrt{\frac{\Sigma(P_i - O_i)^2}{n}} \tag{5.17}$$

where Σ represents the sum of differences between predicted and observed values; P_i is the predicted value for the i_{th} observation; O_i is the observed value for the i_{th} observation; n is the sample size. RMSE helps to perceive the typical distance between predictions and actual values [83].

The coefficient of determination, also known as the R^2 value, quantifies the proportion of variance in the targeted response variable that can be explained by the predicting features; higher R^2 values denote better explanatory power of the model. R^2 can be calculated by:

$$R^2 = 1 - (RSS/TSS) \tag{5.18}$$

RSS represents the sum of squares of residuals, which are distinctions between predicted and observed values; TSS expresses the total sum of squares and, therefore, the variation in the response variable. R^2 helps to assess the extent to which the predictor variables explain this aforementioned variation [73].

RMSE focuses on prediction accuracy, measuring the degree of similarity between predictions and actual values; on the other hand, R^2 emphasises the explanatory power of the model, depicting the proportion how well the features account for the target variation. Both metrics should be considered when evaluating a decision tree regression model, since they provide valuable complimentary insights.

A low R^2 value indicates that a small proportion of the variance in the targeted response variable is due the predicting feature variables. While this might not be ideal generally, a model with low R^2 can still be satisfactory in specific scenarios. Moreover, the domain-specific context should be considered; the intrinsic nature of the problem or domain itself occasionally limits predictability, such as complex instances and interactions where models struggle to explain all variations. A low R^2 can be acceptable during the initial exploratory analysis or hypothesis generation, where models identify potential relationships and patterns requiring further investigation; such a model can also serve as a useful baseline for comparisons. If a more complicated model does not significantly improve R^2, the additional complexity cannot be justified. Achieving high R^2 can be challenging with limited data, affecting model performance due to the presence of greater amounts of noise that are generally present in smaller datasets. Simpler models, including those with low R^2, are often considered more interpretable; for instance, decision trees and linear regression both contribute valuable insights into feature importance.

In this analysis, the target variable is numeric, consisting of death pension amounts accounted for by each record; these values are either zero or the pension amount at the valuation date. Therefore, model performance is assessed based on R^2, attempting to fit each data point well and explain the greatest proportion of target variance. Given the low probability of death events and binary nature of the target variable at the individual record level, the model's estimations should fall between the two possible target values; as a result, it may underestimate the death pension amounts for actual death records and overestimate them for living records. However, the model may still yield a similar overall death pension amount to the actual total.

Correlation between numeric variables in the data are depicted in Figure 5.131. UK and nation-specific (NS) IMD deciles display an extremely high correlation; surprisingly, both IMD deciles are only weakly correlated with pensions at the valuation date, despite higher IMD values suggesting lower levels of deprivation. There is only a very weak correlation for IMD with most of the other variables. The other extremely strong positive correlation is between the pension receipt age relative to SPA and the age at pension receipt date variables; this is unsurprising given that the former is partially derived from the latter. Age at valuation date, the other variable involved in the calculation, shows moderately strong positive correlation with pension duration at valuation date, pension receipt age relative to SPA and age at pension receipt date. SPA and age at valuation date show reasonably strong negative correlation; this is anticipated as SPA has increased with time and is typically higher for younger people. Another moderately strong negative correlation can be viewed between pension receipt age relative to SPA and the SPA itself, where younger people with higher SPA's have more margin regarding earlier retirement.

Unsurprisingly, factors showing the strongest relationships with the target variable, albeit relatively weak, were age at the valuation date, pension at the valuation date and pension duration at the valuation date. Age and pension amount are known to be useful indicators of mortality, as discussed in Chapter 2; other indicators can be identified by examining variables with only strong correlations to the target variable. Factors correlating highly with each other are more inclined to supply overlapping information upon predicting the target variable, reducing the value of their individual contribution towards the estimation. Age at valuation date and pension at valuation date have very weak correlation between each other and relatively strong correlation with the target variable, marking them as a good combination to gauge mortality. On the other hand, the high correlation between age at valuation date and pension duration at valuation date are indicative of a high codependency, resulting in similar information being redundantly provided; therefore, utilising only one of these factors may be preferred as it could hinder the identification of root causes underlying mortality.

IMD does not show substantial correlation with the target variable; this could be due to higher IMD values indicating an increased probability of remaining alive in a given year, represented as the lowest value of zero in the target variable, in addition to a higher pension for deaths, leading to a higher than average target variable value. The correlation between SPA and the target variable is predictably very weak, since numerous people with different characteristics will have an identical SPA. Pension age relative to SPA and age at pension receipt date offer the possibility of a greater contribution towards accurately predicting the target variable, since they are representative of a member's conscious decision, as opposed to a predetermined characteristic, such as SPA; however, while there is greater correlation between these two factors than for SPA, it still remains relatively weak.

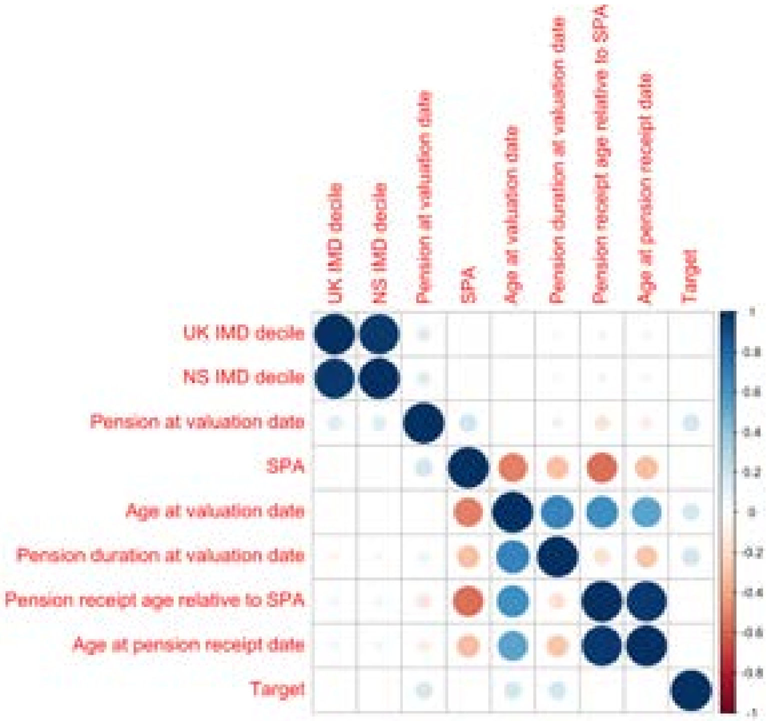


Figure 5.131: Correlation between numeric variables in the data.

Primarily, a regression decision tree ML algorithm was applied to the dataset using all available variables; this aimed to discover the upper limit of the model's potential performance. By identifying the most significant features, further exploration can determine whether fewer variables can achieve a similar performance. Cross-validation was conducted with 3 folds and 2 repeats; each fold was used once as a validation set with the remaining two forming the training set. This approach offers every data point the opportunity to participate in the validation set, with each fold containing approximately one-third of the data. Although there would be a reduction in bias for the model evaluation, an increase in variance may be witnessed due to the relatively small size of the training sets. Repeating the cross-validation means the entire procedure of splitting the data into 3 folds, training and validating the model, is conducted multiple times; this duplication yields more reliable and stable performance estimates by calibrating the results across multiple iterations. Cross-validation more precisely appraises the model's performance regarding unseen data, enhancing its generalisation capability. Moreover, cross-validation efficiently utilises the available data and has the capacity to detect overfitting; if the model consistently performs well, providing accurate predictions across different folds and repeats, overfitting is less likely.

The fit of the decision tree algorithm was optimised in accordance with the R^2 metric; this involved fine-tuning the model's parameters, such as tree depth, cost complexity and the minimum data points required in a node for subsequent splitting. Tree depth refers to the longest path extending from the root node to a leaf node; this parameter sets the maximum depth allowed in the model, forcefully planting terminal nodes at this depth and regulating complexity to the desired level [80].

The cost-complexity measure is a function of the number of terminal nodes and their total misclassification rate; this parameter maintains the balance between tree complexity and accuracy. It is a non-negative value that determines the penalisation of complexity in the model. Larger values result in a simpler tree with fewer splits and terminal nodes, whereas a smaller value denotes a more sophisticated tree with a greater degree of splitting and more terminal nodes.

The parameter for the minimum data points required in a node for subsequent splitting permits additional control and customisation over the decision tree's structure. If a node contains fewer observations than the set value, splitting will be restricted and it will become a terminal node.

Tree depths ranging from 1 to 15 were tested for every selected feature combination to identify the optimal depth that maximises the R^2 metric; this value was consistently maximised by reducing other parameters. As a result, cost complexity was set to 0.0000000001 and the minimum number of data points in a node for further splitting was fixed at 2.

Variables that are considered direct indicators of resulting state, such as reason for exit and exit date, were excluded as predictive features; categorical variables with numerous distinct values, such as postcode and area, were excluded due to their excessive impact on model processing power and complexity. Fitting the saturated model provided insight into the model's potential performance and deduced the most significant variables for rate prediction of the resulting state (see Figure 5.132).

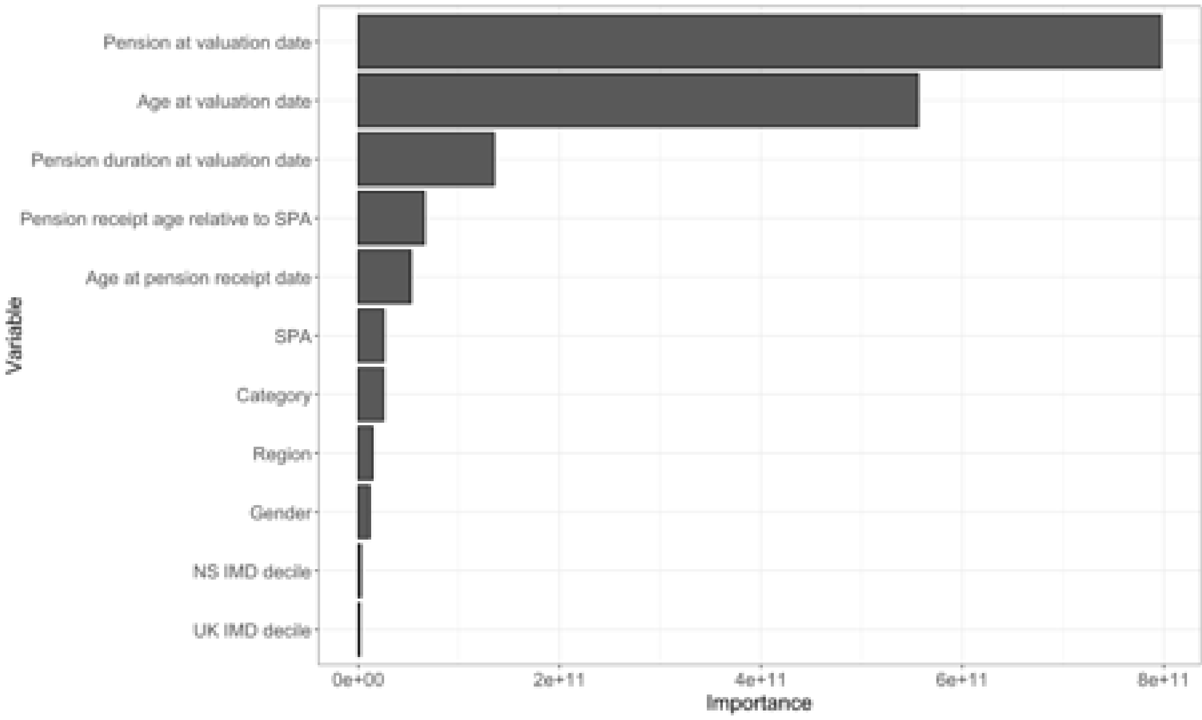


Figure 5.132: Feature importance of all variables used to generate the optimal fitted model.

Given that the target variable is zero for living records, only reflecting the pension amount for deaths, pension at the valuation date was unsurprisingly the most dominant feature in the generation of the decision tree model. Age at the valuation date also shows substantial significance, widely used for differentiating the probability of death. There is a notable drop in influence for all remaining variables, with pension receipt age relative to SPA, age at pension receipt date and SPA ranking next in decreasing prominence. Due to the strong correlation between these variables to age at the valuation date, their usefulness is diminished due to duplicated information. Category, region and gender are character variables that supply additional qualitative information, aiding in the differentiation of outcomes. IMD demonstrated the least impact amongst included variables.

Figure 5.133 presents a scatter plot of residuals against predicted values, a Q-Q plot comparing the distribution of residuals to a normal distribution and the distribution of residuals from the optimal saturated model using all variables. The scatter plot of residuals displays non-random patterns, such as clustering, suggesting that the model may not comprehensively capture all data aspects. The high concentration of residuals at lower predicted values and diffused residuals at higher predicted values indicate heteroscedasticity, where variance of residuals changes with predicted values. As predictions increase, the spread of residuals widens, implying that errors increase in accordance to predictions. Some residuals appear quite far from zero, conveying the model's failure to capture all data nuances; this is as anticipated expected due to the sheer amount of factors influencing mortality and their intricate relationships, which the model is simply unable to fully recreate, especially as the patterns suggest they are highly unlikely to be purely linear. The Q-Q plot shows points deviating from the straight line, especially at the tails, highlighting how the data does not follow a normal distribution; this is evident from the steep curvature at both extremes of the plot, which displays heavier tails and more extreme outlier values than expected in a normal distribution. Points in the middle of the plot stay close to the line, suggesting the central portion of the data approximates a normal distribution. Points far removed from the identity line, especially when positioned around the tails, indicate potential outliers. The model's residuals exhibit a symmetric distribution around zero, with even allocation above and below it; this implies that the model does not systematically make excessive or inadequate predictions. A high peak can be observed at zero, highlighting that many residuals maintain a relatively close proximity to zero, which suggests that many of the model's predictions are accurate; the tails of the distribution have an extensive reach in both directions, indicating some large residuals, which signify the presence of significant errors in predictions for certain cases, despite the

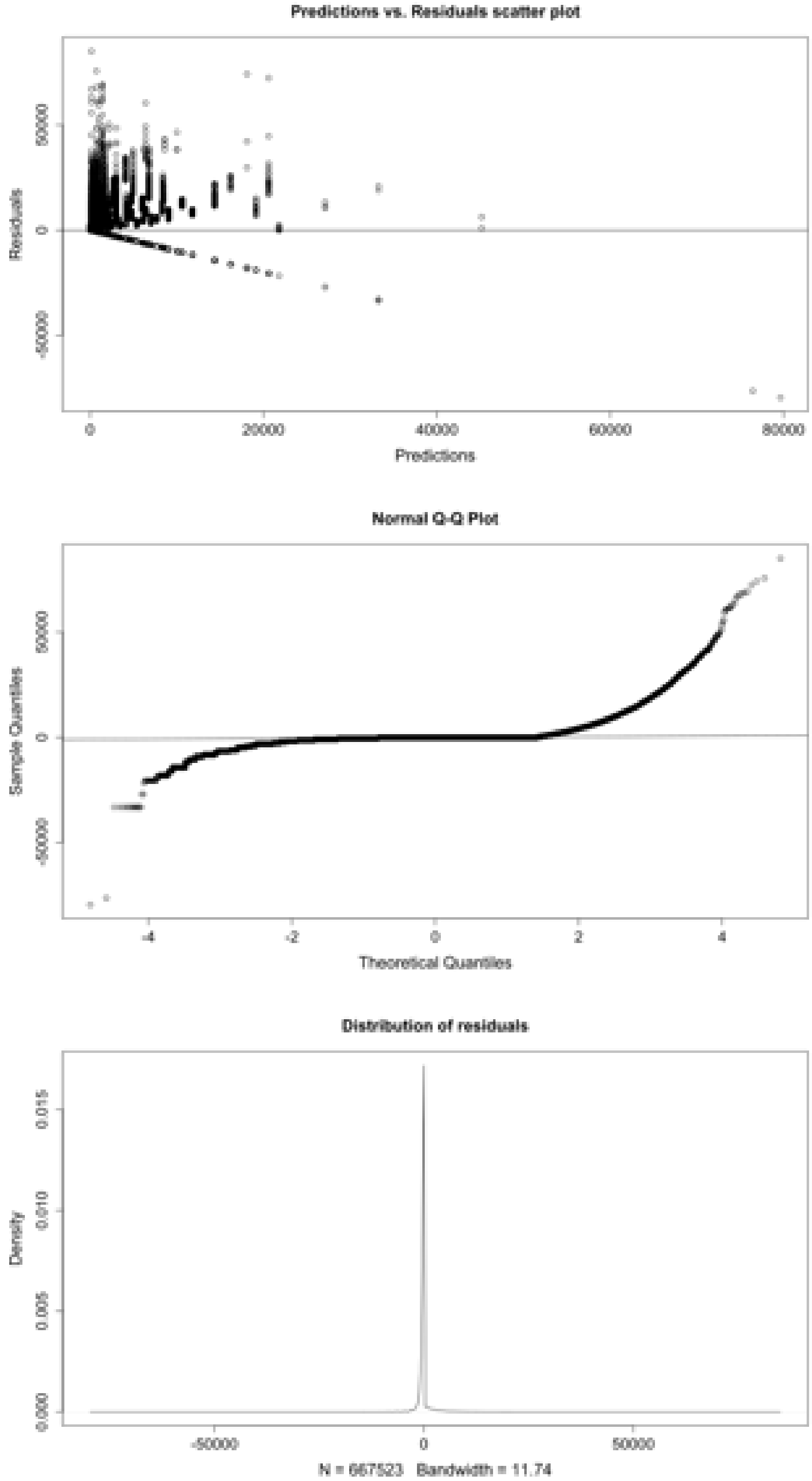


Figure 5.133: Scatter plot of residuals against predicted values, Q-Q plot comparing the distribution of residuals to a normal distribution and distribution of residuals from the optimal fitted model generated using all variables on the test data. Tree depth = 7 and $R^2 = 0.160$ (0.156 on the training data).

model performing well in many circumstances. Given the vast number of observations, it's common to see residuals spread over a broad range; the density values are relatively low, except for the sharp peak at zero, which is a characteristic shape for a plot with many observations [84].

The density plot denotes the model's satisfactory performance in general, though specific areas could be improved, such as the tails. The residual and Q-Q plots, in tandem, insinuate issues regarding model fit, such as non-normality, heteroscedasticity and outliers; they are not necessarily a primary concern of decision trees given that these models are non-parametric, robust to outliers and capable of capturing non-linear relationships, though they should be noted as they could negatively impact the model's predictive performance and reliability. Further fine-tuning of the decision tree's parameters could be investigated to minimise these effects.

Despite using a saturated model, overfitting does not seem to be an issue; this could be attributed to cross-validation increasing the model's robustness by training it across multiple subsets of data. The maximised R^2 value on training data was 0.156 and the resulting R^2 on testing data was 0.160; these relatively low R^2 scores denote a limitation of the decision tree regarding the lack of variance explained. Perhaps this is due to insufficient relevant data used for predicting the target variable, with other unavailable factors influencing outcomes; this is an unsurprising conclusion, given the relatively small number of features per record and complexity of factors affecting mortality. If the model lacks sufficient features to accurately distinguish between living and death records, predictions may be imprecise due to the aggregation of all outcomes, where the average of both zero and non-zero values are considered as opposed to one or the other. However, there may be reasonable accuracy at the group level despite significant deviations in each case, the target variable balances overestimation in living records with underestimation in death records. Alternatively, the model may have assimilated relationships between current variables that are too simplistic to capture underlying patterns, resulting in underfitting. The model does not seem to overfit the training data, as it would typically

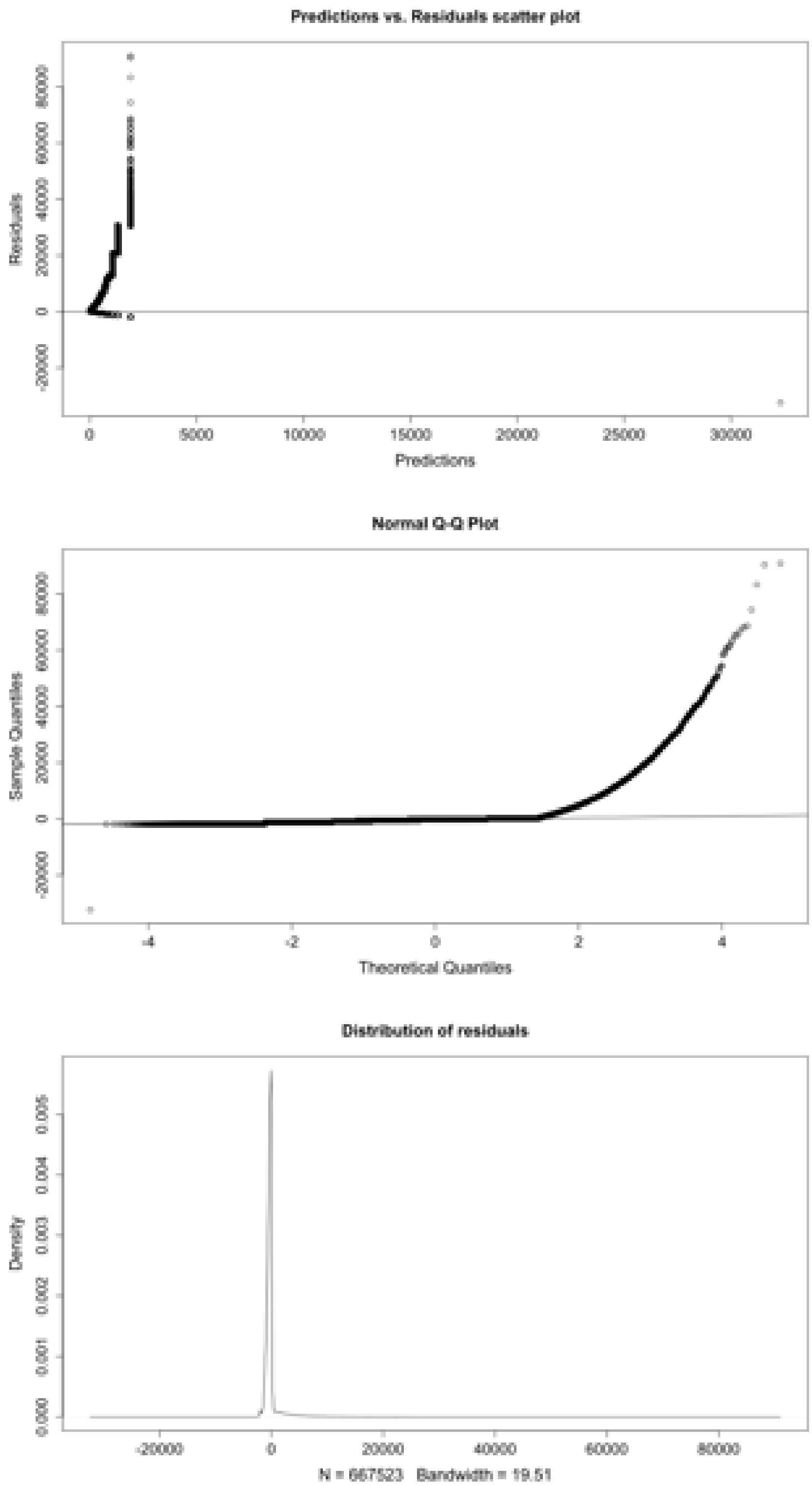


Figure 5.134: Scatter plot of residuals against predicted values, Q-Q plot comparing the distribution of residuals to a normal distribution and distribution of residuals from the optimal fitted model generated using only the pension at valuation date variable on the test data. Tree depth = 4 and $R^2 = 0.031$ (0.031 on the training data).

produce a much higher R^2 value, relative to the testing data. The similar values represent how the model's performance consistently generalises to new and unseen data. The insignificant difference between training and testing scores confirms the model's stability and dependable performance across a variety of datasets; this consistency is generally a favourable sign, though the low R^2 values are reflective of the model's overall predictive shortcomings.

The saturated model reveals the potentially achievable performance of the modelling approach; nevertheless, using all available variables is not particularly efficient as many of them hold little significance. Ideally, maximising efficiency could be achieved by starting with the most valuable feature only, iteratively adding subsequent variables if they sufficiently improve the R^2 value enough to warrant the increased number of parameters. Using only the most relevant variable, pension at valuation date, yields an underwhelming R^2 value of 0.031 in both the training and test data; in light of this considerable under-performance, as determined by the saturated model, this feature is fundamentally insufficient to maximise the available data's potential to predict accurately. Figure 5.134 shows the scatter plot of residuals against predicted values, a Q-Q plot comparing the distribution of residuals to a normal distribution and the distribution of residuals from the optimal model generated using only pension at valuation date. The residuals increase significantly as predicted values increase, displaying heteroscedasticity where residual variance is not constant across predictions; this can be problematic, given the model's error variance dependence on prediction size. The residuals are not symmetrically distributed around the horizontal zero line, with the majority appearing positive; this is especially prevalent for higher predicted values, indicating the model's bias to under-predict larger values, suggesting a bias. There are extreme residuals, usually at the higher end of predicted values, manifesting as data points that the model struggles to forecast with precision. Outliers could be symptomatic of data anomalies or the model's constraints in capturing the complexity of those regions. The pattern of residuals alludes to the model's failure to fully comprehend underlying relationships, particularly for greater val-

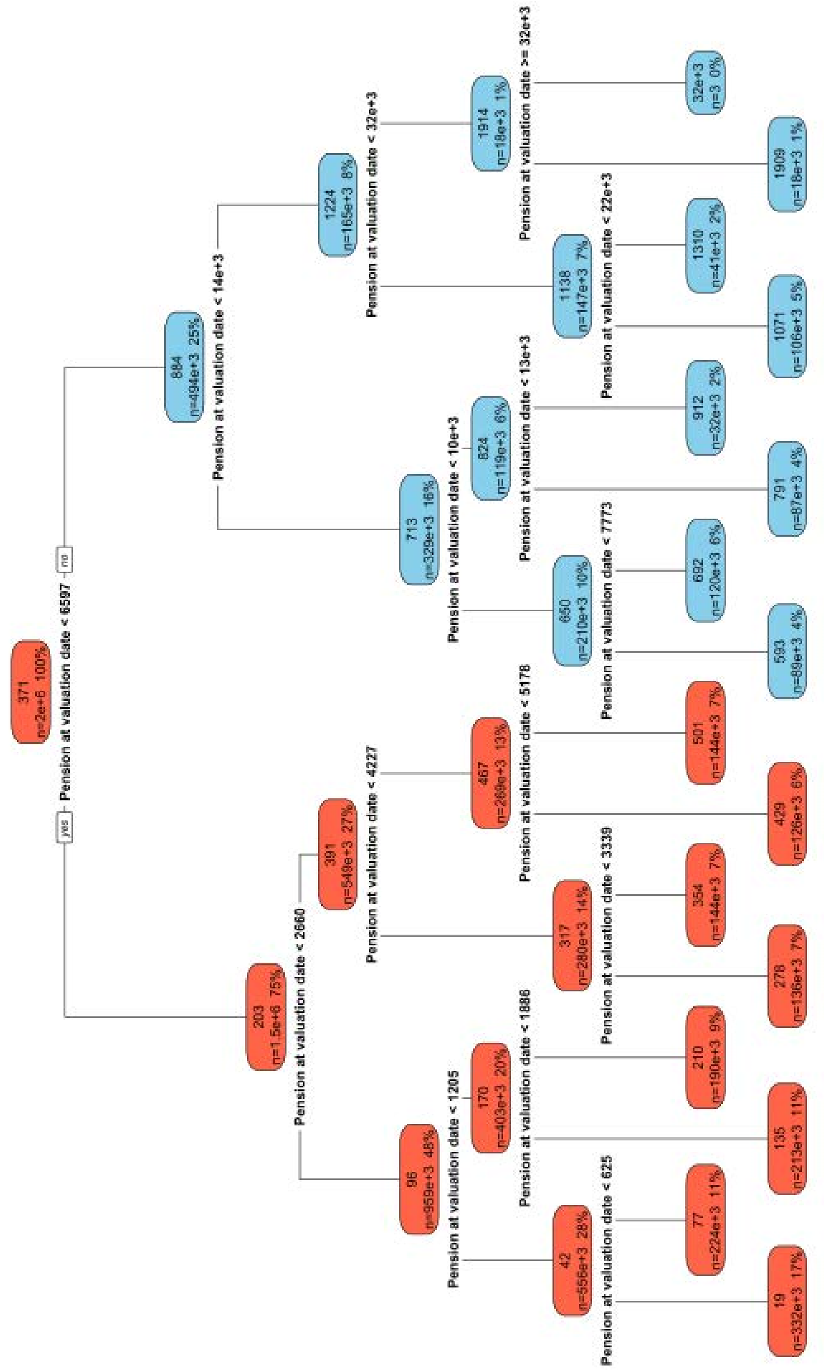


Figure 5.135: Decision tree from the optimal fitted model generated by using only the pension at valuation date variable. Tree depth = 4. $R^2 = 0.031$ (0.031 on the training data). Values inside nodes include the mean value used for predictions (top) followed by number (bottom left) and proportion (bottom right) of observations passing through it from the training data.

ues, expressing a need for a more sophisticated model or additional features to sharpen its predictive power. The concentration of residuals in close proximity to zero for lower predictions denotes the model's reasonable performance for smaller values, deteriorating for larger amounts; these fluctuations in performance signify that the model is not equally or wholly reliable. The Q-Q plot demonstrates significant deviations, where the data does not follow a normal distribution, especially at tails with steep curvature; possible heavy tails can be observed in addition to positive skew, which could be a consequence of the plot's asymmetry with increased deviation and an extended tail on the positive side.

Figure 5.135 provides one example of the decision trees produced using only the pension at valuation date variable; this visualisation depicts the methodology behind generating the decision tree model. This selective approach creates predictions of the target variable by collecting an incremented series of yes and no responses regarding feature variables; the tree depth is fixed at four, reflective of the four layers of decisions within the model. The summit of the tree initiates the process with a singular node, composed of all training data records with each subsequent decision dividing the data into two nodes in the following layer. The number at the top of the nodes depicts the mean response variable for predictions; the amount of observations and relative proportion of the dataset, which is transferred through each node, are also displayed. The data partition is regulated using pension values at the valuation date, above and below £6,597. In consideration of the model's single feature variable, all choices are established using various threshold values of the same item, prompting lower predictions of the response variable at nodes in accordance with lower pension values at the valuation date. Final model predictions are concluded at the deepest node levels. For example, when pension at valuation date falls below £625, in the case of approximately 17% of data, the target variable value is predicted to be £19 using the mean value of such records in the training data. Although this diagram is especially effective in improving knowledge of how the ML algorithm operates, the reasoning behind why particular decisions and thresholds were selected are not always easy to comprehend.

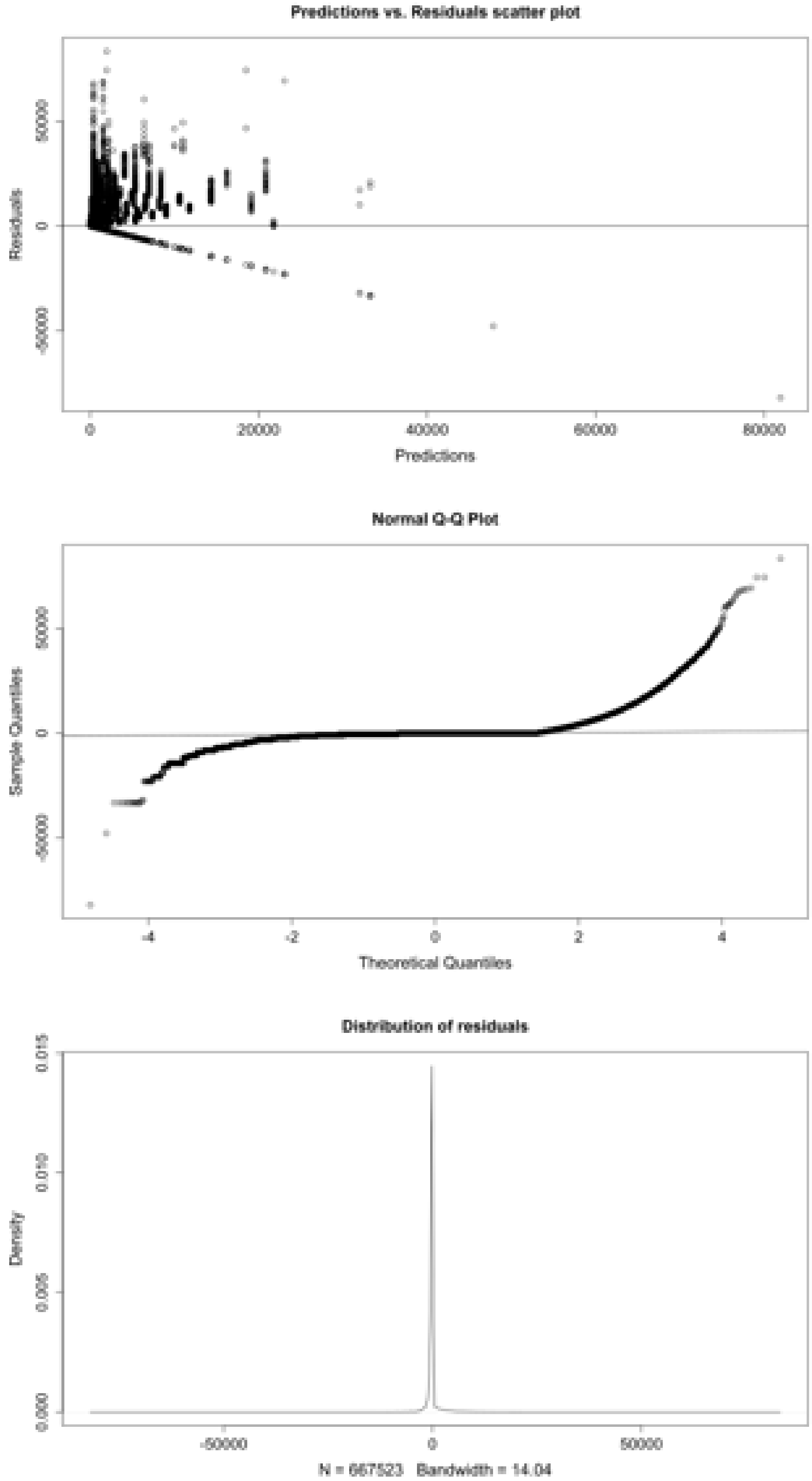


Figure 5.136: Scatter plot of residuals against predicted values, Q-Q plot comparing the distribution of residuals to a normal distribution and distribution of residuals from the optimal fitted model generated using the pension at valuation date and age at valuation date variables on the test data. Tree depth = 7 and R^2 = 0.158 (0.154 on the training data).

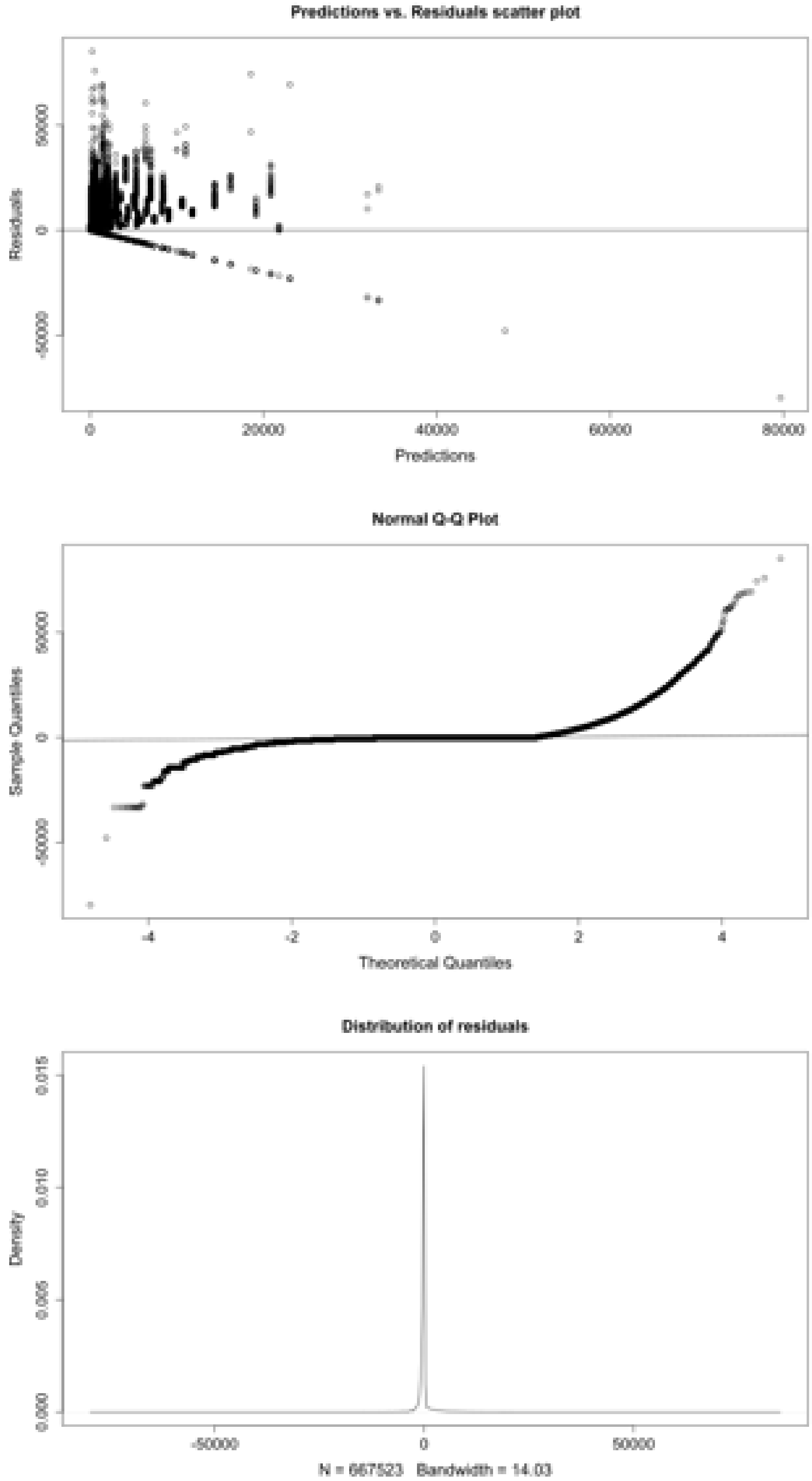


Figure 5.137: Scatter plot of residuals against predicted values, Q-Q plot comparing the distribution of residuals to a normal distribution and distribution of residuals from the optimal fitted model generated using the pension at valuation date, age at valuation date and category variables on the test data. Tree depth = 7 and $R^2 = 0.160$ (0.157 on the training data).

Adding the secondary most important feature, age at the valuation date, significantly improves the model's R^2 value and aligns it closer to the saturated model's performance. This is exemplified in the residual, Q-Q and density plots in Figure 5.136, illustrating similar findings to those in Figure 5.133. The optimal tree depth of 7 results in the potential for 128 predictive nodes, which is simply too elaborate to display; as expected, its comprehensive nature leads to lower target variable estimations for a reduced number of combinations of age and pension at the valuation date.

The next most suitable variable to implement, based on feature importance ranked in Figure 5.132, theoretically should have been pension duration at valuation date; however, gains in the R^2 metric were restrained due to strong correlations with age and pension at valuation date features already actively participating in the model (see Figure 5.131); this limitation is also applicable to pension receipt age relative to SPA, age at pension receipt date and SPA variables. Nevertheless, adding adding category to the model elevated R^2 performance to a level comparable to the saturated model, producing similar residual, Q-Q and density plots as shown in Figure 5.137.

Model description	Optimal R^2 tree depth	Training RMSE	Test RMSE	Training R^2	Test R^2	Expected exit amounts (£)
Saturated	7	1,842	1,832	0.156	0.160	1,076,737,584
Pension only	4	1,973	1,967	0.031	0.031	1,052,571,227
Age added	7	1,845	1,834	0.154	0.158	1,076,241,128
Category added	7	1,842	1,832	0.157	0.160	1,076,719,188

Figure 5.138: RMSE and R^2 metrics on training and testing data sets using optimal R^2 tree depths for various models. The sum of actual exits in the data is £1,064,836,398.

Additional variables showed insubstantial performance gains with Table 5.138 summarising results from saturated and incremented models. Training and test RMSE values were higher than for saturated model in iterations before the category feature was added, indicating a higher deviation between predictions and actual values; this is also verified for R^2 and expected exit amounts, with the exception of lower scores from iterated models. The R^2 progression highlights the disproportionate variance explained in earlier iterations; implementing the category variable produced RMSE and R^2 scores equal to the saturated model rounded to the specified degree. Although the model's total expected exit amounts, inclusive of added category, were closest to the saturated model's values, the actual exits in the data were still overestimated.

5.5.1 ML regression compared to baseline

Development of a regression ML model to fit the data is beneficial in determining the extent of improvement regarding fit in comparison to the SAPS S3 mortality tables used as the baseline analysis.

Category	Best fit scaling factor
Normal health males	1.10
Normal health females	0.77
Ill health males	1.19
Ill health females	0.83
Spouse males	1.11
Spouse females	0.88

Figure 5.139: Proposed optimal scaling factors for the ML algorithm to minimise the least square residuals of actual and expected exit amounts across each age for each category.

The best-fit scaling factors, applicable to the ML model, were calculated for each category by minimising the least square residuals of actual and expected exits across each age; the resulting scaling factors are detailed in Figure 5.139, with large deviations from one indicating a need for further improvements to optimise the ML model for those categories. Specifically, this model appears to balance out the gender impact, yielding male category scaling factors that are substantially above one and female scaling factors considerably below it.

Category	DoF (ML)	DoF (SAPS)	Scale (SAPS)	RSS (ML)	RSS (SAPS)	F	p-value
Normal health males	54	54	0.97	91,374,891,092,586	15,505,133,213,036	0.17	1.00000
Normal health females	58	57	0.95	7,449,341,934,160	2,573,180,486,731	0.35	0.99996
Ill health males	74	74	1.13	3,753,026,961,746	5,803,531,555,305	1.55	0.03135
Ill health females	82	81	1.19	3,865,965,543,504	12,594,080,907,454	3.26	0.00000
Spouse males	87	87	0.98	251,894,909,352	24,976,395,423	0.10	1.00000
Spouse females	94	93	0.94	2,803,895,406,208	1,437,684,646,757	0.51	0.99929

Figure 5.140: F-test comparing the scaled ML model against SAPS S3 CMI tables using optimal scaling factors for each category.

Table 5.140 presents an F-test comparing the optimally scaled regression ML model against SAPS S3 tables with optimal scaling adjustments applied to each category. Four parameters are used for male SAPS S3 categories with female categories employing five [7]; the ML algorithm uses four parameters.

The complexity of the ML regression model for males maintains identical conditions to those deriving SAPS S3 tables, whereas the SAPS S3 table formula uses an additional parameter for females. For normal health and spouse categories, the ML regression model does not have a significantly better fit than the SAPS S3; the extremely high p-value instead conveys that the SAPS S3 table is a superior fit, which is affirmed by the substantially lower RSS value. On the other hand, the ML regression model for ill health categories significantly improves fit to the data in contrast to SAPS S3 tables, particularly for females.

Category	Actual average exit age	Expected exit age (ML)	Expected exit age (SAPS)
Normal health males	81.10	82.19	80.72
Normal health females	81.00	80.75	80.97
Ill health males	75.63	75.59	77.23
Ill health females	73.47	75.03	78.21
Spouse males	78.13	80.04	78.22
Spouse females	87.69	88.38	88.37

Figure 5.141: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Figure 5.145 lends credence to these observations by highlighting the ML regression model's partial capture of higher levels of death at younger ages in ill health categories as well as reducing overestimation of deaths at older ages. This model underestimates actual exits at lower ages for males of normal health and overstates them at higher ages; male spouses suffers from similar, though more dramatic effects, whilst the opposite is true for females of normal health, whereby overestimation is present at lower ages and understatement at higher ages. The visually best-fitting category for the ML model appears to be female spouses where expected values follow actual data reasonably well with a relatively smooth fit. The lines for ML expected exits are consistently not as smooth as their SAPS S3 counterparts across categories, noticeably fluctuating from age to age, which could be resolved by applying smoothing techniques, such as averaging across age groups. Line smoothness improves at older ages where a larger number of exits are prevalent, denoting how greater volumes of exit data, especially at younger ages, could naturally induce smoother rates.

Category	EtR	Actual exits	Expected exits (ML)	Expected exits (SAPS)
Normal health males	20,206,012,254	461,626,826	448,642,944	465,910,222
Normal health females	14,679,818,643	211,641,570	210,162,925	211,738,645
Ill health males	3,807,814,871	147,091,489	149,393,108	140,994,004
Ill health females	3,054,413,132	93,699,818	88,734,146	80,173,577
Spouse males	386,664,214	11,188,474	10,459,551	11,374,389
Spouse females	2,568,006,884	139,475,485	134,706,368	137,244,882

Figure 5.142: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Figure 5.141 further substantiates some of these insights, evidenced by the smaller distance between SAPS S3 expected exit ages and actual exit ages for normal health and spouse categories, though this is not the case for ill health. The expected exit age in the ML model is not similar to the actual value for ill health females, suggesting that the ML model's improvements are likely attributed to the poor fit of SAPS S3 tables as opposed to the ML regression being an excellent fit. Nevertheless, the ML model does show average exit ages that are highly aligned to actual data for males with health issues.

Figure 5.142 draws similar conclusions using expected exits generated by each model in comparison to actual exits. The ML model predictions of total exit amounts are more accurate for those with ill health, though less so for normal health and spouse categories; the ratio of actual to expected exits is displayed in Figure 5.143, signifying that ML regression model values are only closer to one for ill health, having demonstrated more distant values for all other categories. The ML model usually underestimates total exit amounts in most categories, resulting from the application of least squares optimisation scaling.

Figure 5.144 illustrates that there is an imbalance regarding the yearly allocations, despite the relatively reasonable overall expected exit amounts, generated by the ML regression model, in contrast to actual values. The ML algorithm is evidently unable to accurately distinguish whether death occurred for a specific record, causing all these values to be averaged across groups. All living data records correspond to the end of the scheme year in 2015 and 2019, with other years therefore only containing exit records. The observed yearly pattern can be attributed to the overestimation of death amounts in living records, with a value exceeding zero, and the underestimation of deaths, with a smaller value than their pension amount.

Category	Actual/Expected (ML)	Actual/Expected (SAPS)
Normal health males	1.03	0.99
Normal health females	1.01	1.00
Ill health males	0.98	1.04
Ill health females	1.06	1.17
Spouse males	1.07	0.98
Spouse females	1.04	1.02

Figure 5.143: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	Year	EtR	Actual exits	Expected exits (ML)	Expected exits (SAPS)
Normal health males	2013	2,863,256,463	60,156,878	16,696,882	65,681,530
Normal health males	2014	2,951,373,636	66,990,292	18,268,341	68,243,510
Normal health males	2015	3,011,746,941	64,487,375	225,854,047	69,247,443
Normal health males	2016	2,764,492,973	66,015,153	14,401,194	62,309,942
Normal health males	2017	2,823,842,762	68,110,071	14,446,868	64,206,245
Normal health males	2018	2,870,812,873	65,710,707	11,684,740	65,955,561
Normal health males	2019	2,920,486,606	70,156,348	147,290,872	70,265,991
Normal health females	2013	1,858,675,890	25,814,524	4,998,220	27,103,101
Normal health females	2014	1,977,965,613	28,694,138	6,100,362	29,047,772
Normal health females	2015	2,096,792,719	28,212,963	105,499,995	30,504,274
Normal health females	2016	1,998,629,079	30,128,051	4,891,623	28,243,309
Normal health females	2017	2,124,273,832	32,857,702	4,978,291	30,170,129
Normal health females	2018	2,248,124,392	31,734,810	4,278,413	31,940,171
Normal health females	2019	2,375,357,117	34,199,382	79,416,020	34,729,888
Ill health males	2013	604,603,684	19,803,276	4,424,048	20,818,452
Ill health males	2014	592,823,300	20,885,047	4,888,312	20,985,558
Ill health males	2015	576,845,454	20,351,910	72,829,091	20,763,477
Ill health males	2016	533,583,862	21,794,575	4,105,914	19,457,476
Ill health males	2017	517,533,286	22,038,598	4,769,761	19,553,697
Ill health males	2018	499,509,130	21,275,445	3,680,246	19,364,055
Ill health males	2019	482,916,155	20,942,638	54,695,737	20,051,288
Ill health females	2013	465,001,162	11,778,968	1,821,841	10,945,554
Ill health females	2014	463,151,341	12,346,080	2,124,983	11,376,351
Ill health females	2015	459,696,973	12,564,365	43,467,693	11,679,207
Ill health females	2016	419,762,345	13,892,182	1,971,234	10,904,742
Ill health females	2017	418,358,388	14,577,684	2,004,813	11,290,911
Ill health females	2018	415,648,567	14,127,456	1,888,285	11,629,447
Ill health females	2019	412,794,355	14,413,083	35,455,296	12,347,364
Spouse males	2013	45,333,919	1,032,894	204,447	1,209,283
Spouse males	2014	49,748,966	1,564,100	273,509	1,381,808
Spouse males	2015	53,432,820	1,278,867	4,585,062	1,539,730
Spouse males	2016	52,651,614	1,542,505	288,796	1,474,938
Spouse males	2017	57,312,944	1,797,194	328,390	1,682,412
Spouse males	2018	61,894,774	1,877,980	314,279	1,889,887
Spouse males	2019	66,289,177	2,094,934	4,465,068	2,196,332
Spouse females	2013	381,962,648	17,964,934	6,145,181	19,706,449
Spouse females	2014	389,817,028	22,407,052	7,003,945	20,586,442
Spouse females	2015	389,075,830	19,311,757	63,797,887	20,586,957
Spouse females	2016	346,048,812	19,751,019	5,784,280	18,620,087
Spouse females	2017	350,658,082	21,222,412	5,894,634	18,997,254
Spouse females	2018	353,980,137	18,785,934	4,714,850	19,102,908
Spouse females	2019	356,464,348	20,032,377	41,365,591	19,644,785

Figure 5.144: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category and year, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

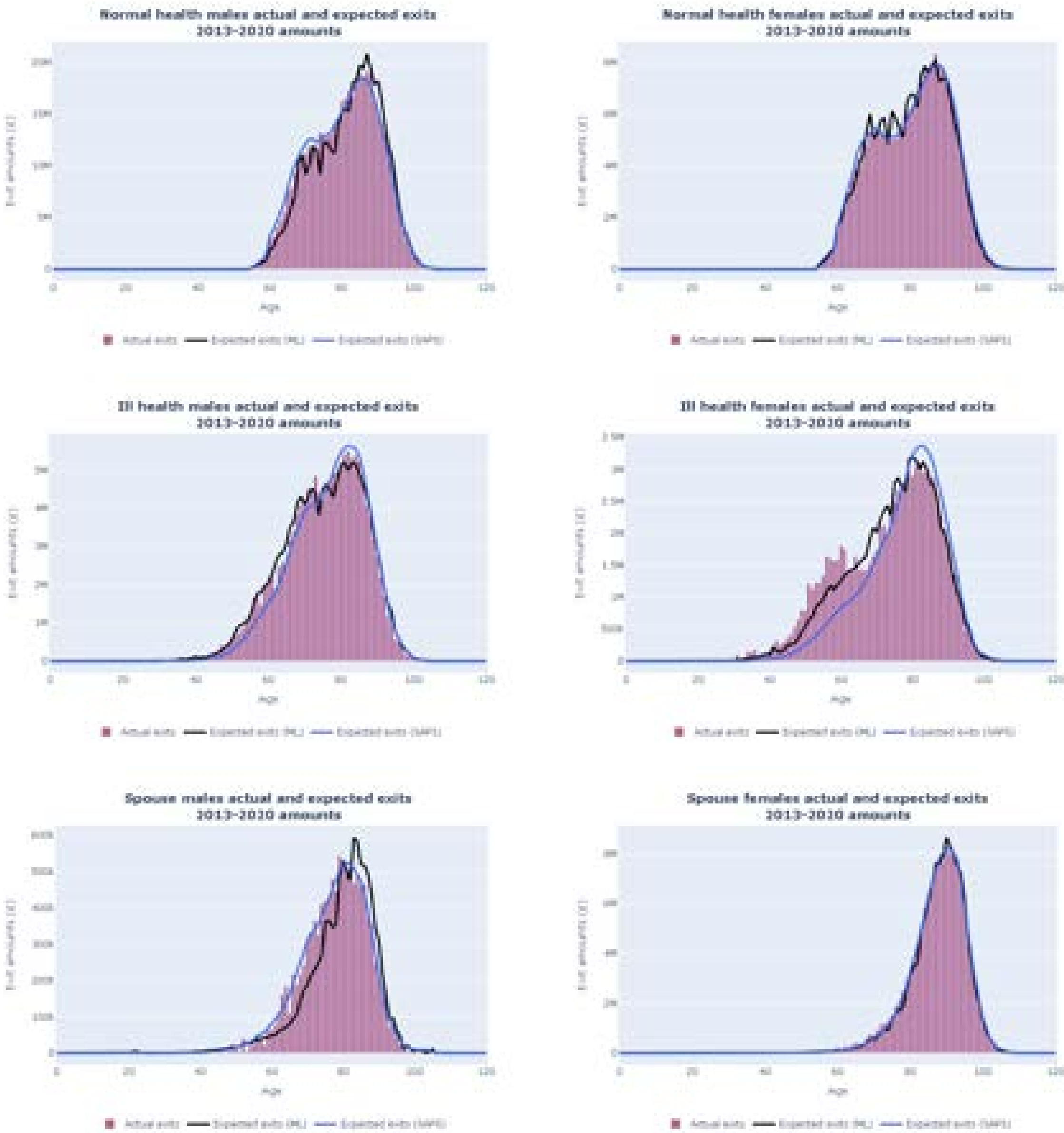


Figure 5.145: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

5.5.2 Adding gender to ML regression compared to baseline

The selected ML model did not incorporate the gender feature, given the lack of significant improvements shown in the algorithm's performance; this omission led to substantial discrepancies in the optimised scaling factors between males and females when applying the model to data categories that differentiated between gender.

Figure 5.146 depicts the minimal impact on R^2 performance upon adding the gender feature to the ML regression model; however, it appears to have a noticeable effect on the optimal scaling factors required to minimise the least squares residuals of actual and expected exit amounts, across each age for each category, as shown in Figure 5.147. The addition of the gender feature decreases male scaling factors, increasing the required female scaling factors, drawing each value closer to one. Regardless of this adjustment, the factors for normal health females, ill health males and female spouses remain relatively distant from the target, highlighting that the model fails to fully capture the nuances within these categories.

The majority of expected exits show marginal improvements for ill health and females of normal health, though they worsened for other categories (Figure 5.148).

Figure 5.149 displays the subsequent F-test metrics from differentiating between the model and SAPS S3 tables. Adding the gender parameter increases the ML model's complexity, reducing the DoF; similar results to the ML model are yielded in the absence of gender.

The charts in Figure 5.150 illustrate a similar performance between the model with and without gender; some subtle changes can be observed, such as an improved fit at younger ages for females of normal and ill health, at the expense of a poorer fit at older ages, particularly for the latter group, where the fit switches from severe overestimation to heavy understatement.

On the whole, there is insufficient evidence to conclude that the current ML regression model is superior with inclusion of the gender variable for this isolated set of data.

Model description	Optimal R^2 tree depth	Training RMSE	Test RMSE	Training R^2	Test R^2	Expected exit amounts (£)
Without gender	7	1,842	1,832	0.157	0.160	1,076,719,188
With gender	7	1,841	1,831	0.158	0.161	1,076,733,850

Figure 5.146: RMSE and R^2 metrics on training and testing data sets using optimal R^2 tree depth models with and without gender. The sum of actual exits in the data is £1,064,836,398.

Category	Best fit scaling factor
Normal health males	0.98
Normal health females	0.87
Ill health males	1.13
Ill health females	0.98
Spouse males	0.99
Spouse females	0.95

Figure 5.147: Proposed optimal scaling factors for the ML algorithm to minimise the least square residuals of actual and expected exit amounts across each age for each category.

Category	EtR	Actual exits	Expected exits (ML)	Expected exits (SAPS)
Normal health males	20,206,012,254	461,626,826	444,881,461	465,910,222
Normal health females	14,679,818,643	211,641,570	211,957,454	211,738,645
Ill health males	3,807,814,871	147,091,489	148,440,567	140,994,004
Ill health females	3,054,413,132	93,699,818	91,837,170	80,173,577
Spouse males	386,664,214	11,188,474	10,380,840	11,374,389
Spouse females	2,568,006,884	139,475,485	134,603,873	137,244,882

Figure 5.148: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	DoF (ML)	DoF (SAPS)	Scale (SAPS)	RSS (ML)	RSS (SAPS)	F	p-value
Normal health males	53	54	0.97	79,382,372,140,054	15,505,133,213,036	0.20	1.00000
Normal health females	57	57	0.95	6,917,754,846,546	2,573,180,486,731	0.37	0.99987
Ill health males	73	74	1.13	3,515,849,141,296	5,803,531,555,305	1.65	0.01666
Ill health females	81	81	1.19	5,414,752,617,371	12,594,080,907,454	2.33	0.00009
Spouse males	86	87	0.98	209,584,729,380	24,976,395,423	0.12	1.00000
Spouse females	93	93	0.94	4,739,488,923,302	1,437,684,646,757	0.30	1.00000

Figure 5.149: F-test comparing the scaled ML model against SAPS S3 CMI tables using optimal scaling factors for each category.

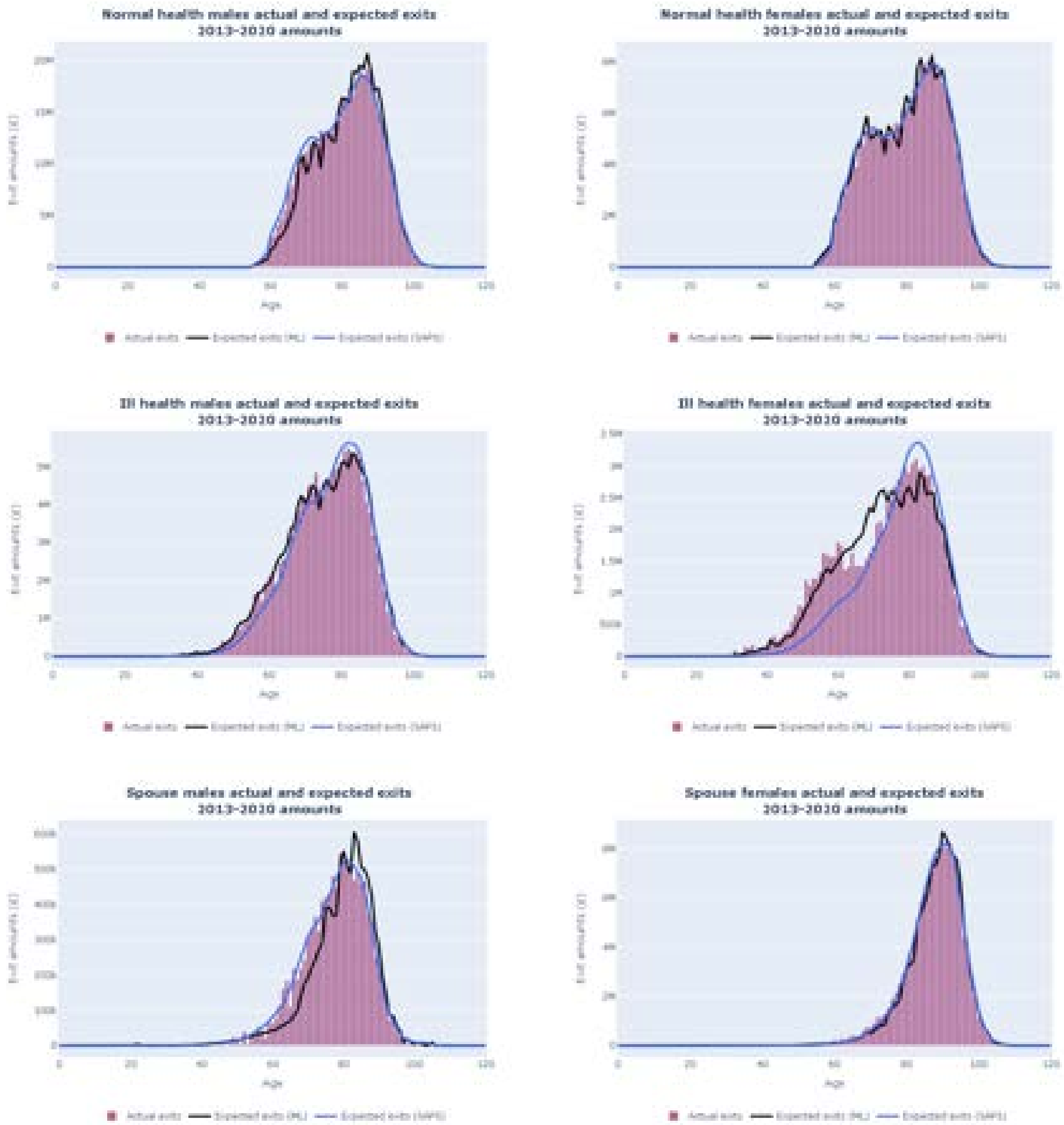


Figure 5.150: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

5.5.3 Adding census data to ML regression compared to baseline

Enhancement of the performance of the ML regression model can be achieved by introducing supplementary variables that refine the algorithm's training capability. Mortality is influenced by numerous factors, with abundant data sources that could potentially provide relevant information; this analysis incorporates variables from the topic summaries of UK census data, dated 21st March 2021, which are provided by the ONS [85]. These factors encompass a range of population characteristics and subject areas; data was utilised at the LSOA granularity level, maintaining consistency with the IMD data already present in the dataset. Further research could aspire to incorporate data types across various fields. The following topic summaries were utilised:

- Population density;
- Households by deprivation dimensions;
- Household size;
- General health;
- Number of households;
- Car or van availability;
- Number of bedrooms;
- Occupancy rating for bedrooms;
- Distance travelled to work;
- Hours worked;
- Highest level of qualification.

These summaries allowed a single value to be determined for each LSOA, either directly or through the application of a weighted mean value across different subgroups. To minimise the bias from scale discrepancies between LSOA's, data pertaining to proportionate metrics in each LSOA were used instead of absolute values.

Augmenting the pension scheme membership data with additional information regarding an individual's area of residence may reveal trends, helping to decipher reasons behind the remaining inexplicable variation between ML regression predictions and actual response variable values. Although these supplementary variables represent averages across an LSOA and may only indirectly pertain to specific circumstances of an individual record, they may divulge relevant information regarding their environment or their general area of residence.

Population density can influence mortality rates through various mechanisms; in densely populated areas, infectious diseases spread more rapidly due to close and frequent proximity forced upon individuals, as observed during the COVID-19 pandemic [86]. High population density often heavily strains healthcare systems, resulting in longer waiting times, overcrowded facilities and reduced quality of care; this negative impact can reach critical levels during health crises. Environmental concerns, such as air and water pollution, are more pronounced in densely populated areas, significantly increasing the risk of respiratory and cardiovascular diseases [87]. Urban areas with highly concentrated populations experience accidents and injuries more frequently due to increased levels of motor and pedestrian congestion, potentially generating greater mortality rates when emergency services are critically overwhelmed [88]. Socio-economic inequalities, often exacerbated by overpopulation, are intertwined with health outcomes, where impoverished areas and residents struggle to regularly access nutritious food, safe housing and healthcare services [89]; furthermore, living in such areas elevates stress levels, compounding physical and mental health issues, in the wake of excessive noise, a lack of privacy and limited access to green spaces [90]. These factors indirectly contribute to increased mortality rates through an inflated risk of chronic diseases and exposure to unhealthy behaviours; however, well established and robust public health infrastructure can mitigate some harmful disadvantages with the implementation of effective sanitation, vaccination programmes and health education to manage and reduce mortality rates. Population density's scope of influence on mortality rates fluctuates depending on the region's economic development, public health infrastructure and social services; cities may potentially experience fewer repercussions than less developed areas.

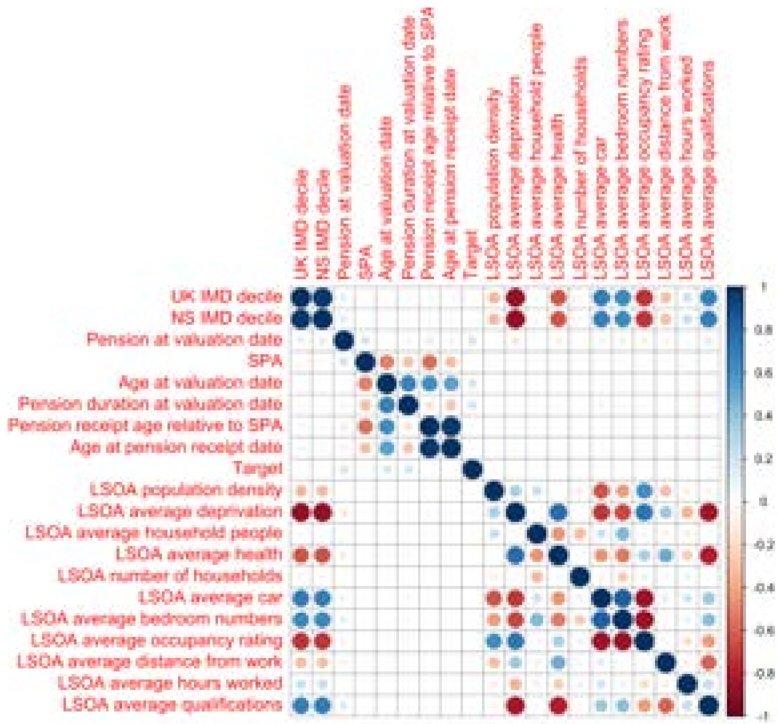


Figure 5.151: Correlation between numeric variables, including those from the additional census data.

Household size affects mortality rates through a variety of social, economic and environmental factors; larger households often foster greater social cohesion, which positively regulates mental health and reduces stress, plausibly lowering mortality rates. Pooled economic resources in sizeable families generally enhance access to healthcare and a nutritious diet, depending on income and number of dependants [91, 92]; on the other hand, the danger of contagion is greater for these households due to the ratio of living space to amount of inhabitants and frequent interactions, facilitating transmission of infectious disease between a wider network of individuals. Overcrowded living conditions can induce poor health by escalating the risk of respiratory and other communicable diseases. Extended family units usually have a more robust infrastructure of available individuals to care for vulnerable members, such as the elderly and those suffering with chronic illnesses, naturally allowing for better health management and lower mortality rates [93]. Health behaviours, such as dietary habits and physical activity levels may be diverse in larger households; age composition also plays a role as family units with a vast proportion of elderly individuals typically have higher mortality rates attributed to age-related health issues. Moreover, cultural and societal norms also notably stimulate these dynamics.

General health impacts mortality rates, encompassing a broad range of physical, mental, emotional and social well-being factors that collectively affect an individual's risk of death. Better overall health typically correlates with lower mortality rates, since healthier individuals are less likely to suffer from chronic diseases, acute illnesses or mental health issues that can precipitate a premature death [94]. Maintaining general wellness is often a consequence of a synergy of health factors, such as consistent nutrition, regular physical activity, adequate sleep and effective stress management; access to quality healthcare services for preventive care, early diagnosis and appropriate medical treatment plays a crucial role in the preservation of good general health and reduction of mortality rates [95]. Conversely, poor general health can significantly increase mortality risk, with ailments leading to complications; these can manifest as heart disease, strokes, respiratory issues and other life-threatening conditions. Additionally, smoking, excessive alcohol consumption, sedentary behaviour and other lifestyle choices exacerbate health risks, contributing to steeper mortality rates.

The number of households in an area affects mortality rates through a network of many interconnected factors. A significant number of households often indicates a dense population, which can burden healthcare providers; this can cause longer wait times and reduced access to medical care, potentially increasing mortality rates. Social support systems can also be affected by household size, with condensed communities offering valuable support and improved health outcomes, as opposed to fragmented communities that experience intense stress levels, more isolation and ill health. Economic disparities can also play a role, whereby wealthier areas usually have better access to healthcare and wholesome living conditions, reducing mortality rates; in contrast, economically deprived areas frequently display heavier mortality, resulting from poorer living conditions and limited healthcare access. Environmental factors, such as air and noise pollution as well as crowded living conditions, are contributors to health risks and accelerate the spread of infectious diseases. Areas with relatively dispersed households may present deficiencies in public services, such as less frequent access to transportation and emergency services, which can generate higher mortality rates [96]. Lifestyle and behaviour patterns, influenced by the number of households, can also impact health. Urban areas may offer enhanced opportunities for physical activity and social engagement, which could be offset by greater levels of stress, crime and unhealthy behaviours.

The availability of cars or vans impacts mortality rates through various direct and indirect factors; enhanced accessibility to healthcare facilities and rapid emergency response times can boost survival rates by procuring appropriate medical treatment and committed management of chronic conditions [97]. Vehicles also facilitate social choices, enabling access to healthier food options and encouraging physical activities, though excessive use can also promote sedentary behaviours triggering obesity [98]. Environmental damage, such as air pollution from vehicle emissions, contributes to respiratory and cardiovascular diseases that correspond to rising mortality rates in high-traffic urban areas. Economic implications should also be noted, given that vehicle ownership often improves employment opportunities and living conditions, thus lowering mortality rates; nonetheless, naturally there is a heightened risk of road traffic accidents with vehicle possession and operation [99].

A residence's number of bedrooms can influence mortality rates through several socioeconomic and health-related factors. Adequate bedroom space generally indicates superior living conditions, thus improving overall health and lowering mortality rates; more bedrooms decrease the probability of overcrowding and risk of contagion between household

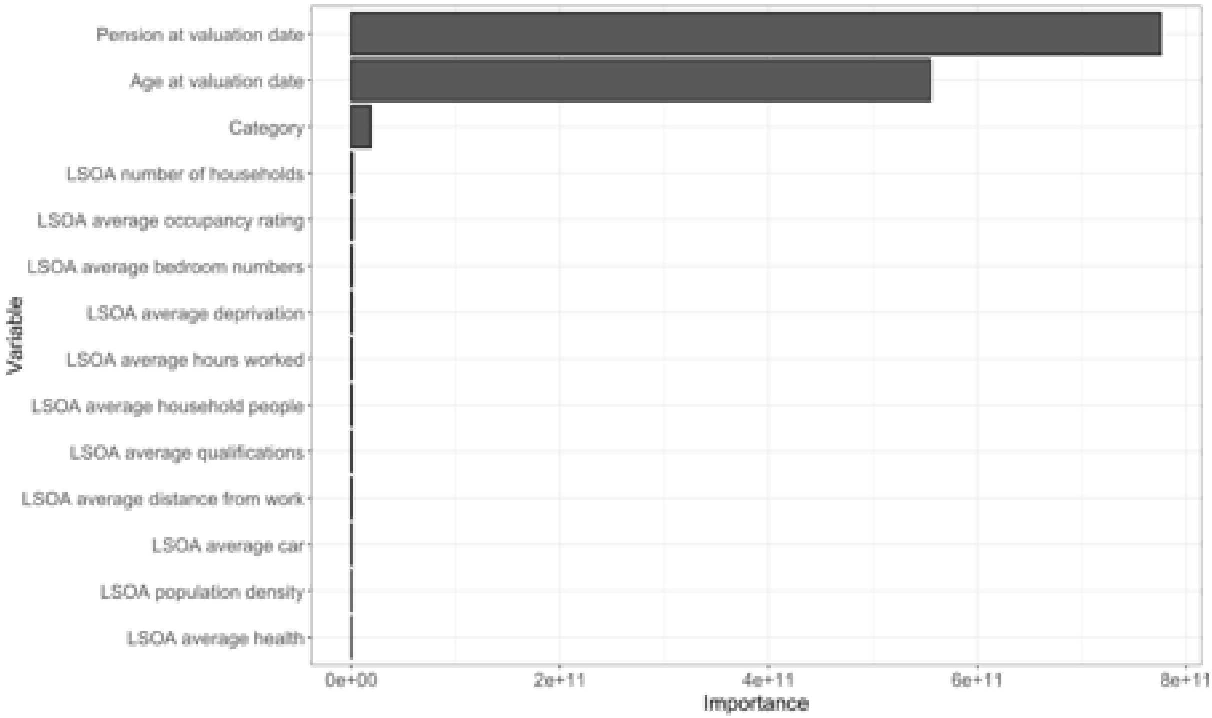


Figure 5.152: Feature importance in the optimal fitted model, generated using additional census data variables on top of the optimal regression model.

Model description	Optimal R^2 tree depth	Training RMSE	Test RMSE	Training R^2	Test R^2	Expected exit amounts (£)
No census data	7	1,842	1,832	0.157	0.160	1,076,719,188
Census data	6	1,841	1,830	0.158	0.154	1,075,867,848

Figure 5.153: RMSE and R^2 metrics on training and testing data sets using optimal R^2 tree depths for models with and without additional census data variables. The sum of actual exits in the data is £1,064,836,398.

Category	Best fit scaling factor
Normal health males	1.10
Normal health females	0.76
Ill health males	1.25
Ill health females	0.87
Spouse males	1.07
Spouse females	0.87

Figure 5.154: Proposed optimal scaling factors for the ML algorithm, using additional census data variables, to minimise the least square residuals of actual and expected exit amounts across each age for each category.

members, as well as supporting emotional well-being and mental health by providing sufficient personal space and lowering stress levels [86]. Homes with more bedrooms often reflect affluence and a higher socio-economic status, associated with more readily available healthcare, nutrition and other essential services positively altering health outcomes; in opposition, overcrowding is often associated with lower socio-economic status, which limits access to quality healthcare, nutritious food and other resources critical to the maintenance of good health, potentially exacerbating mortality rates. Additionally, overcrowding places a strain on household resources, making it challenging to create a healthy living environment; it may also exacerbate existing health conditions, given the tougher challenge in maintaining a clean and hygienic home.

The occupancy rating for bedrooms in a house dictates the adequacy of bedroom space relative to the number of occupants, which impacts mortality rates through various health and socio-economic factors. A favourable occupancy rating, indicating a sufficient amount of bedrooms for household members, generally reflects better living conditions; this leads to lower stress levels, improved mental health and a reduced risk of spreading infectious diseases due to increased personal space. Adequate bedroom size enables superior quality of sleep, which is essential for overall health and well-being, thereby potentially lowering mortality rates; conversely, an unfavourable occupancy rating, with a disproportionate amount of individuals sharing the available bedroom space, is symptomatic of overcrowded living conditions. Overcrowding can contribute to the rapid transmission of infectious diseases such as respiratory infections, thus elevating mortality rates; it also negatively affects mental health due to heightened stress, a lack of privacy and regular sleep disturbance [90].

The distance between home and workplace can affect mortality rates through numerous health, economic and social factors; longer commutes are often linked to greater stress levels, reduced physical activity and less time for sleep, family and leisure [98]; in combination, these factors can negatively impact both physical and mental health over time, gradually increasing mortality rates. Lengthy commutes can also contribute to a sedentary lifestyle, forcing individuals to spend extended periods of inactivity sitting in vehicles or on public transport; insufficient physical activity is associated with a range of health issues, namely obesity, cardiovascular diseases and diabetes, all of which elevate the risk of mortality. The risk of traffic accidents also increases with the distance travelled. Excessive commutes, due to traffic congestion, delays or overall journey length contribute to stress [100]; consequently, there is also less time available for healthy activities, such as preparing nutritious meals, exercising and social engagement, all of which aid a healthy lifestyle. Time constraints can also lead to inferior sleep quality and duration, hindering overall health and increasing the risk of mortality. Individuals with longer commutes generally face higher transportation costs, which may strain financial resources, adversely affecting access to healthcare, nutritious food and other essentials, indirectly affecting health and mortality rates. Further to these economic factors, the environment can have detrimental repercussions on respiratory and cardiovascular health, such as exposure to air pollution during commutes, which contributes to higher mortality rates [87].

Category	EtR	Actual exits	Expected exits (XL)	Expected exits (ML)
Normal health males	20,206,012,254	461,626,826	447,831,008	448,642,944
Normal health females	14,679,818,643	211,641,570	211,810,099	210,162,925
Ill health males	3,807,814,871	147,091,489	150,491,391	149,393,108
Ill health females	3,054,413,132	93,699,818	90,146,703	88,734,146
Spouse males	386,664,214	11,188,474	10,556,237	10,459,551
Spouse females	2,568,006,884	139,475,485	135,177,582	134,706,368

Figure 5.155: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on applying ML models, with and without additional census data variables.

Category	DoF (XL)	DoF (ML)	Scale (ML)	RSS (XL)	RSS (ML)	F	p-value
Normal health males	43	54	1.10	113,486,729,861,510	91,374,891,092,586	0.81	0.77613
Normal health females	47	58	0.77	12,746,438,167,748	7,449,341,934,160	0.58	0.97395
Ill health males	63	74	1.19	8,939,952,180,260	3,753,026,961,746	0.42	0.99981
Ill health females	71	82	0.83	3,593,133,735,274	3,865,965,543,504	1.08	0.37738
Spouse males	76	87	1.11	305,879,938,123	251,894,909,352	0.82	0.81004
Spouse females	83	94	0.88	2,414,942,273,942	2,803,895,406,208	1.16	0.24407

Figure 5.156: F-test comparing scaled ML models with (MLXL) and without (ML) using additional census data variables.

Working hours influence mortality rates through various health, social and economic factors; both excessive and insufficient amounts can have detrimental effects on an individual's physical and mental well-being, resulting in higher mortality rates [101]. Working excessively long hours is often affiliated with heavy stress levels, which can induce health issues, such as cardiovascular diseases, and mental health disorders, such as anxiety and depression. Chronic stress weakens the immune system, increasing susceptibility to illnesses and infections and, thus, the risk of mortality. Extended working hours can also result in sleep deprivation, leaving individuals with less time to relax and recover; poor sleep quality and insufficient rest are linked to a range of health problems, including obesity, diabetes, heart disease and impaired cognitive function, all of which raise mortality rates [102]. Moreover, long working hours significantly minimises the time available for engaging in healthy activities and regular physical exercise, preparing and consuming nutritious meals as well as participating in social and recreational pursuits. Insufficient physical activity is an identified risk factor for numerous health conditions, including obesity, diabetes and cardiovascular diseases; a nutritionally poor diet can lead to multiple deficiencies and health issues. Social isolation and constricted leisure time negatively affects mental health, furthering the probability of premature death. In contrast, insufficient working hours as a result of underemployment or unemployment, can also negatively impact health and mortality rates. Economic insecurity and financial stress from insufficient income strongly restricts access to essential resources such as healthcare, nutritious food and safe housing, with each a crucial element in maintaining good health and lifestyle. The psychological distress of job insecurity and financial strain gives rise to mental health conditions, including depression and anxiety, which adversely affect overall health and longevity. Economic factors play a critical role in the relationship between working hours and mortality rates; individuals working long hours may earn higher incomes, establishing better access to healthcare, healthy food and other necessities promoting good health; however, the negative consequences to their health from working excessively long hours can outweigh these benefits.

The highest level of qualification attained by a person influences mortality rates through many socio-economic, behavioural and health-related factors. Higher achievements in education are generally associated with better health outcomes and lighter mortality rates, whilst lower levels of education can correlate with poorer health and heavier mortality rates. Educational attainment often guides socio-economic status where more qualified individuals are typically granted better job opportunities, higher incomes and greater financial stability [26]; this economic advantage provides better access to healthcare services, healthier food options and safer living conditions, which all contribute to improved health maintenance and lower probability of premature death. Higher educational levels equip individuals with the knowledge and transferable skills to make informed health decisions, understand medical information and navigate healthcare services effectively [103]. Educated individuals are more inclined to engage in healthy lifestyle activities, such as regular exercise, balanced diets and routine medical check-ups, which reduce the risk of chronic diseases and improve overall health. Higher educational attainment is associated with avoidance of adverse behaviours, including lower rates of smoking, excessive alcohol consumption and substance abuse [104]; furthermore, higher professional qualifications often lead to more fulfilling and less physically demanding jobs, which can reduce work-related stress and mental health issues. In contrast, rudimentary or incomplete education is associated with employment that is unstable, more physically taxing and stressful, potentially producing poorer mental health and higher mortality rates. Lower levels of educational attainment tend to limit employment opportunities, resulting in lower incomes and increasing the likelihood of living in disadvantaged areas with restricted access to healthcare and other essential services [105], thus yielding poorer health outcomes and higher mortality rates.

Correlations between these newly introduced numeric variables from the additional census data are shown in Figure 5.151; as expected, they do not exhibit substantial connection with any existing variables, other than for IMD deciles. Variables for number of people per household, number of households, distance from work and hours worked also depict low correlations with the IMD deciles; this is desirable as it highlights the absence of duplicate information within the existing variables. Conversely, all other supplementary variables display a strong correlation with IMD deciles, posing a greater risk of information duplication.

Figure 5.152 illustrates feature importance in the optimal ML regression model resulting from the addition of census variables; unfortunately, all of these variables had an inferior rank than the features used in the optimal fitted model, suggesting they are unlikely to have a significant impact on the ML regression algorithm's performance. These underwhelming results are in part owed to the numbers reflecting characteristics within a geographical area being too large to reasonably represent individual circumstances; it should also be noted that IMD deciles scored poorly without the census data, in terms of feature importance for the optimal ML model, which is unsurprising when taking the identified correlations into account.

The metrics of the ML regression model, after the incorporation of census variables, as shown in Figure 5.153, indicate a marginal improvement in the training R^2 and a decrease in the testing R^2; this suggests that the addition of these hinders the model's ability to generalise to unseen data. Advantages of introducing census variables include a reduction in the optimal tree depth by one level, exhibiting a preference for a simpler tree structure, slightly reduced RMSE scores and a marginal shift of total expected exit amounts towards actual values.

Figure 5.154 displays applicable optimal scaling factors to the ML regression model for each category; only minor adjustments are made to the scaling factors from the model without the census data, except for males of normal health. Scaling factors were raised for ill health categories but reduced for the others.

Figure 5.155 juxtaposes expected exits using the ML regression model with census features (MLXL) to the version without census features (ML); the inclusion of these features brings the expected exits closer to the actual amounts for all females and male spouses but further away for the remaining categories.

Filtering the charts by age in Figure 5.157 highlights the strong similarity between the models' performance. Females with health issues appear to exit more frequently at younger ages, with rates slowing at higher ages in comparison to the model without census data, which more closely conforms to the actual exits. The ill health male model seems to have weakened predictions of actual exits at older ages by generating amounts that are too low. The normal health female model also displays deviations at younger ages that are noticeably higher than the actual amounts. The shape of the expected exits for male spouses also gives less favourable results than the model without census data; although it better aligns to some earlier ages, the middling ages are substantially underestimated.

Figure 5.156 concludes that models with census features are not statistically better than the model without them. The ML algorithm, inclusive of the supplementary census variables, contains fifteen parameters, whereas the model without contains four; despite the increased complexity and reduced DoF in the models with extra census variables, the RSS is only reduced for females of ill health and female spouses.

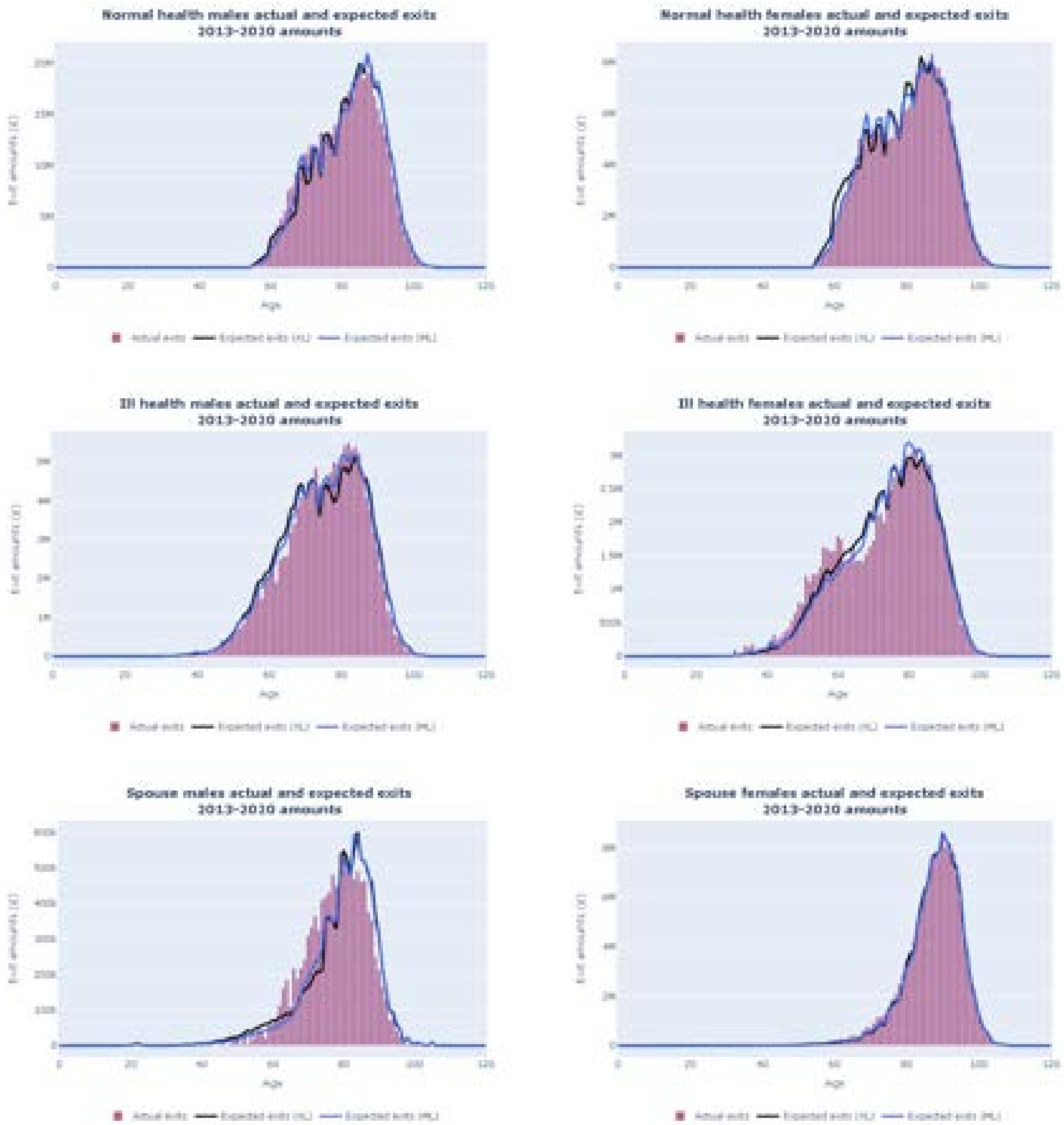


Figure 5.157: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on applying ML models, with and without using additional census data variables.

5.6 ML classification analysis

An alternative approach regards the analysis as a classification problem as opposed to a regression. Decision trees can assign data records to a categorical variable with an alive or dead status, before applying pension amounts to those in the latter group.

In the context of a classification ML algorithm, the model's accuracy is calculated by the proportion of correctly predicted instances out of the total number of cases. The accuracy metric may fall short when dealing with imbalanced datasets whereby one class significantly outweighs another; this can be witnessed with the alive class greatly surpassing the dead class. The accuracy metric also fails to differentiate between false positives and false negatives; therefore, a model only capable of predicting the majority class can mistakenly achieve a misleadingly high accuracy score when the data is imbalanced [106].

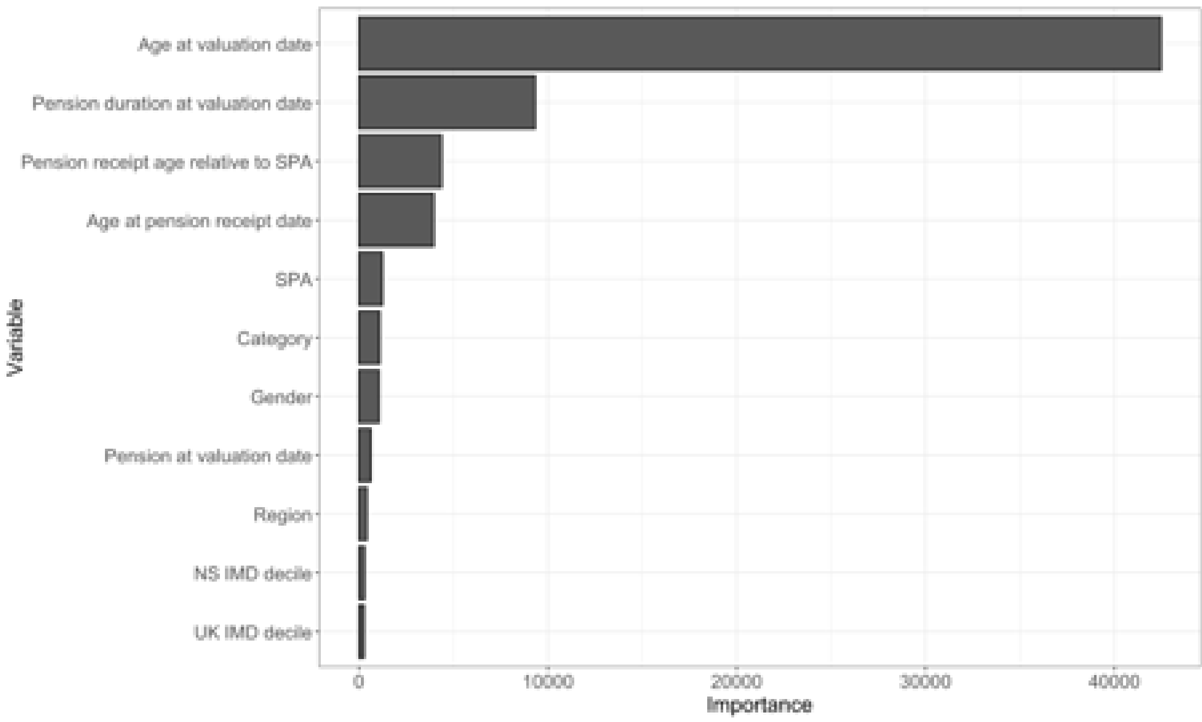


Figure 5.158: Feature importance of all variables used to generate the optimal fitted model.

Instead of relying on the accuracy metric, model performance can be assessed based on the AUC of the ROC curve. AUC scores range from 0, where the model does not correctly predict any of the cases, to 1, which achieves perfect classification. An AUC score of 0.5 suggests that the model performs the equivalent to random estimation, whereas higher values are indicative of a sharper ability to distinguish between positive and negative classes. The ROC curve is a graphical representation of a binary classifier system's performance across various discrimination thresholds; it charts the TPR, also known as sensitivity, against the FPR, otherwise called specificity, at multiple threshold settings. The TPR represents the proportion of actual positive cases that have been correctly identified; the FPR denotes the proportion of actual negative cases that have been misinterpreted as positive by the model. The area under the ROC curve assesses how successfully the model distinguishes between two classes; in this case, these are alive and dead, where the latter is determined as the positive case [107].

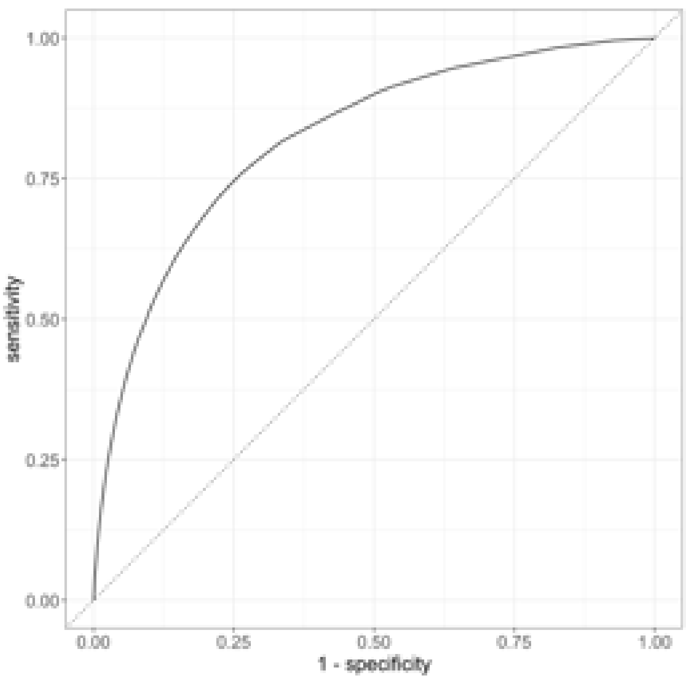


Figure 5.159: ROC curve in the optimal fitted model generated using all variables on the test data. Tree depth = 10 and ROC AUC = 0.820 (0.818 on the training data).

This modelling approach mirrors that used for the ML regression algorithm. Tree depths ranging from 1 to 15 were tested for every selected feature combination to deduce the optimal depth in which the AUC metric is maximised; the AUC value was consistently maximised following the reduction of other parameters. Therefore, cost complexity was set to 0.0000000001 with the minimum number of data points required in a node for further splitting fixed at 2.

Initially, a saturated model was fitted using each available variable in the dataset. Fitting the saturated model shed insight into the model's potential performance, helping to identify the variables and features that are paramount to predicting the resulting state (see Figure 5.158).

The importance of the pension at valuation date feature has diminished substantially from its vital role in the regression approach, given the distinct compositions of the target variable that regression and classification models aim to achieve. The former anticipated producing either pension amounts for death records or zero otherwise, whereas the latter seeks only to distinguish between dead and alive records, yielding identical responses in all cases of death. Consequently, the age at valuation date feature predominantly drives performance in the classification model. Another key difference to the regression model's feature importance is the displacement in relevance of region and gender variables; gender emerges as a more dominant feature than region in the ML classification model, exerting almost as much influence as category.

Figure 5.159 depicts the ROC curve, plotting TPR (y-axis) against FPR (x-axis), across numerous threshold settings for the saturated model with each point constituting a unique classification threshold. The diagonal line from (0,0) to (1,1) represents a random classifier without discriminative power, simply estimating that the group has an AUC of 0.5. Theoretically, the closer the plotted curve is to the top-left corner of the chart, the finer the performance, given that a point located here corresponds to a high TPR and low FPR. The ROC curve sits above the diagonal line, confirming the model's ability to carefully discern between positive and negative classes; considering the curve's swift rise towards the top-left corner, it is apparent that the model accurately distinguishes between positive and negative classes, which is further supported by having an AUC of 0.820. This value means there is an 82% chance that the model correctly distinguishes between a randomly chosen positive instance and its negative counterpart. Although the model performs well in classification tasks for the most part, there is still room for improvement. The curve's smoothness does not directly impact AUC, though it is indicative of the level of consistency in the model's performance across different threshold values; this figure highlights that the ML classification model is stable, reliably discriminating between positive and negative instances. A less smooth or jagged ROC curve would imply that the model's performance sharply changes at different threshold values, proposing an unstable or unreliable ability to classify. Decision trees often produce stepwise ROC curves, given that binary decisions are selected at each node; therefore, it is common for this type of curve to be more jagged as opposed to logistic regression or neural network models, which have a probabilistic nature that fundamentally have infinite outcomes, allowing smoother curves. The ROC curve for the saturated model appears relatively smooth, without remarkable jaggedness or abrupt steps, suggesting consistent model performance across different threshold values. Although minor discrepancies can be observed along the curve, these are subtle and do not detract from its overall smoothness; thus, the model likely exhibits a stable performance, reliably discriminating between positive and negative instances.

Figure 5.160 illustrates the development of the ROC curve, starting with a model that only includes a single feature, which predominantly drives performance, before progressively adding features in descending significance. Similar to regression ML models, variables for pension duration at the valuation date, pension receipt age relative to SPA and SPA were ultimately excluded; their redundancy was a result of high correlation to the age at valuation date variable, which inhibited their ability to provide additional valuable information that would enhance model performance. The age at valuation date feature served as a standalone, reasonable indicator of target status for records, achieving an ROC AUC value of 0.794; thus, there is a 79.4% probability of the model correctly differentiating between a randomly selected positive instance and a randomly selected negative instance. The ROC curve is significantly above the diagonal line, illustrating that the ML

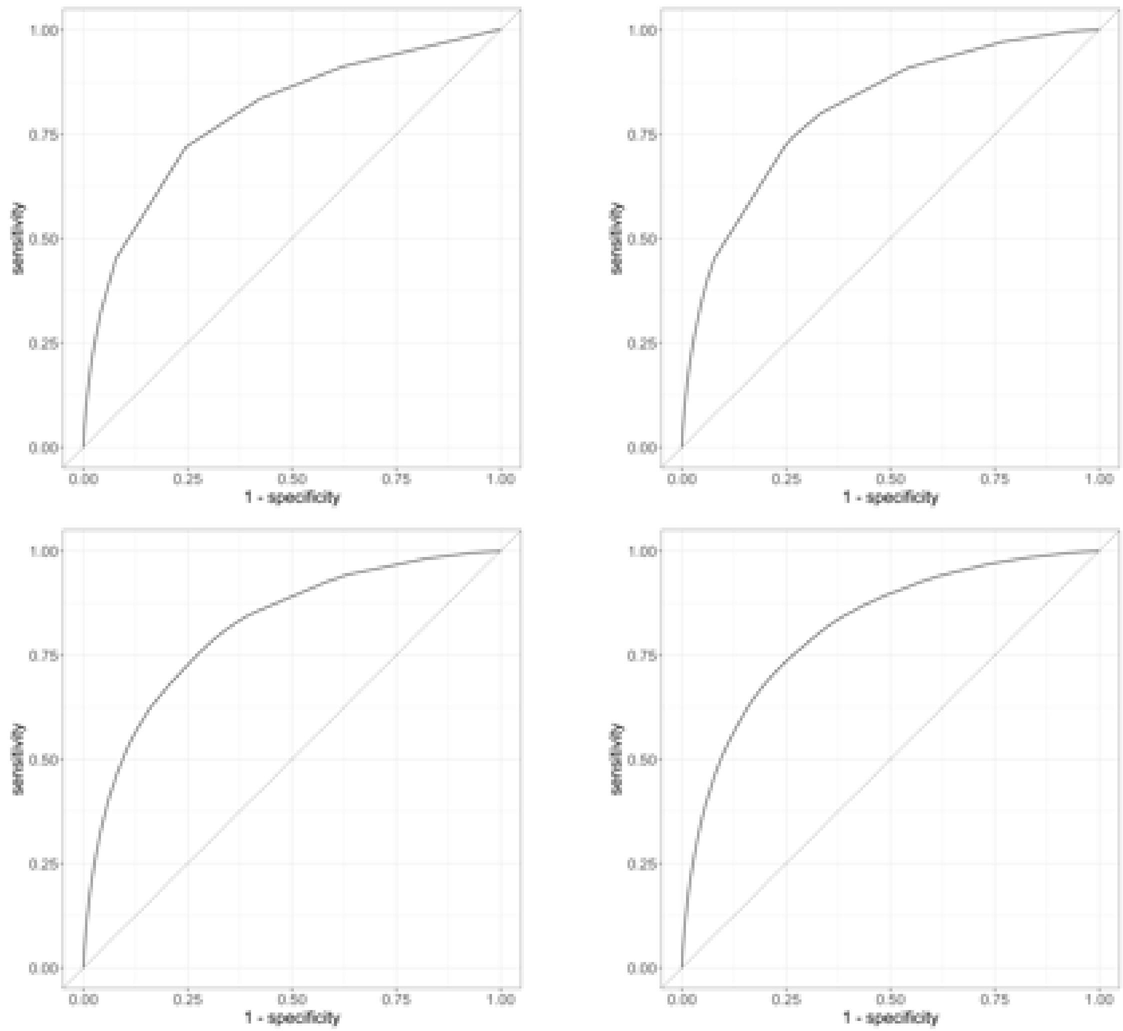


Figure 5.160: ROC curve in optimal fitted models generated using an incremented number of features on test data. Age at valuation date only (top left) ROC AUC = 0.794 (0.792 on the training data), category added (top right) ROC AUC = 0.808 (0.809 on the training data), gender added (bottom left) ROC AUC = 0.815 (0.814 on the training data) and pension at valuation date added (bottom right) ROC AUC = 0.819 (0.816 on the training data).

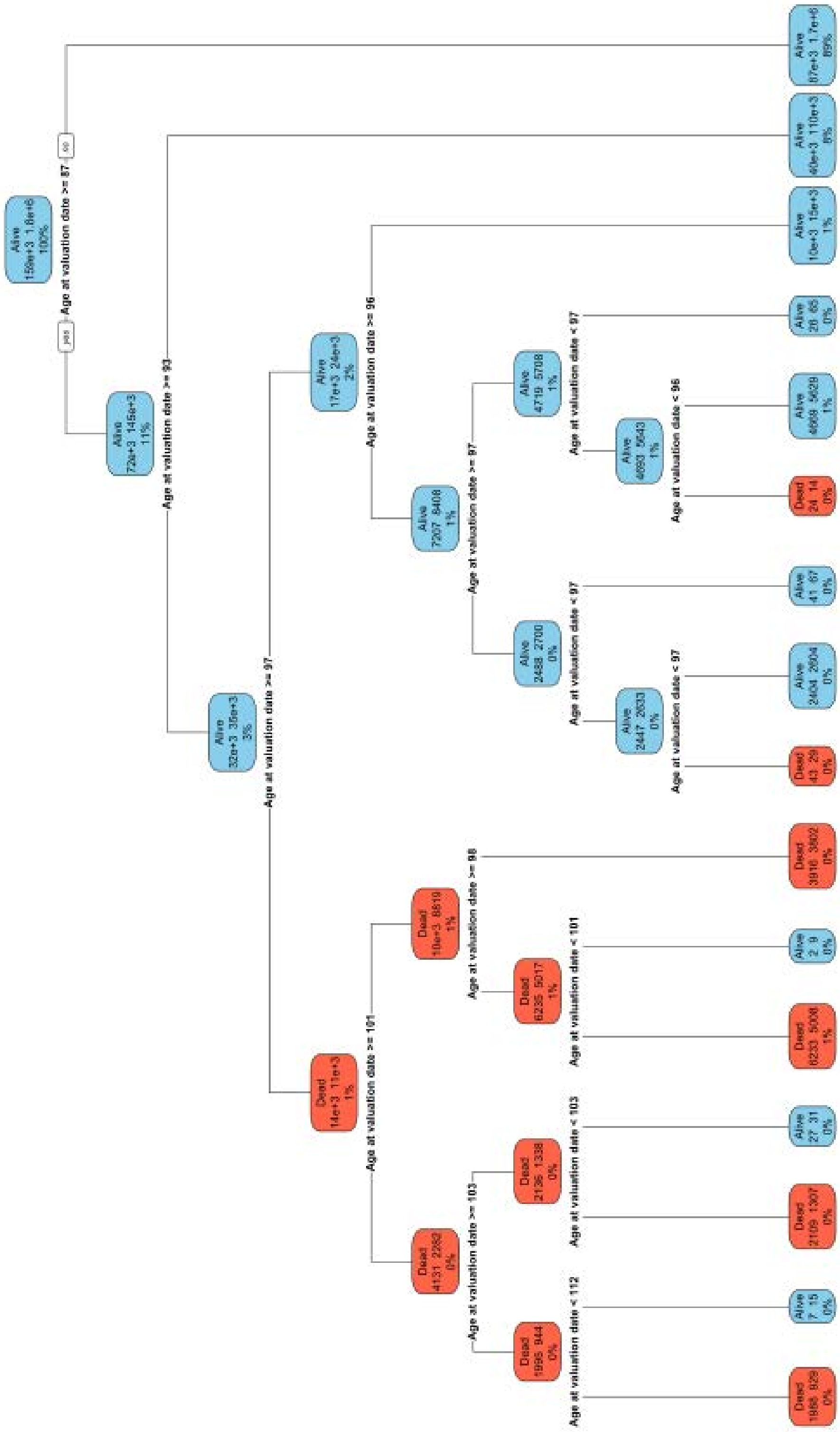


Figure 5.161: Decision tree from the fitted model generated only using the age at valuation date variable. Tree depth = 7. Values inside nodes include the most commonly encountered value(top), which are used for future predictions, followed by number of observations in each status (middle row) and proportion (bottom) of observations passing through it from the training data.

classification model surpasses random guessing. In comparison to the saturated model, the ROC curve for the age only feature is much less smooth, with this irregularity pointing towards fluctuations in the quality of the model's classification predictions. There are a few defining points where notable changes in slope are observed, highlighting that in spite of the model's relatively consistent performance across multiple thresholds, there are key areas provoking an abrupt change in the TPR and FPR. Sections of the ROC curve containing a steep, abrupt slope underline valuable thresholds where the model immediately identifies a much greater proportion of true positives without significantly increasing false positives; these are typically regions where the model is highly effective at correctly distinguishing classes at those specific boundaries. Conversely, flatter sections portray thresholds where the model identifies more false positives without a statistical rise in true positives; these regions have thresholds that are less effective at distinguishing between classes. Points where the curve changes direction more sharply correlate with thresholds that provoke an abrupt change in model performance; these kinks advantageously help to reveal thresholds of critical value to model performance.

Incorporation of the next most useful variable, category, initially relaxes some of the jagged lines, most noticeably at higher FPR values; adding the gender feature induces effective smoothing at lower FPR values; the inclusion of pension at valuation date takes this one step further, targeting the middling values, resulting in a curve of striking visual similarity to the saturated model, as evidenced by an AUC of 0.819.

Classification trees provide decision diagrams of a similar structure to those relating to regression algorithms. The optimal tree depths were too vast for display, although Figure 5.161 provides an example for the age at valuation date only model with a reduced tree depth of seven. The nodes display subtly different information between a classification decision tree and regression algorithm; the statuses located at the node's peak signify the most commonly encountered value of the response variable, which the model would select for predictions regarding future unseen data. The number of observations allocated to each status is shown below the most common value; it is preferable for each node of the model to have a large disparity between positive and negative cases, confirming successful distinction between statuses. Such disparity bolsters confidence regarding the model's predictive accuracy; a node predicts the same outcome regardless of the proportion of values falling into a particular status. The percentage of dataset passing through a node is marked at the bottom of it. Since this model only employs a single feature variable, all decisions are based on different threshold values of the same item; unsurprisingly, the tree denotes that older ages increased the probability of traversing to nodes that are predominantly classified as deceased. The final predictions of the model can be seen at the deepest node levels.

Model description	Optimal ROC AUC tree depth	Training accuracy	Test accuracy	Training ROC AUC	Test ROC AUC
Saturated	10	0.922	0.922	0.818	0.820
Age only	10	0.922	0.922	0.792	0.794
Category added	10	0.922	0.922	0.809	0.808
Gender added	12	0.922	0.922	0.814	0.815
Pension added	10	0.922	0.922	0.816	0.819

Figure 5.162: Accuracy and ROC AUC metrics on training and testing datasets at optimal ROC AUC tree depths for a selection of different models.

Figure 5.162 presents the progression of the ROC AUC metric for the saturated and incremented classification ML models. The age-only model produced training and testing ROC AUC scores significantly below those of the saturated model, indicating that the latter accounts for some of the target variable's discrepancies that the former does not. The training and testing ROC AUC scores are similar for each model, highlighting the similar manner in which both models translate well to unseen data. The identical scores for the accuracy metric exemplify its redundancy as a measure to distinguish performance when predicting the target variable across various models; this is attributed to the formidable imbalance between the amount of positive and negative classes, with the majority of data assigned to the living status. Each incremented variable increased the ROC AUC score until it reached sufficient resemblance to the saturated model's performance after the inclusion of the pension at valuation date feature. Including supplementary variables beyond this point showed an absence of material improvement in performance.

5.6.1 ML classification compared to baseline

The ML classification model is designed to predict a record's status as alive or deceased; predictions of the number of exiting lives can be converted into monetary amounts; for instance, pension at the valuation date are utilised for deceased statuses and zero is assigned to those predicted as alive. Subsequently, the model's outputs can be compared with the standard SAPS S3 mortality tables to determine if the accuracy of predictions have improved.

Category	Best fit scaling factor
Normal health males	1.01
Normal health females	1.00
Ill health males	1.00
Ill health females	1.04
Spouse males	0.97
Spouse females	1.00

Figure 5.163: Proposed optimal scaling factors for the ML algorithm to minimise the least square residuals of actual and expected exit amounts across each age for each category.

Table 5.163 presents the required optimal scaling factors in order to minimise the least square residuals between actual and expected amounts for the ML classification model. These values are remarkably closer to one than those related to ML regression models, underlining that this classification approach has a more robust fit from a categorical analysis perspective. By restricting predictions to the status, the full pension amount at the valuation date is transferred to the target variable prediction for the relevant record, rather than partial contributions from both living and deceased records as witnessed in the regression technique.

Despite necessitating fewer scaling adjustments than the regression ML models, the F-tests shown in Table 5.164 for the ML classification algorithms yield fewer categories that outperform those from the SAPS S3 tables to a statistically significant degree. The most notable difference can be observed with males of ill health, whereby the 5% significance threshold has not been breached. The remainder of the results are similar, with the ill health female category demonstrating a statistically significant improvement in the ML model, while the others appear to have a substantially worse fit. There are four parameters used for male SAPS S3 categories and five for female categories [7]; the ML algorithm uses five parameters.

Category	DoF (ML)	DoF (SAPS)	Scale (SAPS)	RSS (ML)	RSS (SAPS)	F	p-value
Normal health males	52	53	0.97	32,692,370,063,063	15,505,133,213,036	0.47	0.99613
Normal health females	56	56	0.95	6,857,835,504,643	2,573,180,486,731	0.38	0.99983
Ill health males	73	74	1.13	4,031,868,905,276	5,803,531,555,305	1.44	0.06050
Ill health females	81	81	1.19	2,678,435,834,419	12,594,080,907,454	4.70	0.00000
Spouse males	86	87	0.98	116,001,412,973	24,976,395,423	0.22	1.00000
Spouse females	93	93	0.94	2,369,339,018,243	1,437,684,646,757	0.61	0.99158

Figure 5.164: F-test comparing the scaled ML model against SAPS S3 CMI tables using optimal scaling factors for each category.

Figure 5.165 visually confirms that expected exits from the ML classification model align closer to actual exits than the SAPS S3 table for females of ill health, displaying higher exits at younger ages and lower exits at older ages. Male spouses exhibit the poorest fit once again, although there may be marginal improvement from the regression algorithm, underestimating exits at lower ages and overestimating them at higher ages. The fits for other categories appear satisfactory overall; although lines are not as smooth as the SAPS S3 curve, an improvement over the regression models can generally be observed.

Tables 5.166 and 5.167 support the majority of aforementioned observations with the ML classification model, whereby expected exit amounts and mean exit ages are closer to the actual values for ill health categories and not so for most of the others. Female spouses deviate from this since the model provides a slightly closer value despite seemingly being a worse fit.

A similar trend can be witnessed with the ratio of actual to expected exits, as shown in Table 5.168; the ML classification model yields values that are nearer to one for ill health and female spouse categories in comparison to SAPS S3 tables. These figures are also favoured over those derived from the ML regression model, reflecting the classification model's keener accuracy in predicting exit amounts for each category as a whole.

In contrast, Table 5.169 portrays an imbalance between the yearly, similar to the ML regression. Predictions of alive and deceased classifications are based on the dominant status at each decision node; these nodes are not always unanimous in their assignment of

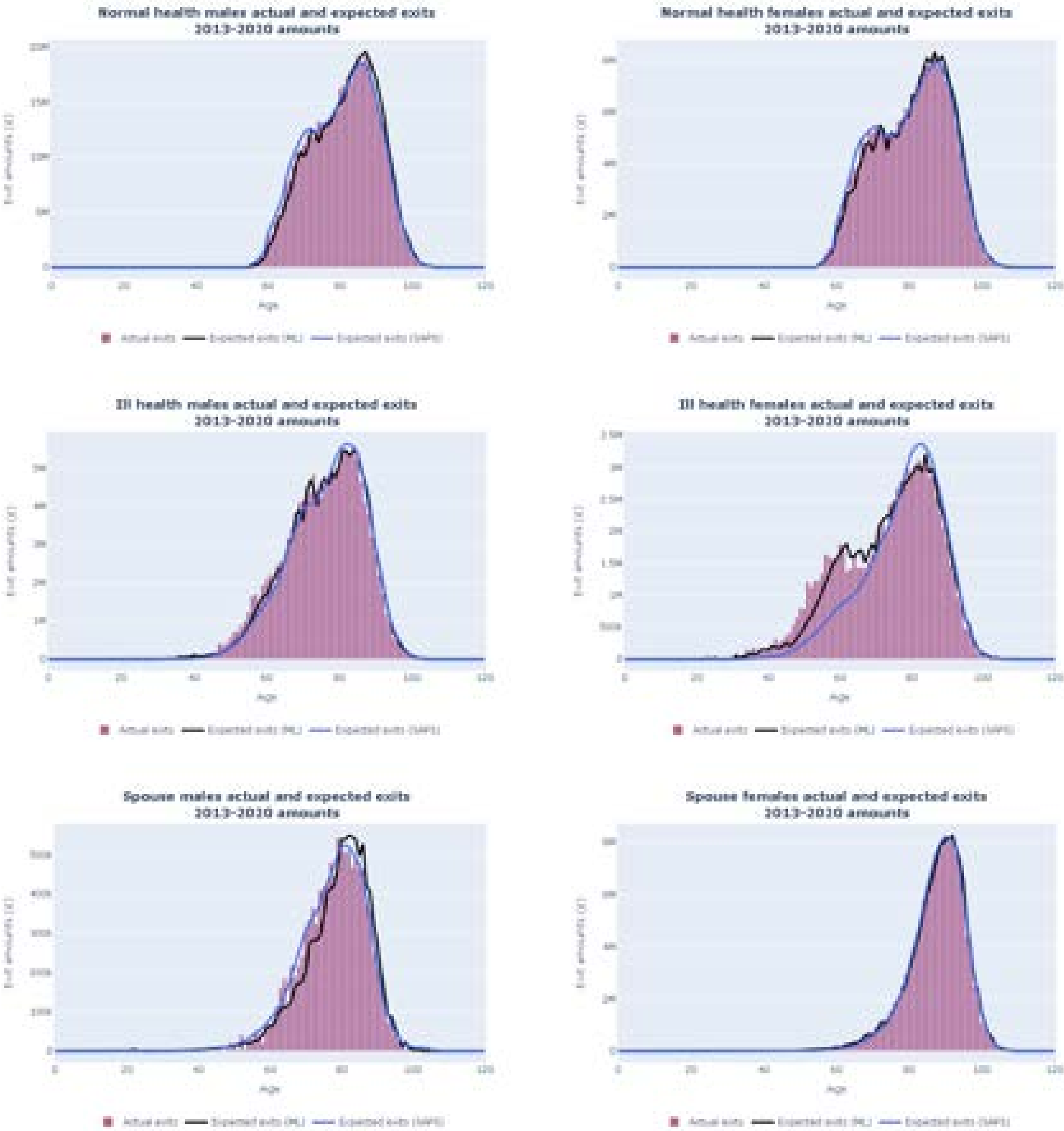


Figure 5.165: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

a particular class, leaving the model unable to accurately differentiate between living and death records. The imbalance in the distribution of alive and deceased data across the years further exacerbates this issue. Since all living records in the data only pertain to 2015 and 2019, other years solely consist of exit records; consequently, it is impossible to produce false positives in these other years with any false negatives reducing the amount of expected exits relative to actual exits. Since these false negatives were seemingly compensated with the introduction of a similar magnitude of false positives in 2015 and 2019, the number of expected exits appeared significantly higher than actual exits in these specific years.

Category	Actual average exit age	Expected exit age (ML)	Expected exit age (SAPS)
Normal health males	81.10	81.83	80.72
Normal health females	81.00	81.77	80.97
Ill health males	75.63	76.74	77.23
Ill health females	73.47	74.96	78.21
Spouse males	78.13	79.40	78.22
Spouse females	87.69	88.23	88.37

Figure 5.166: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	EtR	Actual exits	Expected exits (ML)	Expected exits (SAPS)
Normal health males	20,206,012,254	461,626,826	455,724,845	465,910,222
Normal health females	14,679,818,643	211,641,570	209,056,915	211,738,645
Ill health males	3,807,814,871	147,091,489	143,657,919	140,994,004
Ill health females	3,054,413,132	93,699,818	90,971,012	80,173,577
Spouse males	386,664,214	11,188,474	10,809,514	11,374,389
Spouse females	2,568,006,884	139,475,485	138,212,663	137,244,882

Figure 5.167: EtR and the actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	Actual/Expected (ML)	Actual/Expected (SAPS)
Normal health males	1.01	0.99
Normal health females	1.01	1.00
Ill health males	1.02	1.04
Ill health females	1.03	1.17
Spouse males	1.04	0.98
Spouse females	1.01	1.02

Figure 5.168: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	Year	EtR	Actual exits	Expected exits (ML)	Expected exits (SAPS)
Normal health males	2013	2,863,256,463	60,156,878	16,061,783	65,681,530
Normal health males	2014	2,951,373,636	66,990,292	17,558,976	68,243,510
Normal health males	2015	3,011,746,941	64,487,375	221,812,819	69,247,443
Normal health males	2016	2,764,492,973	66,015,153	14,917,784	62,309,942
Normal health males	2017	2,823,842,762	68,110,071	15,098,283	64,206,245
Normal health males	2018	2,870,812,873	65,710,707	12,268,410	65,955,561
Normal health males	2019	2,920,486,606	70,156,348	158,006,789	70,265,991
Normal health females	2013	1,858,675,890	25,814,524	5,626,016	27,103,101
Normal health females	2014	1,977,965,613	28,694,138	6,591,578	29,047,772
Normal health females	2015	2,096,792,719	28,212,963	101,760,145	30,504,274
Normal health females	2016	1,998,629,079	30,128,051	5,596,206	28,243,309
Normal health females	2017	2,124,273,832	32,857,702	5,745,031	30,170,129
Normal health females	2018	2,248,124,392	31,734,810	4,806,219	31,940,171
Normal health females	2019	2,375,357,117	34,199,382	78,931,721	34,729,888
Ill health males	2013	604,603,684	19,803,276	4,377,157	20,818,452
Ill health males	2014	592,823,300	20,885,047	4,734,008	20,985,558
Ill health males	2015	576,845,454	20,351,910	67,964,067	20,763,477
Ill health males	2016	533,583,862	21,794,575	4,311,529	19,457,476
Ill health males	2017	517,533,286	22,038,598	4,974,197	19,553,697
Ill health males	2018	499,509,130	21,275,445	3,865,367	19,364,055
Ill health males	2019	482,916,155	20,942,638	53,431,592	20,051,288
Ill health females	2013	465,001,162	11,778,968	1,981,289	10,945,554
Ill health females	2014	463,151,341	12,346,080	2,223,755	11,376,351
Ill health females	2015	459,696,973	12,564,365	42,820,127	11,679,207
Ill health females	2016	419,762,345	13,892,182	2,245,368	10,904,742
Ill health females	2017	418,358,388	14,577,684	2,286,768	11,290,911
Ill health females	2018	415,648,567	14,127,456	2,020,303	11,629,447
Ill health females	2019	412,794,355	14,413,083	37,393,403	12,347,364
Spouse males	2013	45,333,919	1,032,894	186,934	1,209,283
Spouse males	2014	49,748,966	1,564,100	254,753	1,381,808
Spouse males	2015	53,432,820	1,278,867	4,510,338	1,539,730
Spouse males	2016	52,651,614	1,542,505	292,207	1,474,938
Spouse males	2017	57,312,944	1,797,194	343,288	1,682,412
Spouse males	2018	61,894,774	1,877,980	318,943	1,889,887
Spouse males	2019	66,289,177	2,094,934	4,903,051	2,196,332
Spouse females	2013	381,962,648	17,964,934	6,301,216	19,706,449
Spouse females	2014	389,817,028	22,407,052	7,117,613	20,586,442
Spouse females	2015	389,075,830	19,311,757	62,777,491	20,586,957
Spouse females	2016	346,048,812	19,751,019	6,316,785	18,620,087
Spouse females	2017	350,658,082	21,222,412	6,408,996	18,997,254
Spouse females	2018	353,980,137	18,785,934	5,064,362	19,102,908
Spouse females	2019	356,464,348	20,032,377	44,226,199	19,644,785

Figure 5.169: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category and year, in addition to expected exit amounts based on applying a ML algorithm in contrast to those relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

5.6.2 Adding census data to ML classification compared to baseline

After the identification of the optimal ML classification model, using variables exclusively from the membership data, performance improvements through the incorporation of supplementary external data can be analysed.

The significance of each feature upon its incorporation to the optimal ML classification model is illustrated in Figure 5.170; once again, the additional census data variables appear to be of the least importance, with a marginally more noteworthy impact than observed in ML regressions.

Table 5.171 indicates that the training ROC AUC has slightly improved upon the inclusion of these extra features; on the other hand, the testing ROC AUC decreases by a small degree more, underlining how these modifications impair the model's ability to generalise predictions for new, unseen data.

The optimal scaling factors applicable to the model for each category are presented in Table 5.172; factors for females of normal health and female spouses have shifted further from one, whereas they are a closer fit for females with health issues and male spouses.

Expected exits with the census data variables, in Table 5.173, only yield similar total expected exit amounts for spouses. Figure 5.175 highlights close approximations of exits for models with and without additional census data variables by age for each category; the predominant differences are a tendency to underestimate exit amounts at older ages for females in ill health, whilst the reverse is true for male spouses at similar ages.

Figure 5.174 presents the F-test juxtaposing expected exits for models with and without extra census data variables; the ML algorithm, inclusive of the supplementary census variables, contains sixteen parameters, whereas the model without contains five; the additional census data causes a reduction of the RSS across all categories. However, the p-values greatly exceed the critical 5% threshold; thus, the additional complexity introduced by these variables cannot be justified since there is insufficient evidence concluding the model's superiority.

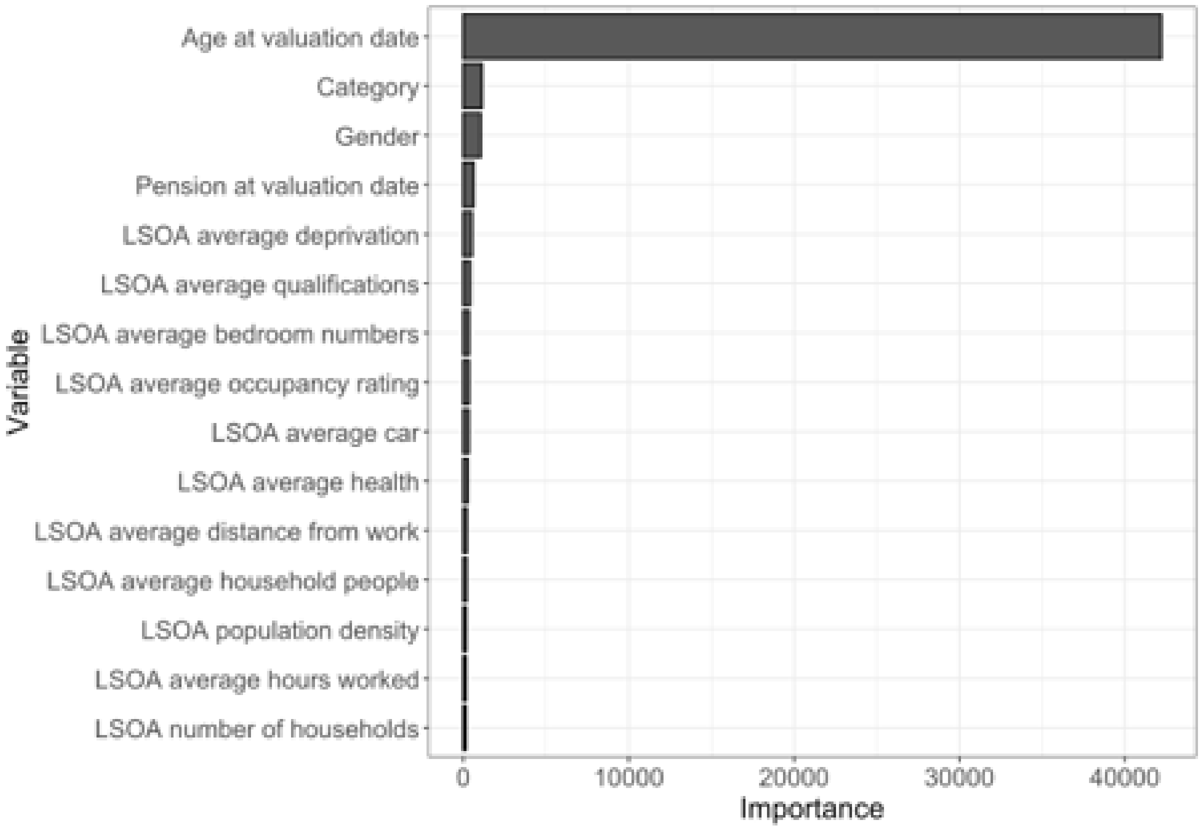


Figure 5.170: Feature importance in the optimal fitted model, generated using the additional census data variables on top of the optimal classification model.

Model description	Optimal ROC AUC tree depth	Training accuracy	Test accuracy	Training ROC AUC	Test ROC AUC
No census data	10	0.922	0.922	0.816	0.819
Census data	10	0.922	0.922	0.817	0.816

Figure 5.171: Accuracy and ROC AUC metrics on training and testing data sets at optimal ROC AUC tree depths for models with and without the added census data variables.

Category	Best fit scaling factor
Normal health males	0.99
Normal health females	0.98
Ill health males	1.00
Ill health females	0.99
Spouse males	0.99
Spouse females	0.99

Figure 5.172: Proposed optimal scaling factors for the ML algorithm, using additional census data variables, to minimise the least square residuals of actual and expected exit amounts across each age for each category.

Category	EtR	Actual exits	Expected exits (XL)	Expected exits (ML)
Normal health males	20,206,012,254	461,626,826	455,970,821	461,782,551
Normal health females	14,679,818,643	211,641,570	208,841,089	213,886,536
Ill health males	3,807,814,871	147,091,489	143,840,439	144,218,429
Ill health females	3,054,413,132	93,699,818	89,785,040	94,085,573
Spouse males	386,664,214	11,188,474	10,567,547	10,345,709
Spouse females	2,568,006,884	139,475,485	139,562,375	141,288,696

Figure 5.173: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on applying a ML algorithm in contrast to those from relevant SAPS S3 CMI tables, as used by GAD in the 2020 valuations, with optimal scaling factors applied.

Category	DoF (XL)	DoF (ML)	Scale (ML)	RSS (XL)	RSS (ML)	F	p-value
Normal health males	42	53	1.01	32,709,894,629,852	33,720,958,101,491	1.03	0.46324
Normal health females	46	57	1.00	7,871,056,831,573	8,580,168,262,747	1.09	0.38372
Ill health males	62	73	1.00	3,820,563,766,318	3,824,439,841,434	1.00	0.50100
Ill health females	70	81	1.04	4,910,580,604,711	5,338,141,808,682	1.09	0.36148
Spouse males	74	85	0.97	167,302,864,261	169,043,697,057	1.01	0.48377
Spouse females	82	93	1.00	1,833,445,548,086	1,951,963,877,565	1.06	0.38712

Figure 5.174: F-test comparing scaled ML models with (MLXL) and without (ML) the added variables from census data for each category.

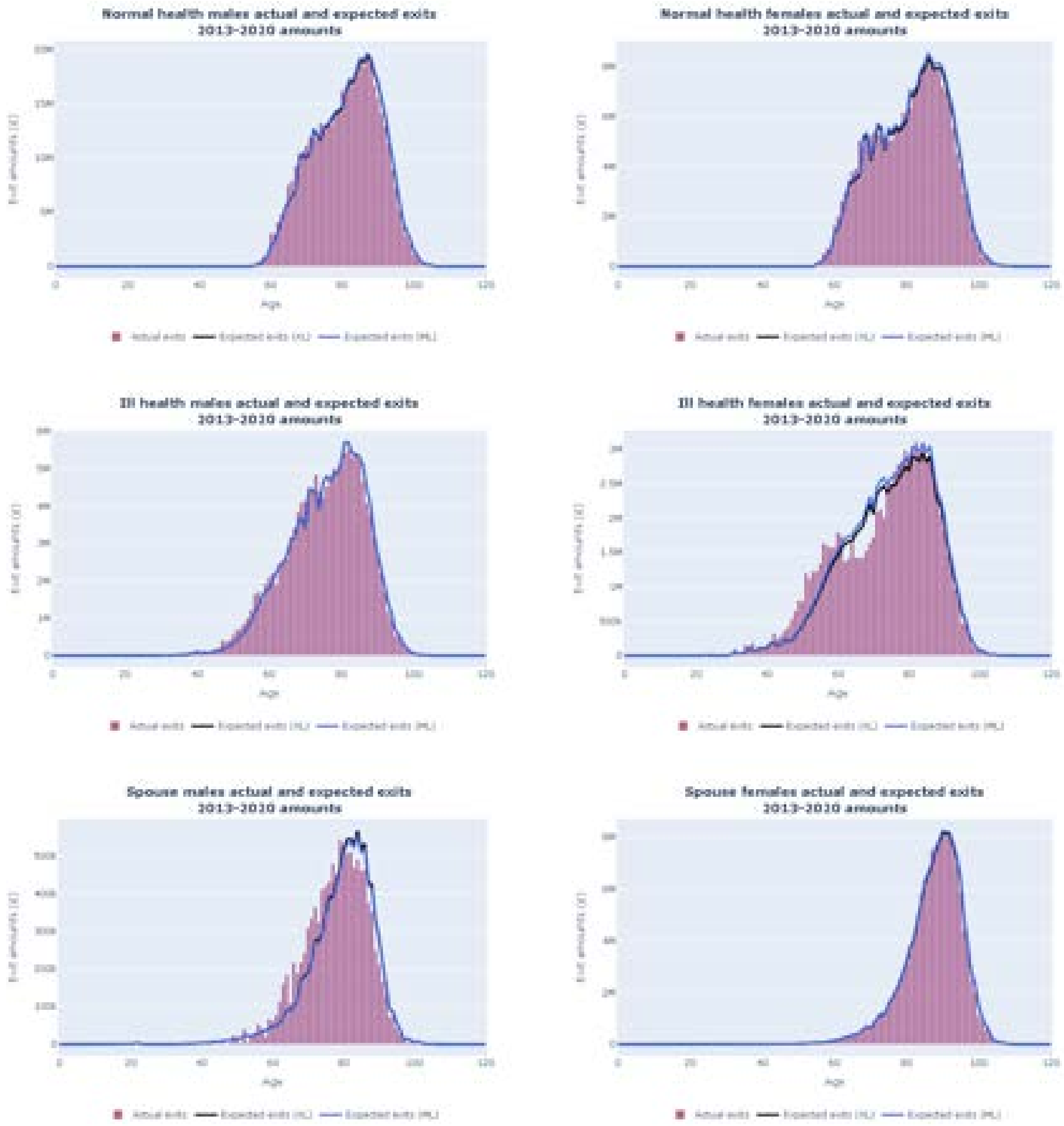


Figure 5.175: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on applying ML models, with and without added variables from the census data.

Chapter 6

Conclusions and recommendations

Chapter 2 explored the most prominent factors commonly known to influence mortality rates, whilst accounting for their relative availability in pension scheme membership data. The relationships are complex with the impact of each factor continually evolving over time.

Chapter 3 discussed the general objectives of pension scheme valuations for various involved stakeholders, as well as highlighting regulations and uncertainties in the environment in which they operate. After defining the role of AoS, the aims of an AoE were also underlined with a focus on how scheme size particularly affects usefulness.

Delving deeper into experience analysis, Chapter 4 discusses the necessary variables and implications of poor quality data. Different types of mortality models were briefly reviewed before focusing on select methods utilised specifically for this investigation. Concluding this chapter was an overview of calculations and terminology related to conducting an AoE in order to determine mortality rates.

Chapter 5 applied the general information covered by previous chapters to a specific set of pension scheme data. The analysis began by summarising membership data to comprehend its profile, confirming its suitability for analysis. EtR methodology was explained before its implementation in an AoE with applicable SAPS S3 series mortality tables. After examining the relative prudence of recommended assumptions from the most recent scheme valuations, best estimate assumptions were derived to be used as a baseline for comparison against other AoE techniques. The consequences from further subdivision using postcode data by IMD and region were explored, in addition to the impact caused by IMD-based SAPS S4 tables. Subsequently, mortality rates were derived by graduating them from the data in accordance to a formula from the Gompertz-Makeham family. The final sections investigated more experimental and crude approaches to analysing mortality experience data using ML regression and classification methods. Initial attempts were made to improve model performance by supplementing membership data with external variables at the LSOA granularity level.

All methods of mortality experience analysis usually produced reasonable results; SAPS S3 series assume that age, gender, pension type and pension amounts are the most relevant factors influencing mortality. These tables fit the data very well, when scaled by an appropriate percentage, except for females of ill health. Incorporating additional IMD subdivision highlighted that models fitted specifically to extremely high or low values tended to perform significantly better than the original model fitted to the overall dataset, which underestimated exit amounts by age for lower IMDs and overestimated them at higher IMDs. In contrast, additional regional subdivision produced a vaguer picture where there was an absence of significant performance improvements after the development of region-specific models. A subtle differentiation can be observed between northern and southern regions, which could be further explored. The analysis with S4 IMD-based tables generated models that had a far superior performance for most categories, especially with exceptionally low and high groupings. Subsequent IMD-based mortality tables distinguishing between normal and ill health records could improve performance for ill health categories; normal health categories already performed to a satisfactory degree despite this lack of segregation. The graduated mortality models fitted specifically to data by age were statistically superior to optimised SAPS tables; the optimal and preferred graduation formulae were simpler than those generating SAPS S3 series tables for spouses, though more complex for ill health categories. Normal health categories produced tables of similar complexity to those underlying SAPS S3 tables, although it resulted in a Gompertz-Makeham formula for females, as opposed to a Gompertz type. The ML regression and classification models denoted a remarkable improvement to overall fit for some ill health categories; nonetheless, they also provided seemingly worse fits for others. Furthermore, a substantial deterioration was witnessed regarding the smoothness of the fits; further investigation could be pursued through additional data or model tuning to enhance this aspect of the curves. Age, pension type, gender and pension value were amongst the preferred variable choices for ML models. Although available census data was initially employed to supplement membership records, these variables did not substantially enhance model performance. Closer examination of alternative data sources could reveal more if a broader range of features, with suspected links to mortality, is experimented with. Surprisingly, differentiating mortality by IMD was not particularly useful in decision tree ML models.

Despite offering the possibility of more accurate estimations at an individual level, ML regression and classification models in this analysis only provided reasonable outcomes at a group level. Decision tree methods were used for their simplicity and ease of interpretation; they offer a good foundation for understanding the interactions between features and their relative importance in predicting the target variable. The classification method yielded moderate model performance scores overall, partly due to the inherent imbalance between the number of positive and negative statuses within the data. The shape of estimated exits and, thus, underlying mortality rates, fitted well against the whole dataset, only requiring minor adjustments to optimise performance for each membership category. Attempts to directly predict exit amounts with a regression ML algorithm compromised model performance scores, necessitating more substantial scaling adjustments in order to optimise the fit at a category level. Estimated exits produced by ML models were not as smooth as those generated by other techniques; more comprehensive research could achieve smoother models. For instance, subsequent tuning of the model hyperparameters or using additional training data, either through including more member records or new features with stronger associations to the target variable than the census data used in this analysis. The ML approach allows for feature importance to be regularly monitored, assessing whether the factors contributing to mortality are changing over time in the selected population, as well as the identification, reaction and adaptation to the evolution of this ranking if and when it occurs. However, ML algorithms are still reliant on the availability of necessary variables in the dataset to identify such trends; therefore, this approach still potentially offers a more engaging analysis, revealing relevant knowledge and understanding pertaining to the population in question. Smaller schemes, with insufficient data to reliably analyse using more traditional methods, could benefit from a ML model that prompts more accurate predictions based on features at an individual level; if there were appropriate features with high mortality correlation that could be accurately captured, training could be processed solely using larger scheme data without reliance on specific experience data. More sophisticated techniques such as random forests and XGboost algorithms might be more suitable in handling intricate relationships. With the rise in popularity of ML algorithms in mortality and pensions work, more appropriate features and refined methods may emerge when analysing mortality experience.

Most methods form assumptions regarding significant model features at the outset, assuming their relevance; SAPS tables currently provide differentiated mortality rates by age, gender, health status, pension amount and IMD. The recent inclusion of additional IMD SAPS tables demonstrates the CMI's awareness concerning the impact of alternative factors aiding the evolution of mortality, keeping SAPS tables as relevant as possible to the current landscape. Naturally, the CMI will likely add subsequent mortality tables, which relate to other factors should their relative prominence increase; this wider range of options enable schemes to refine the experience analysis to their unique data and membership profile. From the user perspective, while this would likely capture and acknowledge the most current features of significance at a national level, the analysis would still probably fail to identify specific trends relevant to the population under consideration; thus, the insight gained from such an analysis may be overly restricted and narrow in application. Conversely, some dominant mortality factors, such as age, are almost guaranteed to have persistent relevance indefinitely; therefore, the incremental gains from other features may not sufficiently warrant further experimentation, given that the analysis is already adequately reliable, which justifies the continuation of the current approach. SAPS tables have a broader dataset allowing them to more reliably capture current mortality trends for the general population in comparison to the historic experience from scheme-specific data.

From a practical perspective, using SAPS tables is very convenient to apply to pension scheme data and allows for an easily adjustable analysis. Generating the EtR can present some complications, particularly if using an exact initial EtR; as discussed, there are approximations and alternative approaches that significantly simplify or bypass these calculations without materially detracting from the analysis. Whilst there is not an exhaustive selection of SAPS tables to choose from, experimentation can be achieved with different SAPS table variants to decipher the optimal fit to the shape of the mortality data.

Although further subdivision of the data by IMD or region does not affect the inherent methodology of calculations or their complexity, it increases the amount of categories to analyse from six to sixty-six. Consequently, there is a substantial inflation in human processing time to prepare data, confirm postcodes are correctly mapped where possible and interpret the results. The deeper examination also challenges the identification and communication of essential findings that other stakeholders require for decision making. Subsequent employment of IMD-based SAPS tables is relatively straightforward, using fixed and concise mappings, albeit necessitating adjustments to align pension amounts to the correct dates; these tables also beneficially reduce the number of groups to twenty-four, since records are allocated to four groups instead of IMD deciles. Currently, there is an absence of alternative IMD SAPS table variant options, though users could circumvent this issue by blending rates to customise their own weighted tables.

Graduations offer the opportunity to start the analysis afresh, adjusting the level of complexity as needed; since numerous types of mortality formula could be applied, technical knowledge and expertise are required in order to ascertain an appropriate selection. Experimentation with multiple formula types and parameter values is necessary to determine the most appropriate choice to use for the assumptions. Techniques might be required to fix or extend the fits at the tails where insufficient data prohibits the development of a reliable and smooth graduation, which can usually only be achieved for a limited number of ages in the middle of the required range. Obtaining graduated tables for the full age range may therefore be impractical in terms of time and resources; the process can be partially automated, if the family of the model is known, and fixed, due to the formulaic nature of fitting curves; nonetheless, additional judgement may be more useful in the selection phases in comparison to other methods. However, taking the time to develop the model's most pertinent form, using the most precise factors in its design, would yield superior attainments in contrast to automatically fitting the most relevant SAPS series table as the basis for mortality assumptions.

Uniquely, there is an opportunity for ML models to dismiss the need to derive exposures at each age, working directly with the data, subject to the prerequisite transformations to ensure quality. Technical knowledge and expertise are vital in selecting the correct type of algorithm, which is dependent on the data available and the properties of the response variable being predicted. Professional judgement is essential throughout the analysis to ascertain valuable model features and verify that the model's logic in distinguishing different outcomes adheres to common sense or falls within the realms of reasonable expectations based on evidence. Although it is possible to build a working model relatively quickly, fine-tuning it to optimise performance still remains a challenge; in conclusion, the ML approach currently offers the most practical potential, with additional risks and pitfalls to overcome.

Graduations of mortality rates are driven by a formulaic approach. The differentiation is again determined by the subdivision of data, specifying the number of models to generate as well as which variables are included for each one. Age, the most intuitive contributor to mortality rates, is routinely included in the graduation formula; other factors, based on descending significance, are accounted for with the subsequent subdivision of data to increase the number of models generated. Whilst this method provides the freedom to explore a broader set of potential features, common sense dictates a prevailing reliance to anchor the analysis to prominent features that are published by other organisations; this tendency may also be borne out of difficulties associated with limited membership data as well as the vast array of possible variables to explore, which have uncertain or complex relationships with mortality.

Many factors that influence mortality are not typically incorporated in pension scheme data, limiting the potential scope and quality of analysis; supplementary data from these underdeveloped areas may elevate model performance. Although it would be beneficial to include more variables relating to direct or indirect factors that impact mortality in an AoE analysis, this is unrealistic at an individual level given the sensitive and personal nature of this type of information; thus, it may be possible to circumvent such issues by gathering information at group levels. Additional investigation and judgement would identify relevant information and discern the optimal level of granularity required, respectively.

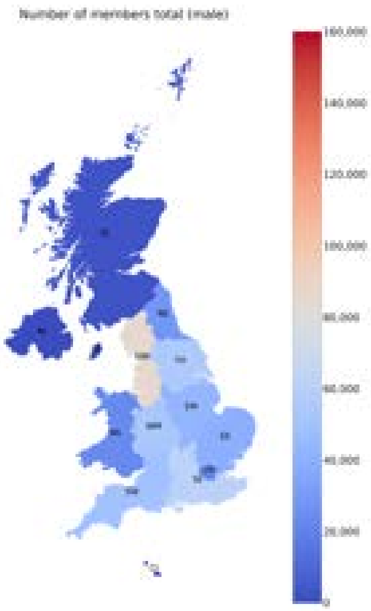
Although models have been tested against historic data to deduce the best fit, it should be noted that models with the closest fit to past experience will not necessarily produce more accurate predictions about future experience; factors deemed prevalent in the previous data may become more or less influential over time. Moreover, it is also possible that the experience in the past data was not representative of typical behaviour due to unique extenuating circumstances; in this case, it may be more useful to compare data against other pension schemes and wider datasets to observe other common and distinguishing traits; the use of standardised mortality rates and ratios might aid such comparisons.

Overall, models fitted based on past data still provide valuable insights that can be a foundation for the adaptation of assumptions relating to future outlooks. A closer fitting model should provide a more reliable starting point from which to make adaptations that allow for known trends and circumstantial deviations. Mortality assumptions, which better reflect the relevant population, can generate more accurate estimations of cashflow requirements, which need to be contributed towards pensions and lump sums, in addition to their potential duration. Subject to the purpose of the analysis, such as the type of pension scheme valuation being executed, there may be a desire to diverge from the best estimate assumptions, by an arbitrary amount, to establish a required degree of prudence and contingency. Reduced uncertainty regarding the accuracy of baseline assumptions, in combination with a greater understanding of the potential evolution of scheme membership, could minimise the required degree of prudence; thus, the allocation of money could be optimised, fulfilling the objectives of all stakeholders involved in the scheme.

Chapter 7

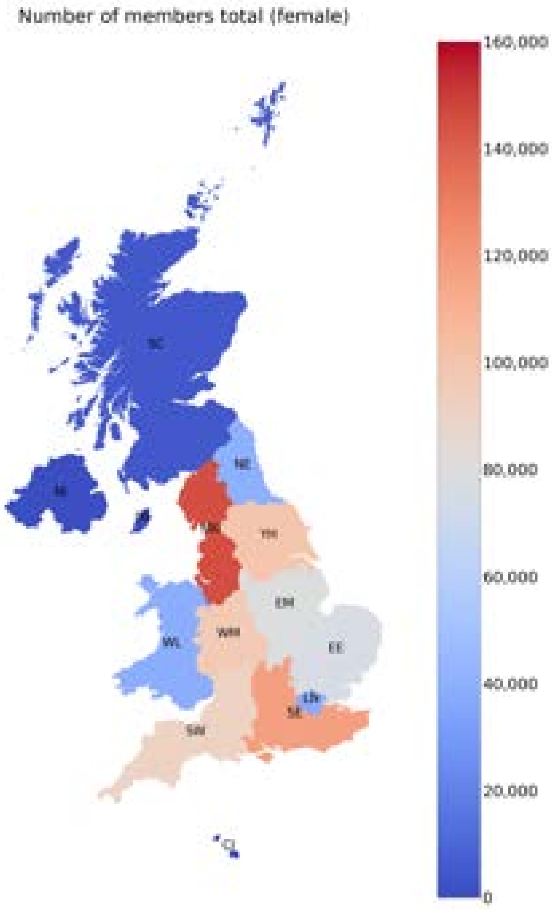
Appendix

7.1 Membership at valuation date



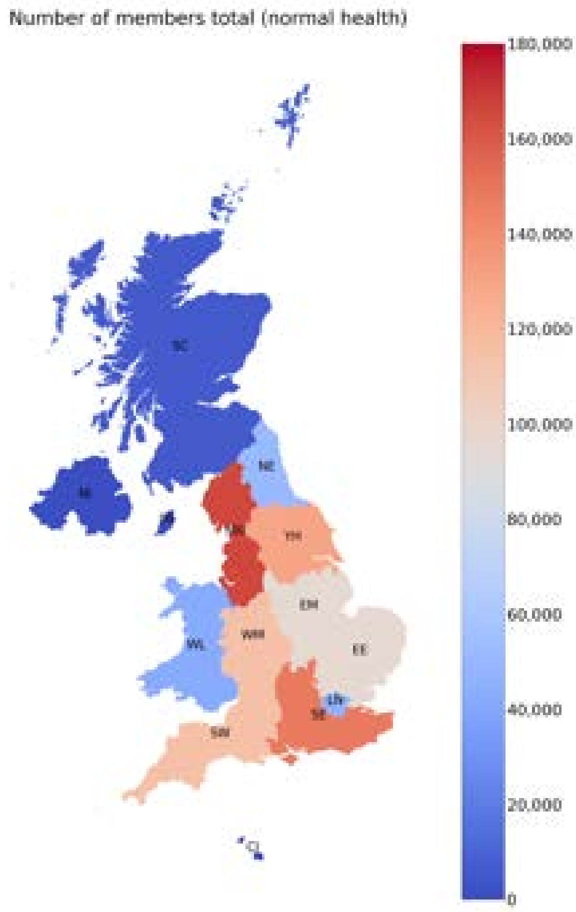
Region label	Region	Number of members total (male)
NW	North West (England)	86,034
SE	South East (England)	61,574
YH	Yorkshire and The Humber	55,242
WM	West Midlands (England)	52,598
SW	South West (England)	49,893
EM	East Midlands (England)	43,113
EE	East of England	39,979
NE	North East (England)	28,610
WL	Wales	23,279
LN	London	21,836
	Unknown	12,903
SC	Scotland	3,065
NI	Northern Ireland	466
IM	Isle of Man	159
CI	Channel Islands	64

Figure 7.1: Number of male pensioner and dependant scheme members at 31st March 2020 by region.



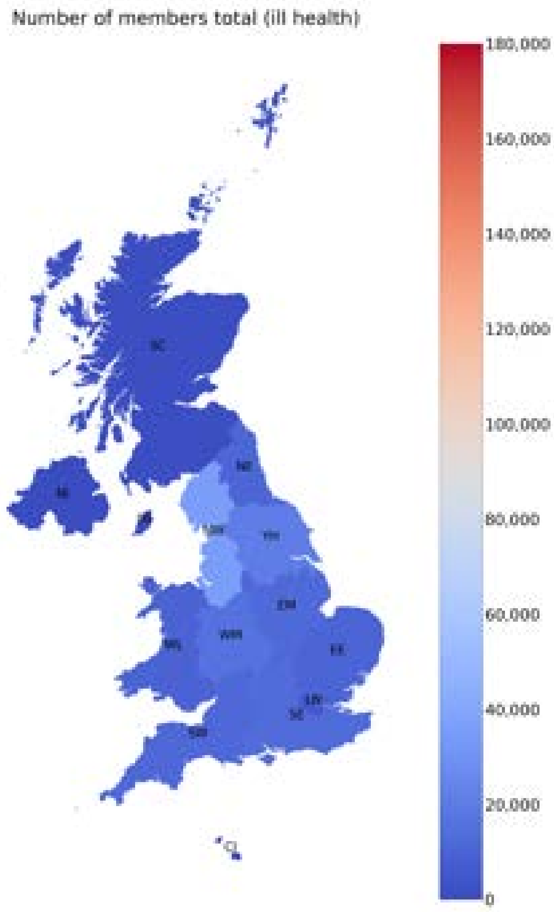
Region label	Region	Number of members total (female)
NW	North West (England)	145,374
SE	South East (England)	117,906
YH	Yorkshire and The Humber	99,686
WM	West Midlands (England)	96,578
SW	South West (England)	90,772
EM	East Midlands (England)	79,718
EE	East of England	76,439
NE	North East (England)	40,577
WL	Wales	38,763
LN	London	37,905
	Unknown	19,596
SC	Scotland	4,849
NI	Northern Ireland	665
IM	Isle of Man	243
CI	Channel Islands	142

Figure 7.2: Number of female pensioner and dependant scheme members at 31st March 2020 by region.



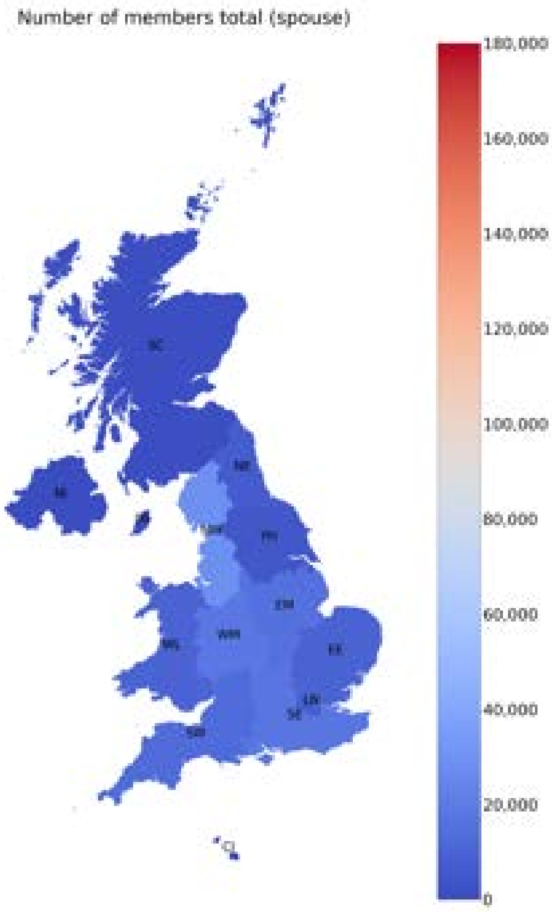
Region label	Region	Number of members total (normal health)
NW	North West (England)	165,663
SE	South East (England)	147,842
YH	Yorkshire and The Humber	129,471
SW	South West (England)	115,050
WM	West Midlands (England)	114,809
EE	East of England	96,109
EM	East Midlands (England)	94,960
NE	North East (England)	51,247
LN	London	46,185
WL	Wales	42,425
	Unknown	24,343
SC	Scotland	6,147
NI	Northern Ireland	858
IM	Isle of Man	305
CI	Channel Islands	171

Figure 7.3: Number of normal health retired pensioner scheme members at 31st March 2020 by region.



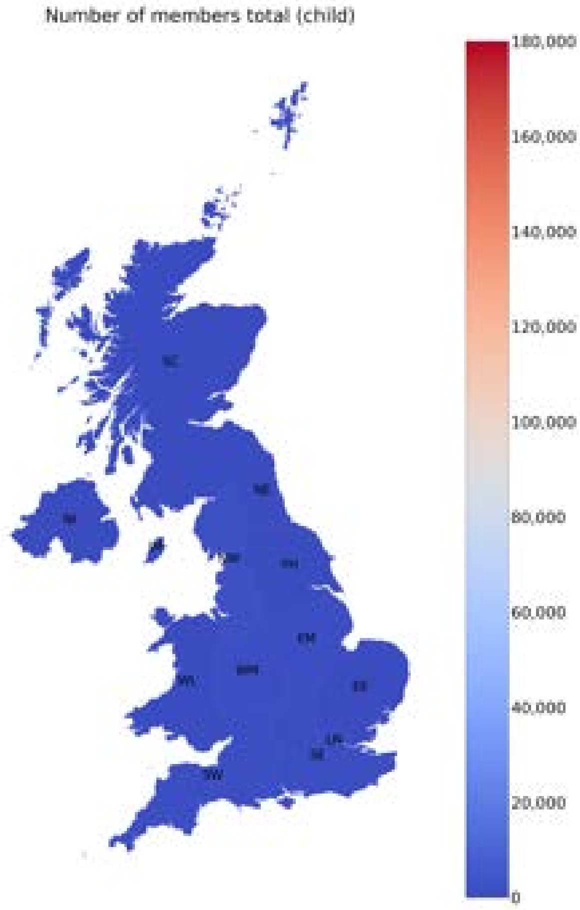
Region label	Region	Number of members total (ill health)
NW	North West (England)	35,561
YH	Yorkshire and The Humber	20,467
WM	West Midlands (England)	15,585
SE	South East (England)	13,218
EM	East Midlands (England)	12,052
SW	South West (England)	11,550
NE	North East (England)	10,939
EE	East of England	10,259
WL	Wales	9,594
LN	London	6,042
	Unknown	3,923
SC	Scotland	971
NI	Northern Ireland	148
IM	Isle of Man	49
CI	Channel Islands	17

Figure 7.4: Number of ill health retired pensioner scheme members at 31st March 2020 by region.



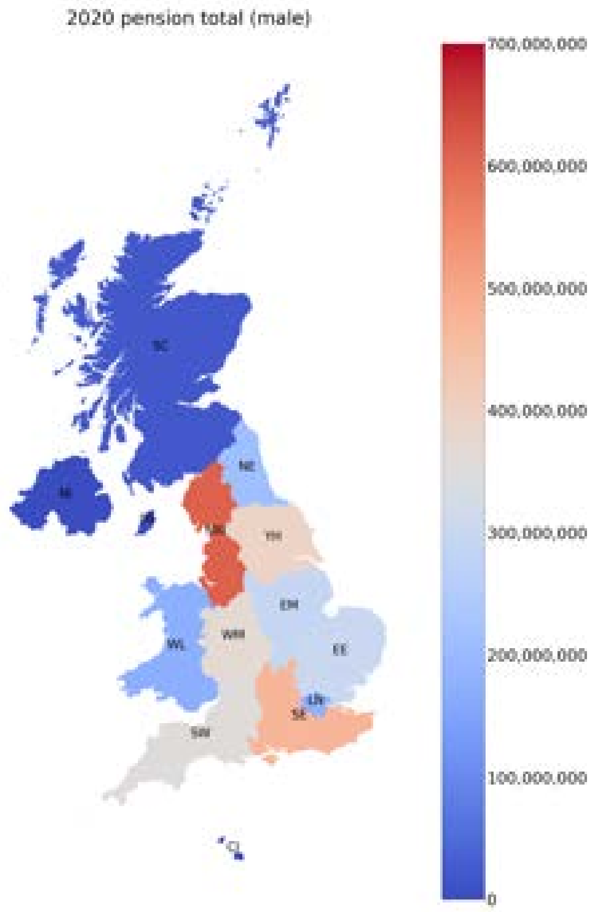
Region label	Region	Number of members total (spouse)
NW	North West (England)	28,651
WM	West Midlands (England)	17,929
SE	South East (England)	17,452
EM	East Midlands (England)	14,974
SW	South West (England)	13,333
EE	East of England	9,523
WL	Wales	9,500
LN	London	7,115
NE	North East (England)	6,595
YH	Yorkshire and The Humber	4,745
	Unknown	3,968
SC	Scotland	761
NI	Northern Ireland	116
IM	Isle of Man	44
CI	Channel Islands	18

Figure 7.5: Number of dependant spouse scheme members at 31st March 2020 by region.



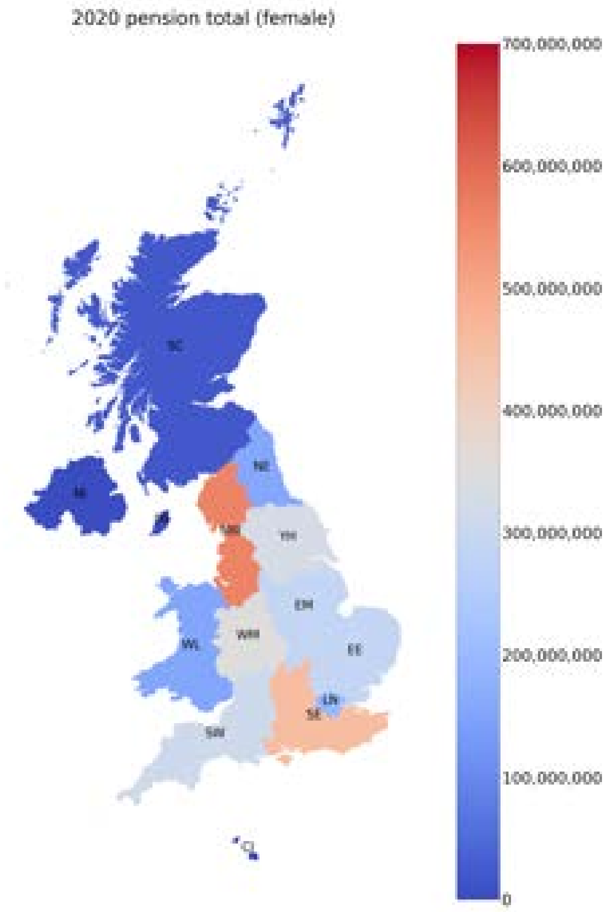
Region label	Region	Number of members total (child)
NW	North West (England)	1,533
SE	South East (England)	968
WM	West Midlands (England)	853
EM	East Midlands (England)	845
SW	South West (England)	732
EE	East of England	527
WL	Wales	523
NE	North East (England)	406
LN	London	399
	Unknown	265
YH	Yorkshire and The Humber	245
SC	Scotland	35
NI	Northern Ireland	9
IM	Isle of Man	4

Figure 7.6: Number of dependant child scheme members at 31st March 2020 by region.



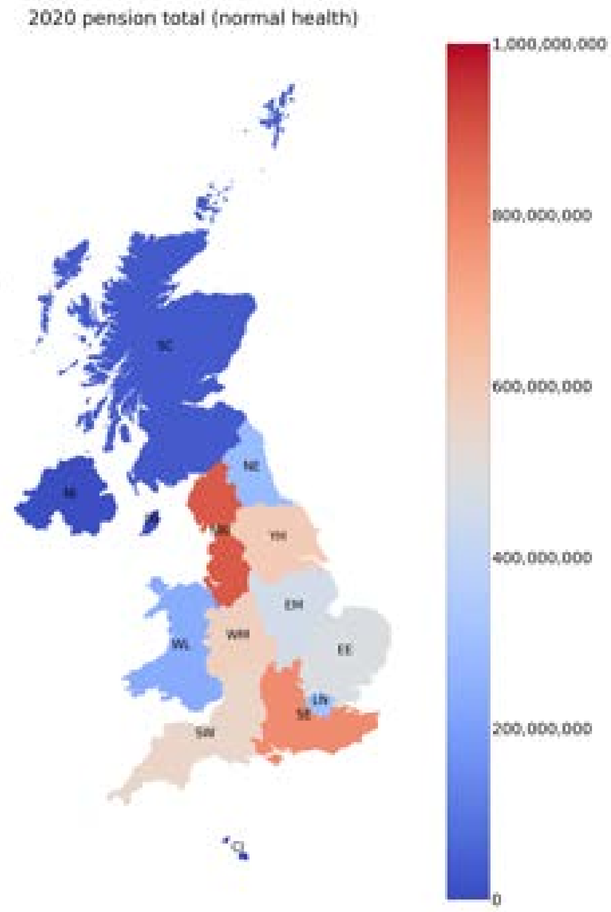
Region label	Region	2020 pension total (male)
NW	North West (England)	610,516,987
SE	South East (England)	471,598,517
YH	Yorkshire and The Humber	394,169,206
WM	West Midlands (England)	370,241,300
SW	South West (England)	357,500,714
EE	East of England	299,610,647
EM	East Midlands (England)	290,511,475
NE	North East (England)	210,760,744
LN	London	177,546,371
WL	Wales	173,752,637
	Unknown	76,106,778
SC	Scotland	20,181,627
NI	Northern Ireland	2,633,172
IM	Isle of Man	1,066,818
CI	Channel Islands	476,857

Figure 7.7: Total pension values (in 2020 terms) of male pensioner and dependant scheme members at 31st March 2020 by region.



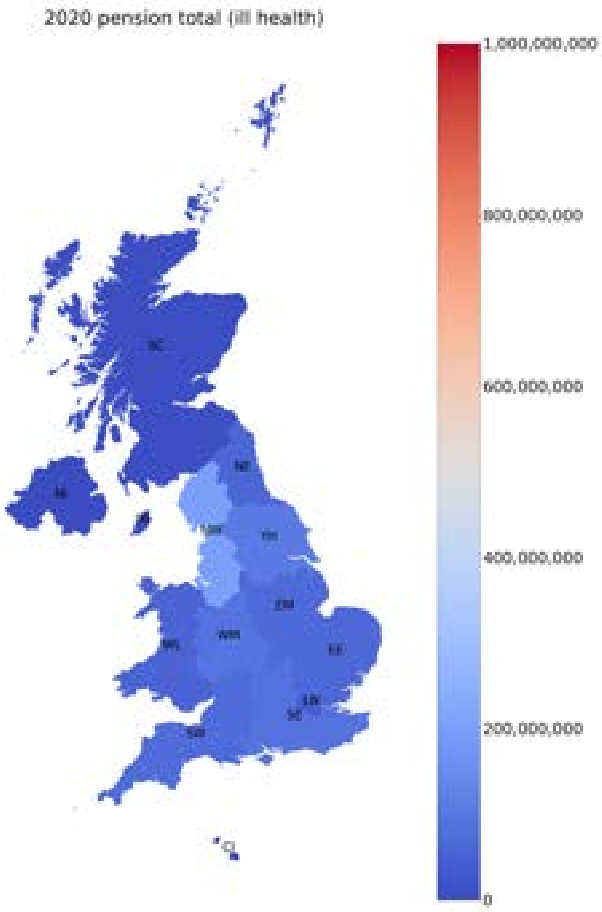
Region label	Region	2020 pension total (female)
NW	North West (England)	562,988,422
SE	South East (England)	458,907,455
WM	West Midlands (England)	352,418,074
YH	Yorkshire and The Humber	329,737,840
SW	South West (England)	310,264,697
EE	East of England	285,527,870
EM	East Midlands (England)	284,865,159
LN	London	194,446,153
NE	North East (England)	158,029,027
WL	Wales	153,184,911
	Unknown	70,169,318
SC	Scotland	20,150,040
NI	Northern Ireland	2,535,399
IM	Isle of Man	951,915
CI	Channel Islands	435,686

Figure 7.8: Total pension values (in 2020 terms) of female pensioner and dependant scheme members at 31st March 2020 by region.



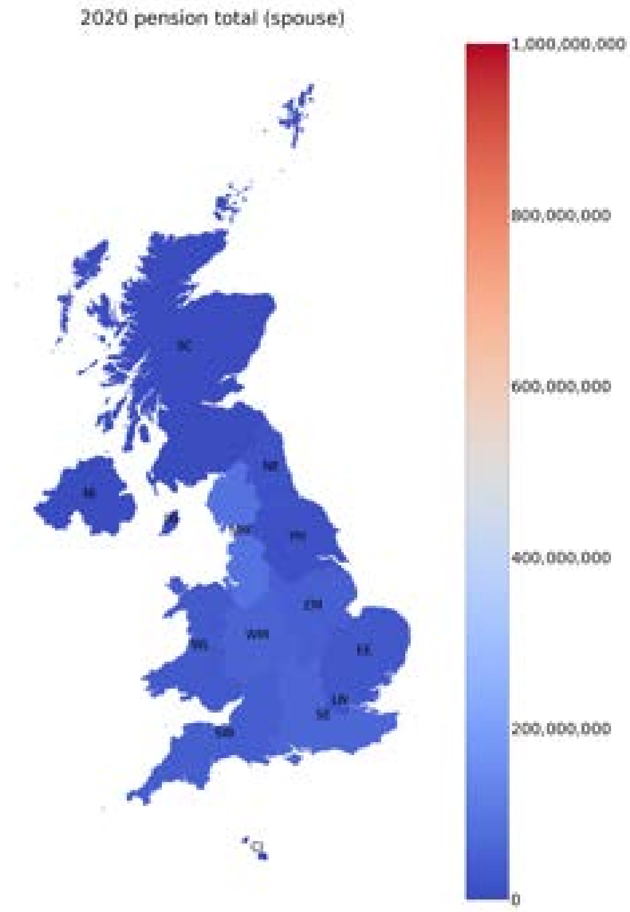
Region label	Region	2020 pension total (normal health)
NW	North West (England)	884,856,534
SE	South East (England)	783,514,662
YH	Yorkshire and The Humber	599,590,543
WM	West Midlands (England)	578,282,717
SW	South West (England)	554,131,612
EE	East of England	489,172,093
EM	East Midlands (England)	459,595,808
LN	London	306,113,599
NE	North East (England)	288,461,457
WL	Wales	239,714,521
	Unknown	110,577,960
SC	Scotland	31,446,131
NI	Northern Ireland	4,040,485
IM	Isle of Man	1,537,544
CI	Channel Islands	758,250

Figure 7.9: Total pension values (in 2020 terms) of normal health retired pensioner scheme members at 31st March 2020 by region.



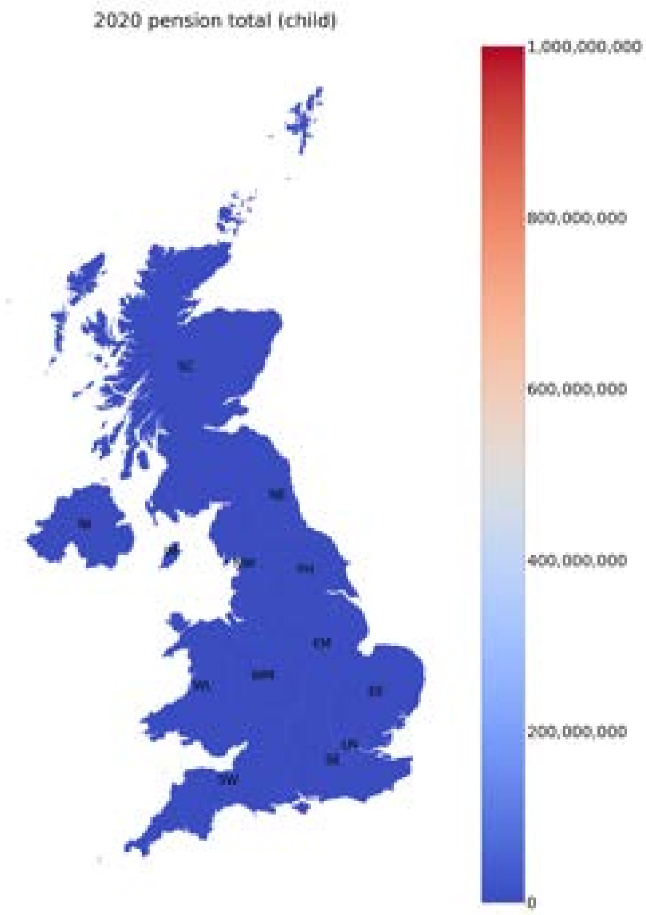
Region label	Region	2020 pension total (ill health)
NW	North West (England)	200,109,958
YH	Yorkshire and The Humber	109,276,884
WM	West Midlands (England)	90,309,485
SE	South East (England)	87,013,953
SW	South West (England)	71,024,672
EM	East Midlands (England)	68,179,199
EE	East of England	63,995,769
NE	North East (England)	60,466,063
WL	Wales	54,678,101
LN	London	39,793,848
	Unknown	23,684,488
SC	Scotland	6,428,356
NI	Northern Ireland	786,536
IM	Isle of Man	325,339
CI	Channel Islands	96,695

Figure 7.10: Total pension values (in 2020 terms) of ill health retired pensioner scheme members at 31st March 2020 by region.



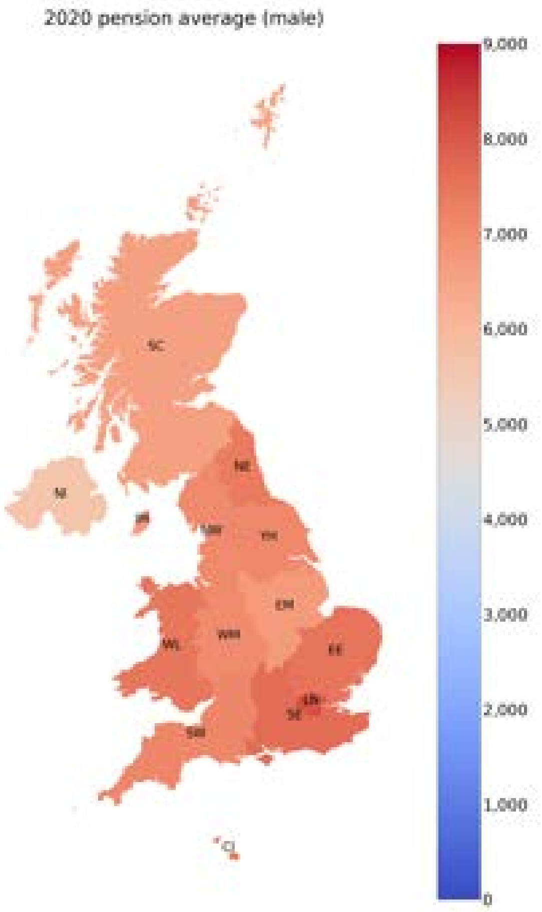
Region label	Region	2020 pension total (spouse)
NW	North West (England)	86,264,311
SE	South East (England)	58,629,935
WM	West Midlands (England)	52,658,210
EM	East Midlands (England)	46,347,336
SW	South West (England)	41,706,331
WL	Wales	31,665,260
EE	East of England	31,173,489
LN	London	25,352,505
NE	North East (England)	19,234,118
YH	Yorkshire and The Humber	14,771,739
	Unknown	11,675,413
SC	Scotland	2,420,495
NI	Northern Ireland	331,149
IM	Isle of Man	154,341
CI	Channel Islands	57,598

Figure 7.11: Total pension values (in 2020 terms) of dependant spouse scheme members at 31st March 2020 by region.



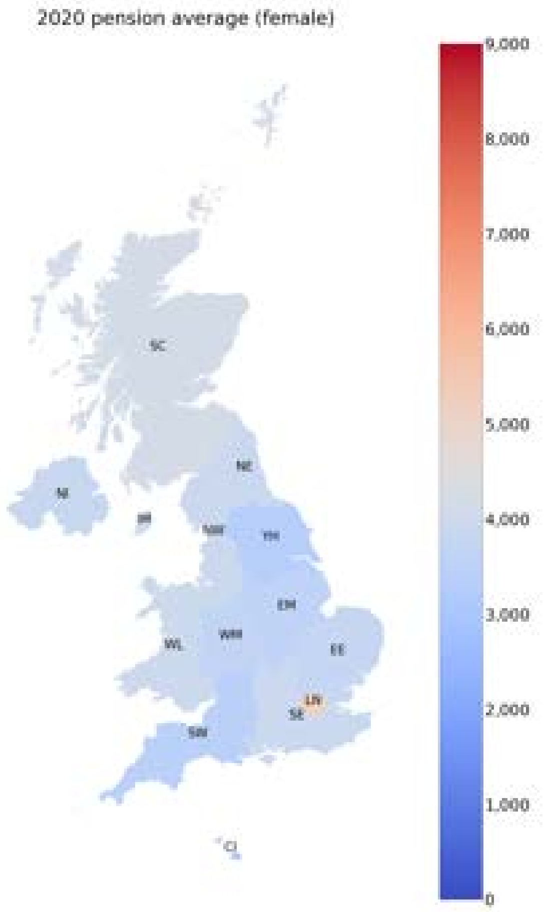
Region label	Region	2020 pension total (child)
NW	North West (England)	2,274,605
WM	West Midlands (England)	1,408,963
SE	South East (England)	1,347,423
EM	East Midlands (England)	1,254,290
SW	South West (England)	902,796
WL	Wales	879,666
EE	East of England	797,165
LN	London	732,572
NE	North East (England)	628,133
	Unknown	338,235
YH	Yorkshire and The Humber	267,881
SC	Scotland	36,684
NI	Northern Ireland	10,400
IM	Isle of Man	1,508

Figure 7.12: Total pension values (in 2020 terms) of dependant child scheme members at 31st March 2020 by region.



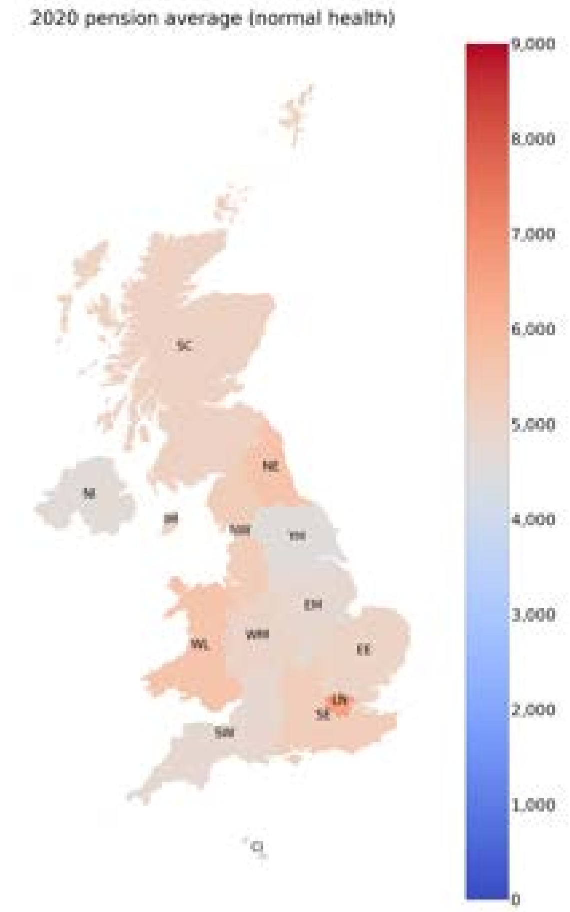
Region label	Region	2020 pension average (male)
LN	London	8,131
SE	South East (England)	7,659
EE	East of England	7,494
WL	Wales	7,464
CI	Channel Islands	7,451
NE	North East (England)	7,367
SW	South West (England)	7,165
YH	Yorkshire and The Humber	7,135
NW	North West (England)	7,096
WM	West Midlands (England)	7,039
EM	East Midlands (England)	6,738
IM	Isle of Man	6,710
SC	Scotland	6,585
	Unknown	5,898
NI	Northern Ireland	5,651

Figure 7.13: Mean pension values (in 2020 terms) of male pensioner and dependant scheme members at 31st March 2020 by region.



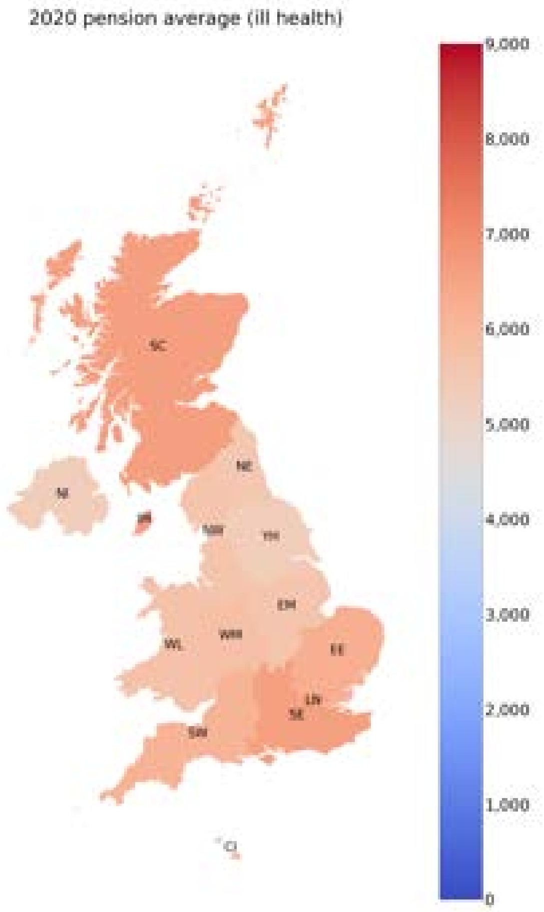
Region label	Region	2020 pension average (female)
LN	London	5,130
SC	Scotland	4,156
WL	Wales	3,952
IM	Isle of Man	3,917
NE	North East (England)	3,895
SE	South East (England)	3,892
NW	North West (England)	3,873
NI	Northern Ireland	3,813
EE	East of England	3,735
WM	West Midlands (England)	3,649
	Unknown	3,581
EM	East Midlands (England)	3,573
SW	South West (England)	3,418
YH	Yorkshire and The Humber	3,308
CI	Channel Islands	3,068

Figure 7.14: Mean pension values (in 2020 terms) of female pensioner and dependant scheme members at 31st March 2020 by region.



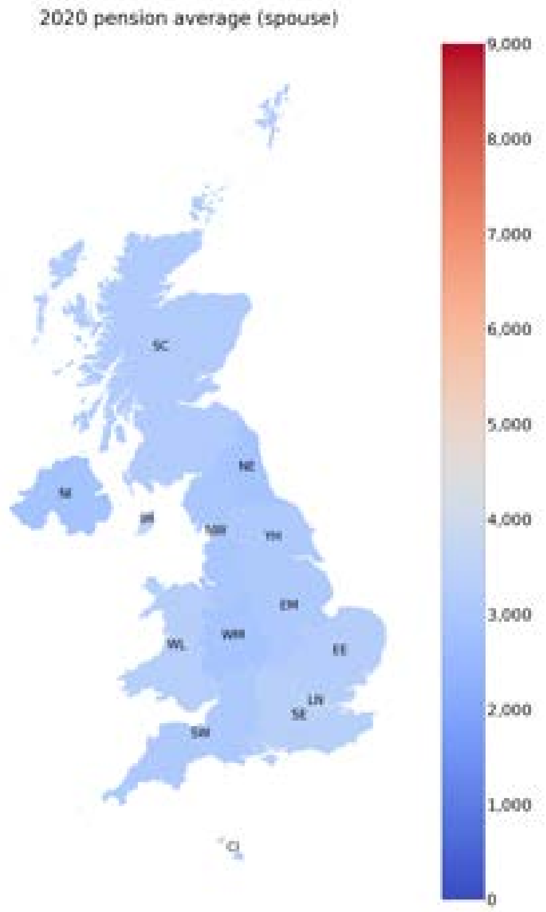
Region label	Region	2020 pension average (normal health)
LN	London	6,628
WL	Wales	5,650
NE	North East (England)	5,629
NW	North West (England)	5,341
SE	South East (England)	5,300
SC	Scotland	5,116
EE	East of England	5,090
IM	Isle of Man	5,041
WM	West Midlands (England)	5,037
EM	East Midlands (England)	4,840
SW	South West (England)	4,816
NI	Northern Ireland	4,709
YH	Yorkshire and The Humber	4,631
	Unknown	4,542
CI	Channel Islands	4,434

Figure 7.15: Mean pension values (in 2020 terms) of normal health retired pensioner scheme members at 31st March 2020 by region.



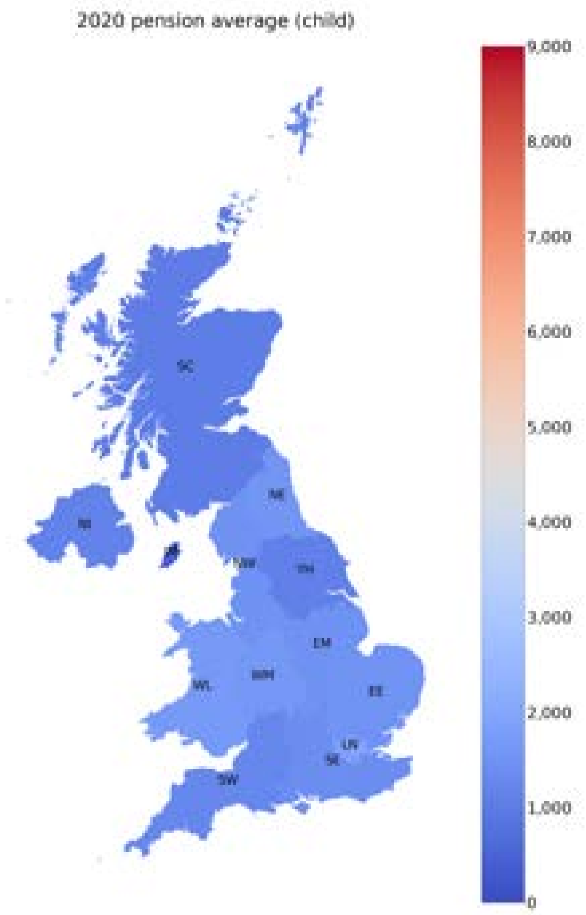
Region label	Region	2020 pension average (ill health)
IM	Isle of Man	6,640
SC	Scotland	6,620
LN	London	6,586
SE	South East (England)	6,583
EE	East of England	6,238
SW	South West (England)	6,149
	Unknown	6,037
WM	West Midlands (England)	5,795
WL	Wales	5,699
CI	Channel Islands	5,688
EM	East Midlands (England)	5,657
NW	North West (England)	5,627
NE	North East (England)	5,528
YH	Yorkshire and The Humber	5,339
NI	Northern Ireland	5,314

Figure 7.16: Mean pension values (in 2020 terms) of ill health retired pensioner scheme members at 31st March 2020 by region.



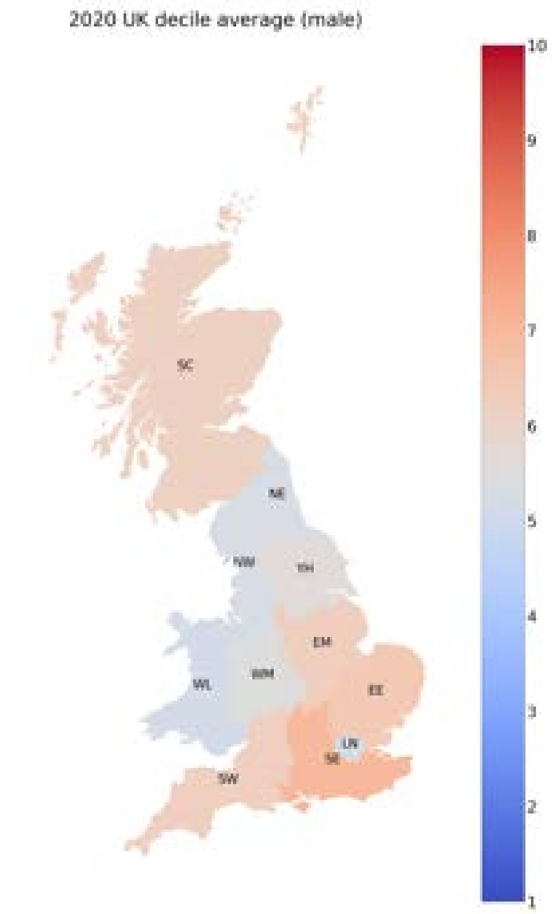
Region label	Region	2020 pension average (spouse)
LN	London	3,563
IM	Isle of Man	3,508
SE	South East (England)	3,359
WL	Wales	3,333
EE	East of England	3,273
CI	Channel Islands	3,200
SC	Scotland	3,181
SW	South West (England)	3,128
YH	Yorkshire and The Humber	3,113
EM	East Midlands (England)	3,095
NW	North West (England)	3,011
	Unknown	2,942
WM	West Midlands (England)	2,937
NE	North East (England)	2,916
NI	Northern Ireland	2,855

Figure 7.17: Mean pension values (in 2020 terms) of dependant spouse scheme members at 31st March 2020 by region.



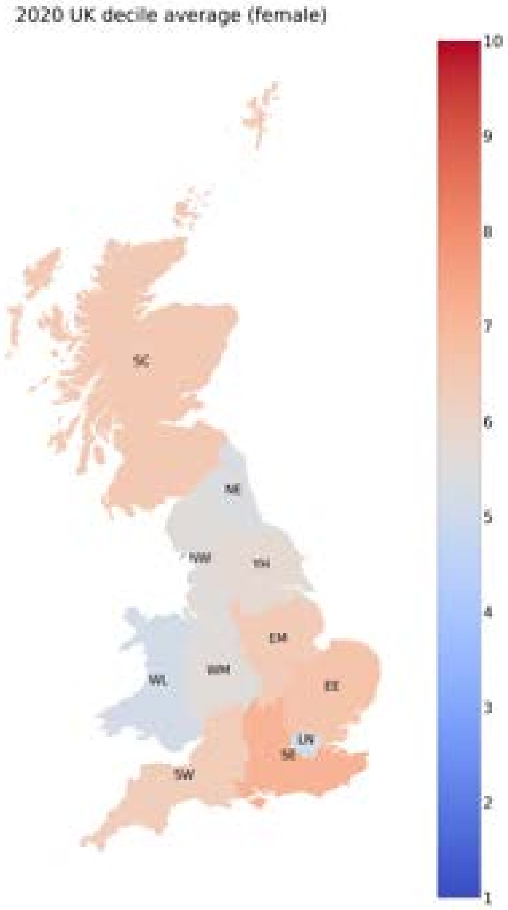
Region label	Region	2020 pension average (child)
LN	London	1,836
WL	Wales	1,682
WM	West Midlands (England)	1,652
NE	North East (England)	1,547
EE	East of England	1,513
EM	East Midlands (England)	1,484
NW	North West (England)	1,484
SE	South East (England)	1,392
	Unknown	1,276
SW	South West (England)	1,233
NI	Northern Ireland	1,156
YH	Yorkshire and The Humber	1,093
SC	Scotland	1,048
IM	Isle of Man	377

Figure 7.18: Mean pension values (in 2020 terms) of dependant child scheme members at 31st March 2020 by region.



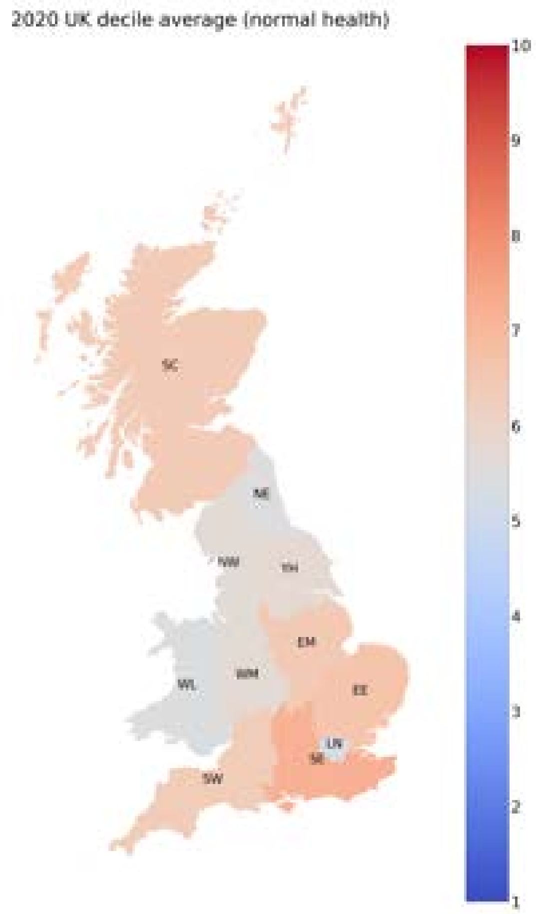
Region label	Region	2020 UK decile average (male)
SE	South East (England)	6.99
EE	East of England	6.47
EM	East Midlands (England)	6.26
SW	South West (England)	6.21
SC	Scotland	6.15
YH	Yorkshire and The Humber	5.63
WM	West Midlands (England)	5.52
NW	North West (England)	5.31
NE	North East (England)	5.28
LN	London	5.20
WL	Wales	5.16

Figure 7.19: Mean UK decile of male pensioner and dependant scheme members at 31st March 2020 by region.



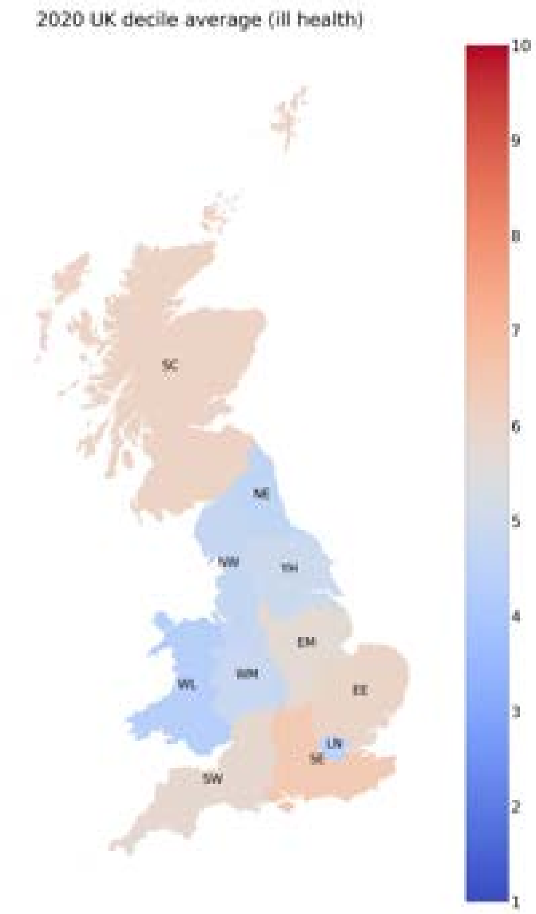
Region label	Region	2020 UK decile average (female)
SE	South East (England)	7.15
EE	East of England	6.64
SC	Scotland	6.45
EM	East Midlands (England)	6.40
SW	South West (England)	6.34
YH	Yorkshire and The Humber	5.76
WM	West Midlands (England)	5.62
NW	North West (England)	5.59
NE	North East (England)	5.40
WL	Wales	5.18
LN	London	5.15

Figure 7.20: Mean UK decile of female pensioner and dependant scheme members at 31st March 2020 by region.



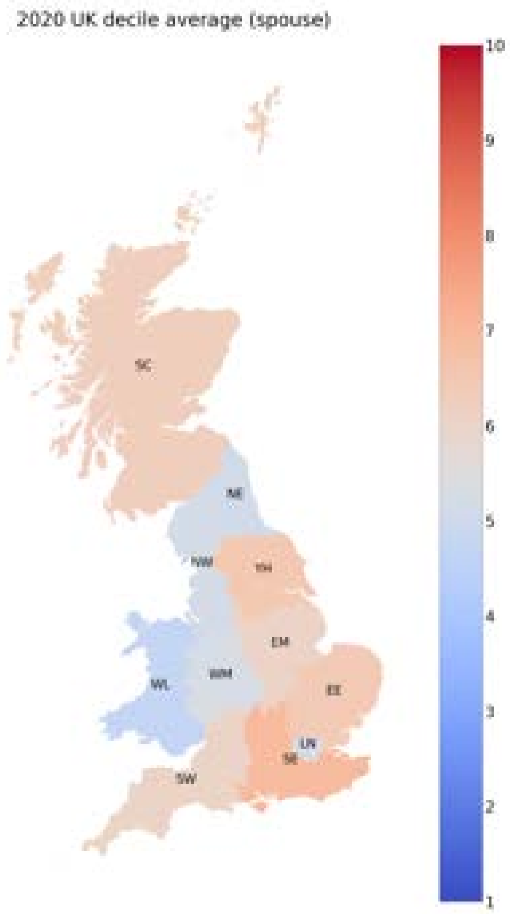
Region label	Region	2020 UK decile average (normal health)
SE	South East (England)	7.17
EE	East of England	6.66
EM	East Midlands (England)	6.48
SC	Scotland	6.39
SW	South West (England)	6.37
YH	Yorkshire and The Humber	5.80
WM	West Midlands (England)	5.74
NW	North West (England)	5.71
NE	North East (England)	5.53
WL	Wales	5.46
LN	London	5.24

Figure 7.21: Mean UK decile of normal health retired pensioner scheme members at 31st March 2020 by region.



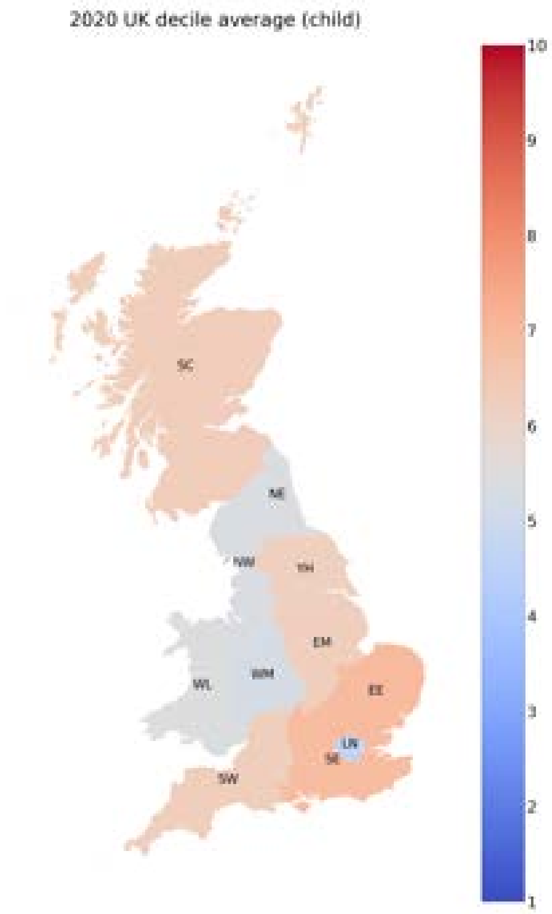
Region label	Region	2020 UK decile average (ill health)
SE	South East (England)	6.44
SC	Scotland	6.07
EE	East of England	6.00
SW	South West (England)	5.84
EM	East Midlands (England)	5.71
YH	Yorkshire and The Humber	4.97
WM	West Midlands (England)	4.84
NW	North West (England)	4.74
NE	North East (England)	4.65
LN	London	4.52
WL	Wales	4.36

Figure 7.22: Mean UK decile of ill health retired pensioner scheme members at 31st March 2020 by region.



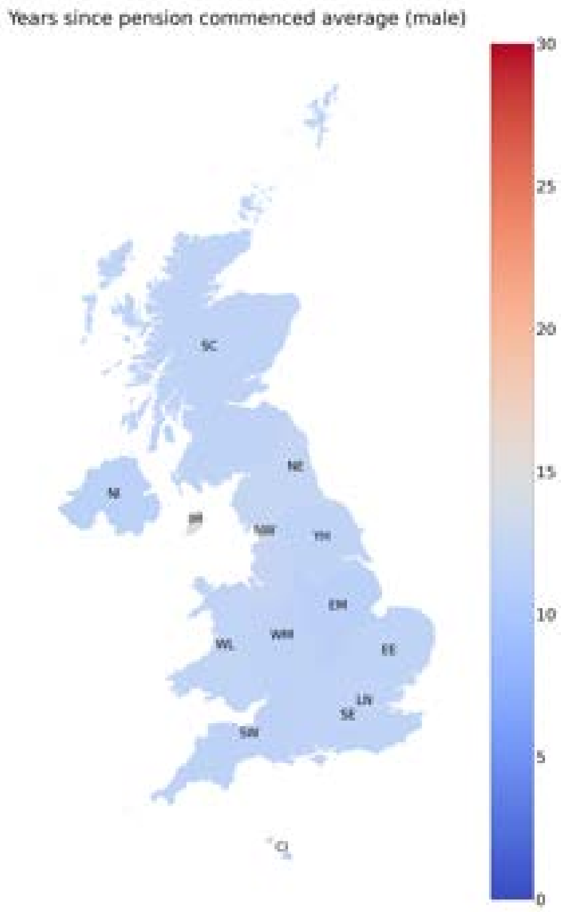
Region label	Region	2020 UK decile average (spouse)
SE	South East (England)	6.91
YH	Yorkshire and The Humber	6.50
EE	East of England	6.44
SC	Scotland	6.27
SW	South West (England)	6.05
EM	East Midlands (England)	6.04
WM	West Midlands (England)	5.29
LN	London	5.28
NW	North West (England)	5.17
NE	North East (England)	5.11
WL	Wales	4.69

Figure 7.23: Mean UK decile of dependant spouse scheme members at 31st March 2020 by region.



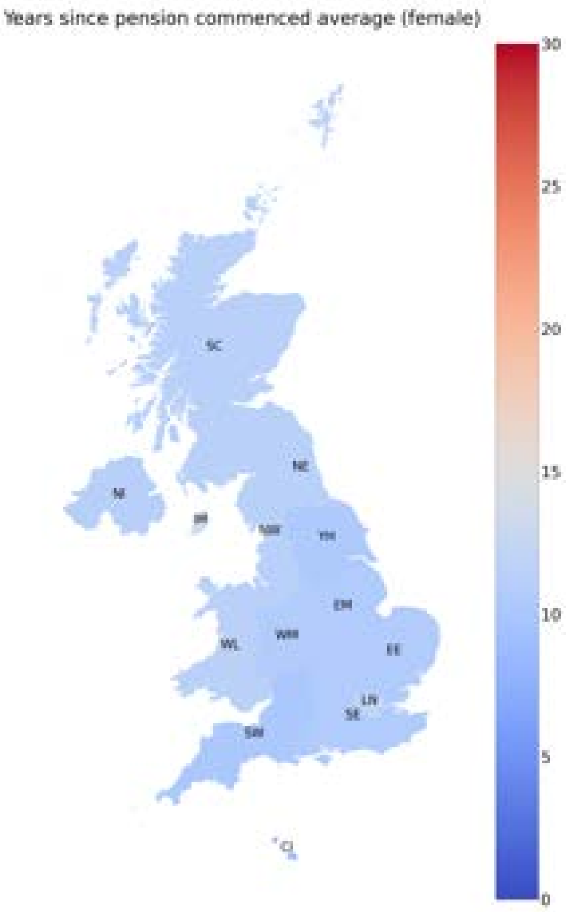
Region label	Region	2020 UK decile average (child)
EE	East of England	6.92
SE	South East (England)	6.90
SC	Scotland	6.29
SW	South West (England)	6.27
EM	East Midlands (England)	6.26
YH	Yorkshire and The Humber	6.20
WL	Wales	5.45
NE	North East (England)	5.42
NW	North West (England)	5.36
WM	West Midlands (England)	5.24
LN	London	4.61

Figure 7.24: Mean UK decile of dependant child scheme members at 31st March 2020 by region.



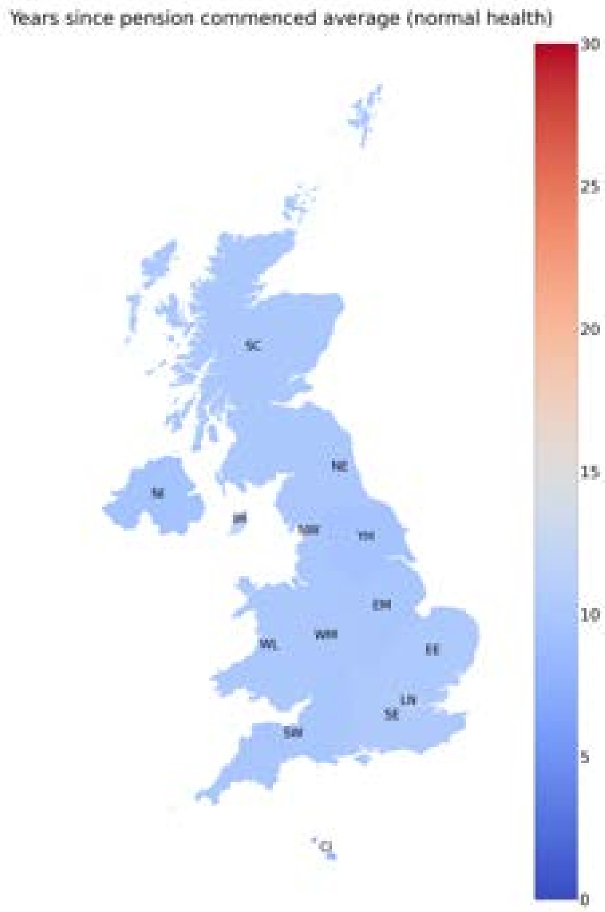
Region label	Region	Years since pension commenced average (male)
IM	Isle of Man	14.22
	Unknown	13.67
NE	North East (England)	12.38
NW	North West (England)	12.37
LN	London	12.17
WL	Wales	12.03
YH	Yorkshire and The Humber	11.93
SC	Scotland	11.89
SE	South East (England)	11.86
EE	East of England	11.84
SW	South West (England)	11.83
CI	Channel Islands	11.82
WM	West Midlands (England)	11.80
NI	Northern Ireland	11.51
EM	East Midlands (England)	11.38

Figure 7.25: Mean number of years since pension commencement of male pensioner and dependant scheme members at 31st March 2020 by region.



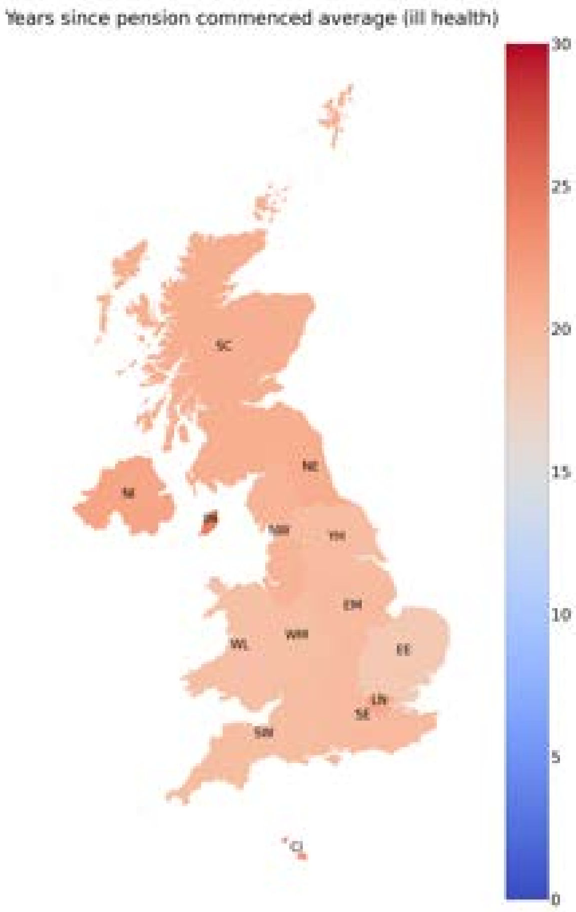
Region label	Region	Years since pension commenced average (female)
IM	Isle of Man	13.08
	Unknown	12.90
LN	London	12.03
WL	Wales	11.37
NW	North West (England)	11.29
NE	North East (England)	11.28
SC	Scotland	11.26
NI	Northern Ireland	11.13
EM	East Midlands (England)	10.66
WM	West Midlands (England)	10.61
SE	South East (England)	10.60
EE	East of England	10.60
YH	Yorkshire and The Humber	10.35
SW	South West (England)	10.18
CI	Channel Islands	9.13

Figure 7.26: Mean number of years since pension commencement of female pensioner and dependant scheme members at 31st March 2020 by region.



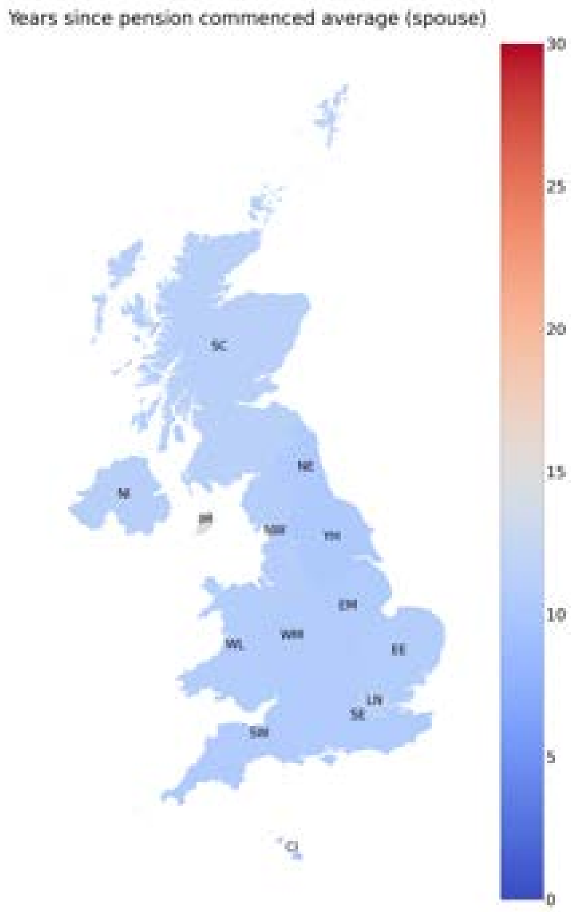
Region label	Region	Years since pension commenced average (normal health)
	Unknown	12.02
IM	Isle of Man	11.61
LN	London	11.00
SE	South East (England)	10.34
EE	East of England	10.30
WL	Wales	10.12
SC	Scotland	10.11
NW	North West (England)	10.01
WM	West Midlands (England)	9.99
NE	North East (England)	9.99
SW	South West (England)	9.94
EM	East Midlands (England)	9.89
YH	Yorkshire and The Humber	9.56
NI	Northern Ireland	9.55
CI	Channel Islands	8.72

Figure 7.27: Mean number of years since pension commencement of normal health retired pensioner scheme members at 31st March 2020 by region.



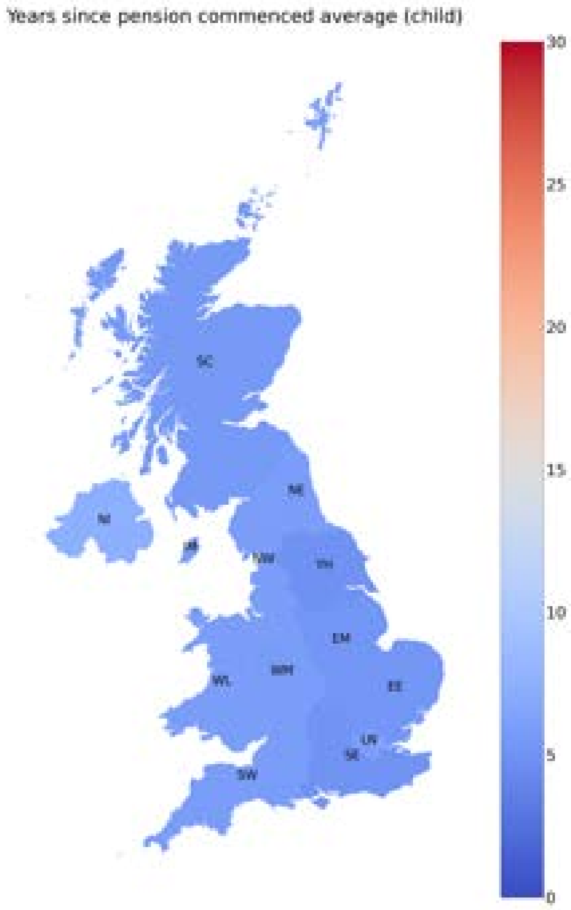
Region label	Region	Years since pension commenced average (ill health)
IM	Isle of Man	24.59
CI	Channel Islands	22.60
	Unknown	22.20
NI	Northern Ireland	21.64
LN	London	21.16
NE	North East (England)	20.91
SC	Scotland	20.74
NW	North West (England)	20.30
SE	South East (England)	19.77
EM	East Midlands (England)	19.74
YH	Yorkshire and The Humber	19.67
SW	South West (England)	19.48
WM	West Midlands (England)	19.35
WL	Wales	19.33
EE	East of England	18.29

Figure 7.28: Mean number of years since pension commencement of ill health retired pensioner scheme members at 31st March 2020 by region.



Region label	Region	Years since pension commenced average (spouse)
IM	Isle of Man	15.21
	Unknown	12.10
LN	London	11.73
SC	Scotland	11.26
NI	Northern Ireland	11.21
NW	North West (England)	11.00
EE	East of England	10.88
WL	Wales	10.83
WM	West Midlands (England)	10.69
SW	South West (England)	10.62
SE	South East (England)	10.61
EM	East Midlands (England)	10.58
NE	North East (England)	10.45
YH	Yorkshire and The Humber	10.42
CI	Channel Islands	9.91

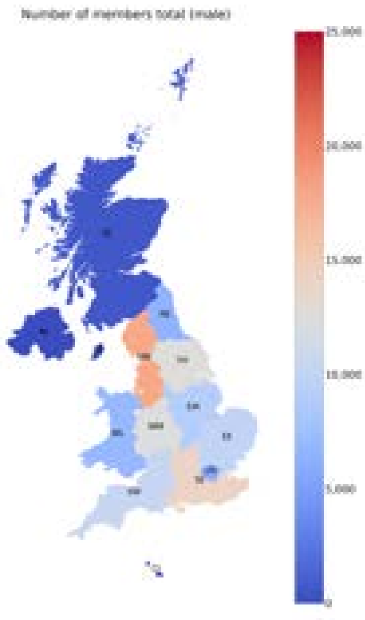
Figure 7.29: Mean number of years since pension commencement of dependant spouse scheme members at 31st March 2020 by region.



Region label	Region	Years since pension commenced average (child)
NI	Northern Ireland	7.47
NW	North West (England)	5.92
WL	Wales	5.91
NE	North East (England)	5.91
LN	London	5.87
	Unknown	5.79
IM	Isle of Man	5.79
WM	West Midlands (England)	5.77
SW	South West (England)	5.76
SC	Scotland	5.60
EM	East Midlands (England)	5.38
EE	East of England	5.33
SE	South East (England)	5.14
YH	Yorkshire and The Humber	4.98

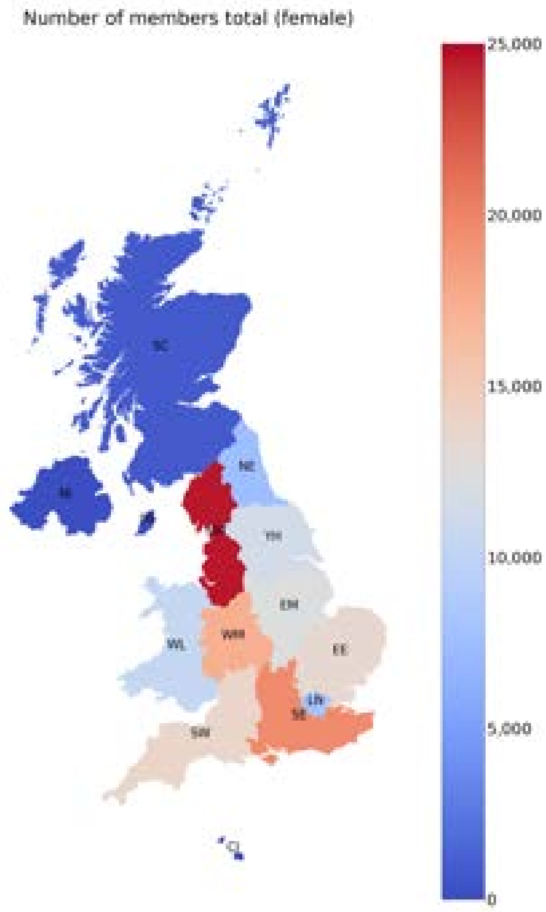
Figure 7.30: Mean number of years since pension commencement of dependant child scheme members at 31st March 2020 by region.

7.2 Intervaluation period movements



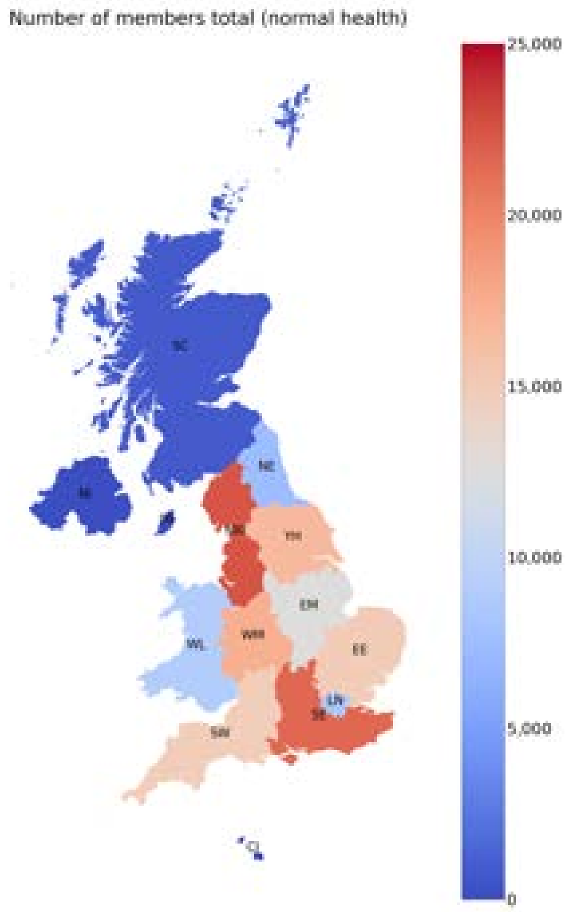
Region label	Region	Number of members total (male)
NW	North West (England)	17,846
SE	South East (England)	14,139
YH	Yorkshire and The Humber	12,660
WM	West Midlands (England)	12,494
SW	South West (England)	10,520
EE	East of England	10,117
EM	East Midlands (England)	9,120
	Unknown	7,930
WL	Wales	6,994
NE	North East (England)	6,527
LN	London	5,806
SC	Scotland	625
NI	Northern Ireland	83
IM	Isle of Man	27
CI	Channel Islands	11

Figure 7.31: Number of male pensioner and dependant scheme members, exiting between 01st April 2013 and 31st March 2020 by region.



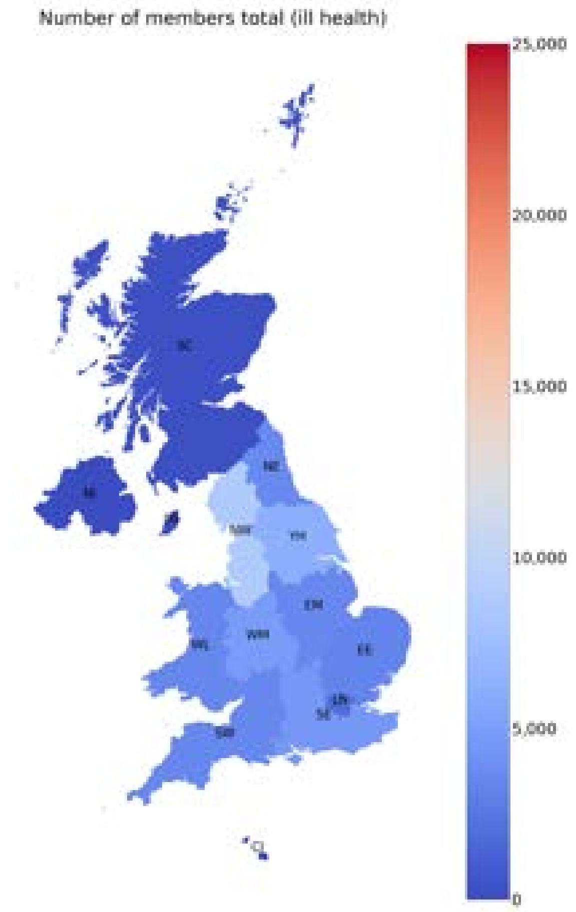
Region label	Region	Number of members total (female)
NW	North West (England)	24,576
SE	South East (England)	19,650
WM	West Midlands (England)	17,357
SW	South West (England)	13,798
EE	East of England	13,458
EM	East Midlands (England)	12,479
YH	Yorkshire and The Humber	11,456
WL	Wales	10,431
	Unknown	10,268
LN	London	7,880
NE	North East (England)	7,486
SC	Scotland	800
NI	Northern Ireland	113
IM	Isle of Man	47
CI	Channel Islands	22

Figure 7.32: Number of female pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



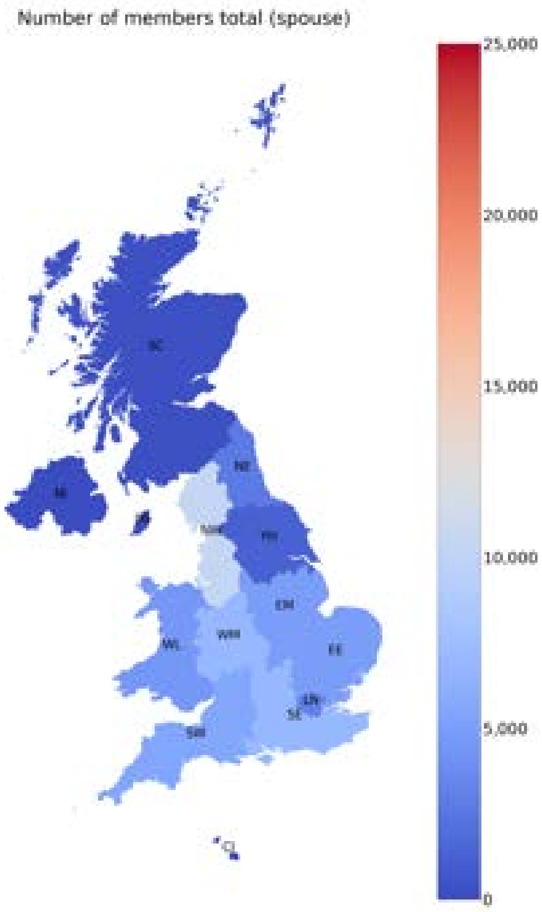
Region label	Region	Number of members total (normal health)
NW	North West (England)	22,370
SE	South East (England)	21,493
WM	West Midlands (England)	17,374
YH	Yorkshire and The Humber	16,580
EE	East of England	14,967
SW	South West (England)	14,786
EM	East Midlands (England)	12,584
	Unknown	10,719
WL	Wales	8,917
LN	London	8,290
NE	North East (England)	7,399
SC	Scotland	866
NI	Northern Ireland	111
IM	Isle of Man	41
CI	Channel Islands	16

Figure 7.33: Number of normal health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



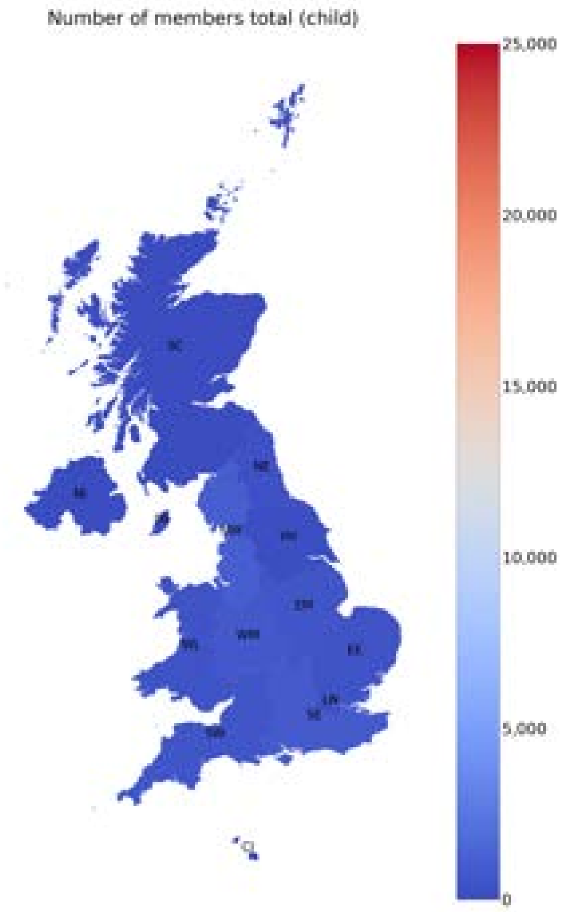
Region label	Region	Number of members total (ill health)
NW	North West (England)	8,716
YH	Yorkshire and The Humber	6,216
WM	West Midlands (England)	4,824
SE	South East (England)	4,522
NE	North East (England)	3,625
EM	East Midlands (England)	3,588
WL	Wales	3,496
SW	South West (England)	3,487
EE	East of England	3,278
	Unknown	2,850
LN	London	1,933
SC	Scotland	241
NI	Northern Ireland	44
IM	Isle of Man	12
CI	Channel Islands	3

Figure 7.34: Number of ill health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



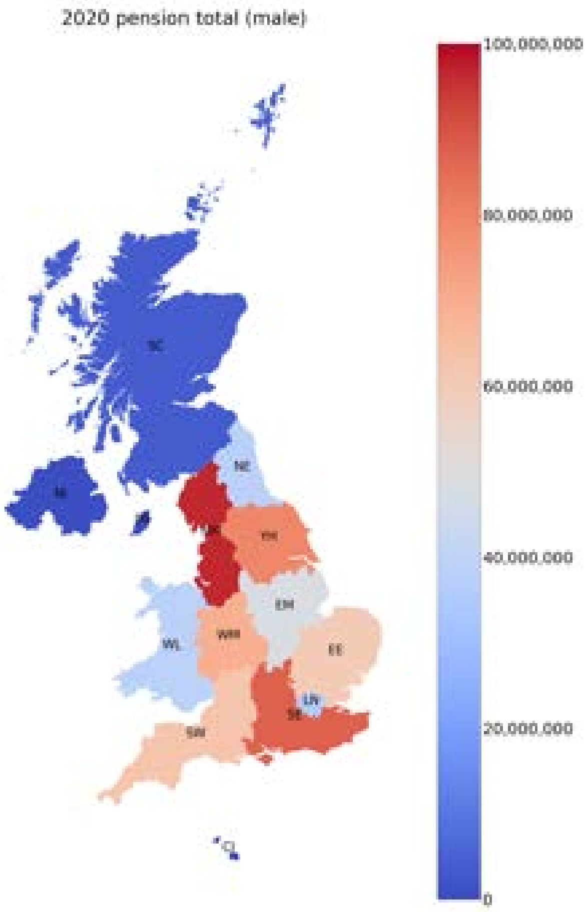
Region label	Region	Number of members total (spouse)
NW	North West (England)	10,328
SE	South East (England)	6,944
WM	West Midlands (England)	6,893
SW	South West (England)	5,475
EM	East Midlands (England)	4,926
EE	East of England	4,891
WL	Wales	4,576
	Unknown	4,193
LN	London	3,074
NE	North East (England)	2,693
YH	Yorkshire and The Humber	1,187
SC	Scotland	281
NI	Northern Ireland	33
IM	Isle of Man	20
CI	Channel Islands	10

Figure 7.35: Number of dependant spouse scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



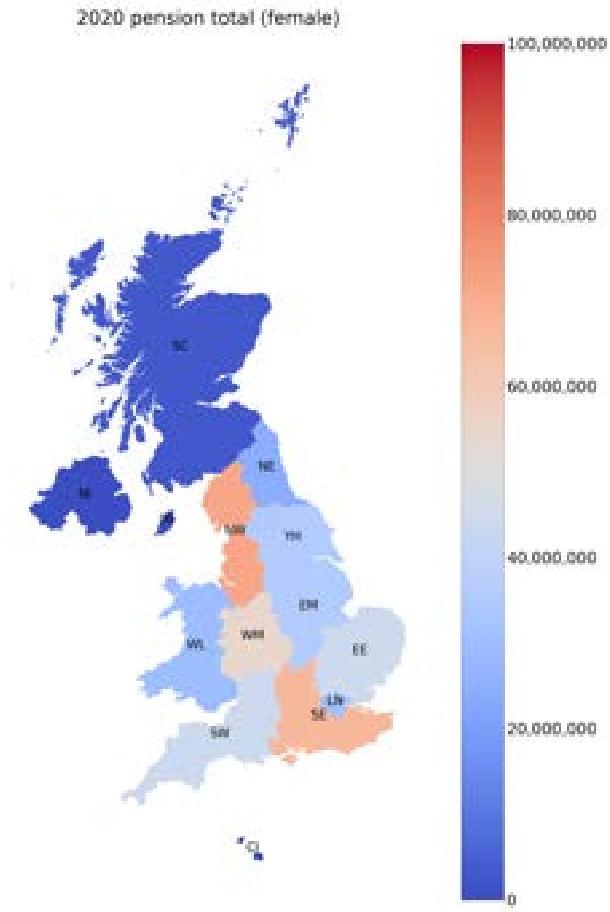
Region label	Region	Number of members total (child)
NW	North West (England)	1,008
SE	South East (England)	830
WM	West Midlands (England)	760
SW	South West (England)	570
EM	East Midlands (England)	501
EE	East of England	439
WL	Wales	436
	Unknown	436
LN	London	389
NE	North East (England)	296
YH	Yorkshire and The Humber	133
SC	Scotland	37
NI	Northern Ireland	8
CI	Channel Islands	4
IM	Isle of Man	1

Figure 7.36: Number of dependant child scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



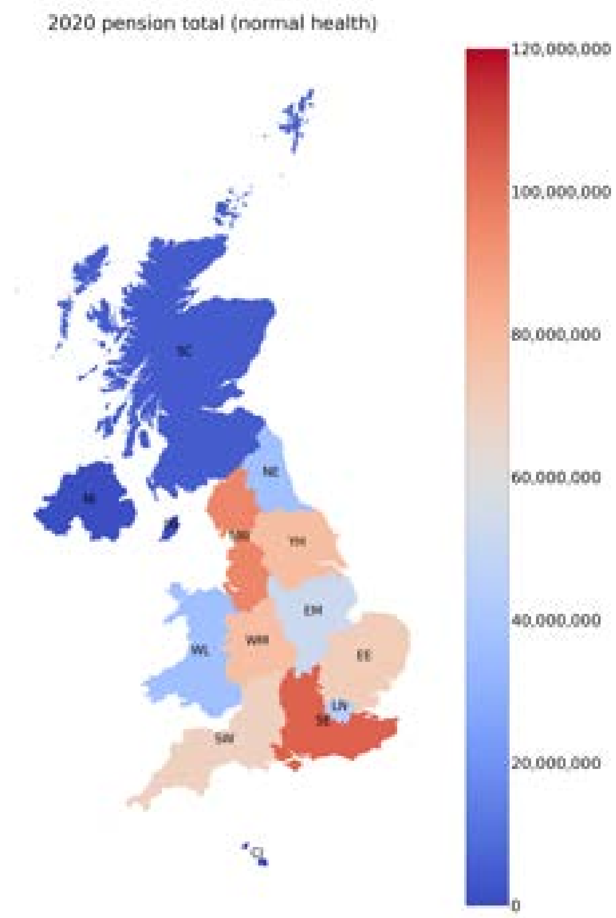
Region label	Region	2020 pension total (male)
NW	North West (England)	96,841,269
SE	South East (England)	87,184,764
YH	Yorkshire and The Humber	79,418,652
WM	West Midlands (England)	69,242,596
SW	South West (England)	62,828,802
EE	East of England	60,342,357
EM	East Midlands (England)	48,435,265
	Unknown	46,805,793
WL	Wales	38,648,828
NE	North East (England)	38,245,831
LN	London	35,320,730
SC	Scotland	3,851,758
NI	Northern Ireland	392,088
IM	Isle of Man	244,157
CI	Channel Islands	74,358

Figure 7.37: Total pension values (in 2020 terms) of male pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



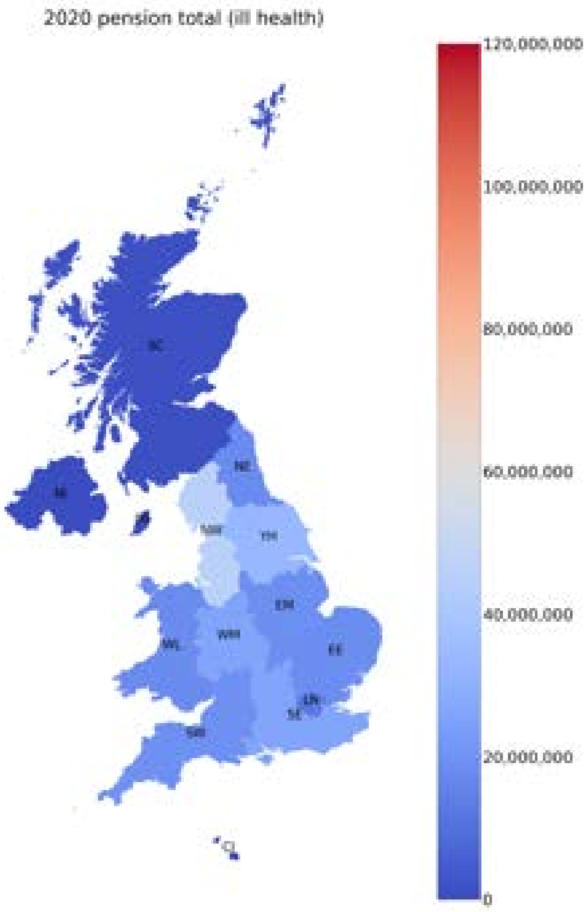
Region label	Region	2020 pension total (female)
NW	North West (England)	71,518,938
SE	South East (England)	67,031,905
WM	West Midlands (England)	54,362,250
EE	East of England	44,031,984
SW	South West (England)	43,415,639
YH	Yorkshire and The Humber	35,468,471
EM	East Midlands (England)	35,356,823
	Unknown	34,910,667
LN	London	30,768,928
WL	Wales	29,616,094
NE	North East (England)	24,533,425
SC	Scotland	2,813,839
NI	Northern Ireland	405,997
IM	Isle of Man	123,100
CI	Channel Islands	77,888

Figure 7.38: Total pension values (in 2020 terms) of female pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



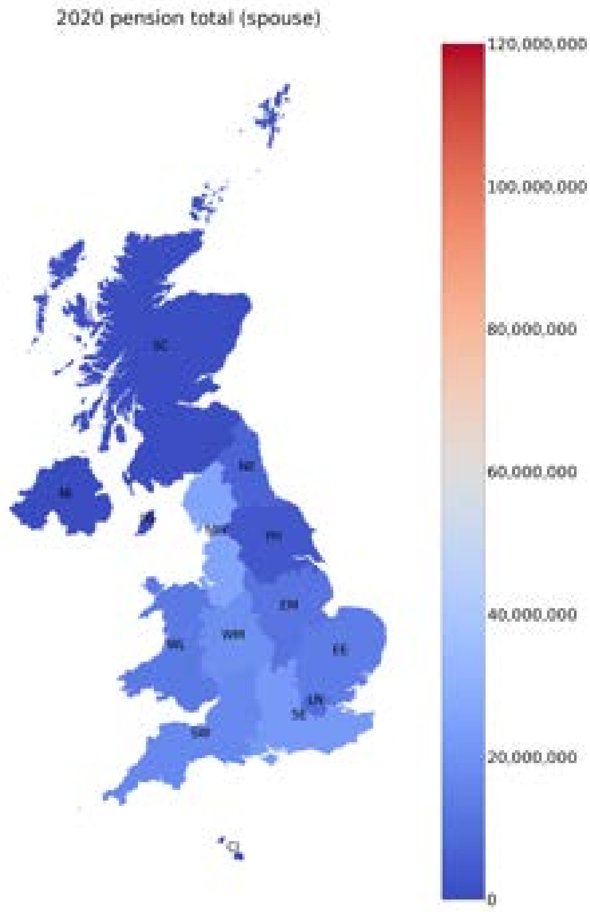
Region label	Region	2020 pension total (normal health)
SE	South East (England)	104,577,747
NW	North West (England)	95,514,457
YH	Yorkshire and The Humber	77,964,528
WM	West Midlands (England)	77,424,193
EE	East of England	71,398,975
SW	South West (England)	68,978,274
	Unknown	52,864,131
EM	East Midlands (England)	52,418,753
LN	London	44,237,419
NE	North East (England)	37,013,079
WL	Wales	36,567,170
SC	Scotland	4,297,875
NI	Northern Ireland	477,283
IM	Isle of Man	202,409
CI	Channel Islands	77,013

Figure 7.39: Total pension values (in 2020 terms) of normal health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



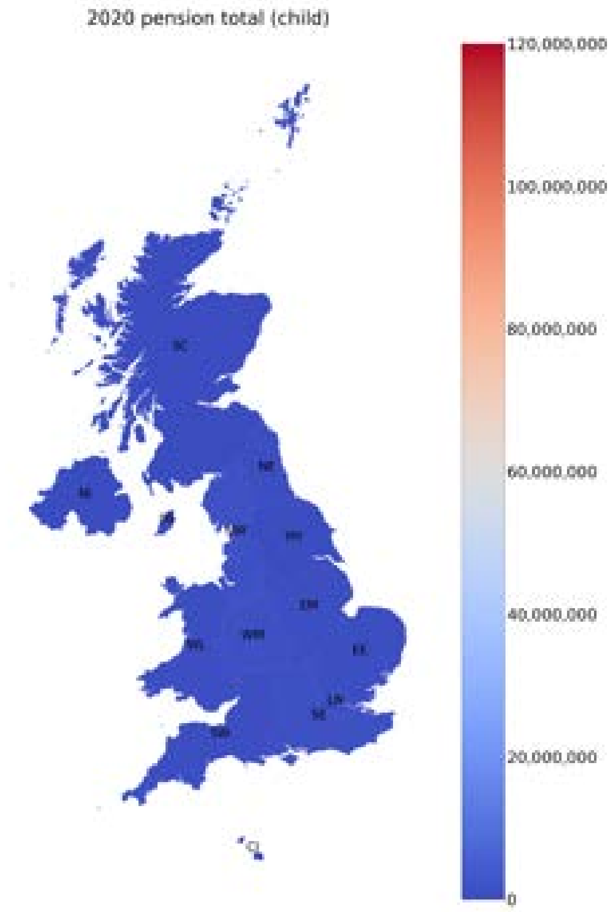
Region label	Region	2020 pension total (ill health)
NW	North West (England)	45,450,524
YH	Yorkshire and The Humber	33,239,466
WM	West Midlands (England)	26,085,561
SE	South East (England)	25,899,819
SW	South West (England)	19,464,344
EM	East Midlands (England)	18,989,754
EE	East of England	18,307,191
NE	North East (England)	18,276,938
WL	Wales	18,104,541
	Unknown	16,361,405
LN	London	11,776,394
SC	Scotland	1,402,961
NI	Northern Ireland	239,827
IM	Isle of Man	110,504
CI	Channel Islands	19,489

Figure 7.40: Total pension values (in 2020 terms) of ill health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



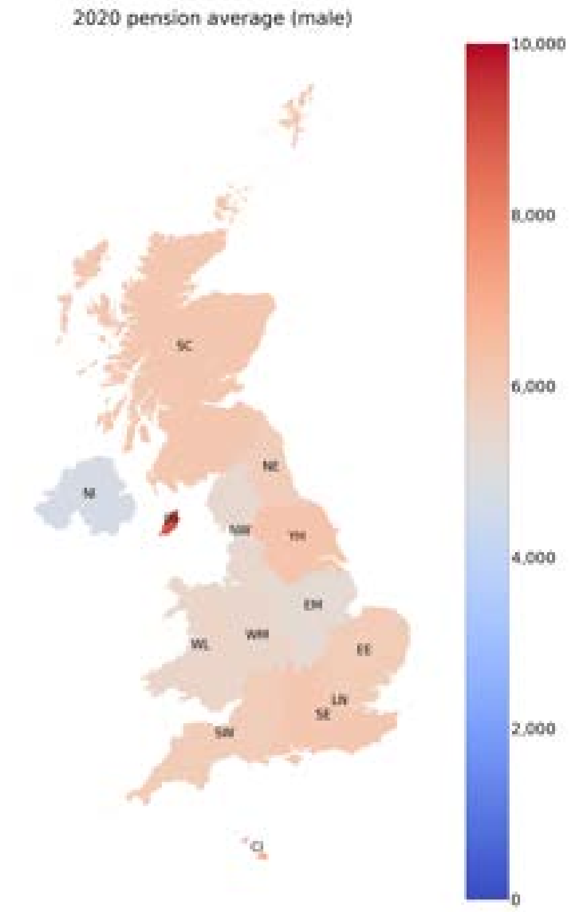
Region label	Region	2020 pension total (spouse)
NW	North West (England)	25,945,952
SE	South East (England)	22,452,924
WM	West Midlands (England)	18,853,922
SW	South West (England)	17,018,226
EE	East of England	14,035,206
WL	Wales	12,808,817
	Unknown	11,840,387
EM	East Midlands (England)	11,653,502
LN	London	9,319,767
NE	North East (England)	7,002,854
YH	Yorkshire and The Humber	3,491,641
SC	Scotland	904,427
NI	Northern Ireland	73,745
IM	Isle of Man	52,389
CI	Channel Islands	50,271

Figure 7.41: Total pension values (in 2020 terms) of dependant spouse scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



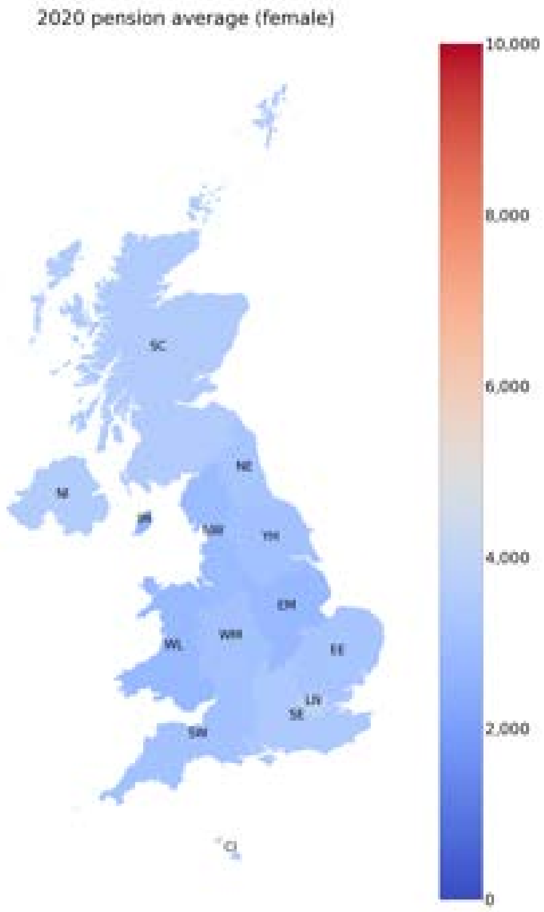
Region label	Region	2020 pension total (child)
NW	North West (England)	1,449,274
SE	South East (England)	1,286,178
WM	West Midlands (England)	1,241,169
WL	Wales	784,395
SW	South West (England)	783,598
LN	London	756,079
EM	East Midlands (England)	730,080
	Unknown	650,537
EE	East of England	632,969
NE	North East (England)	486,385
YH	Yorkshire and The Humber	191,487
SC	Scotland	60,334
NI	Northern Ireland	7,229
CI	Channel Islands	5,473
IM	Isle of Man	1,956

Figure 7.42: Total pension values (in 2020 terms) of dependant child scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



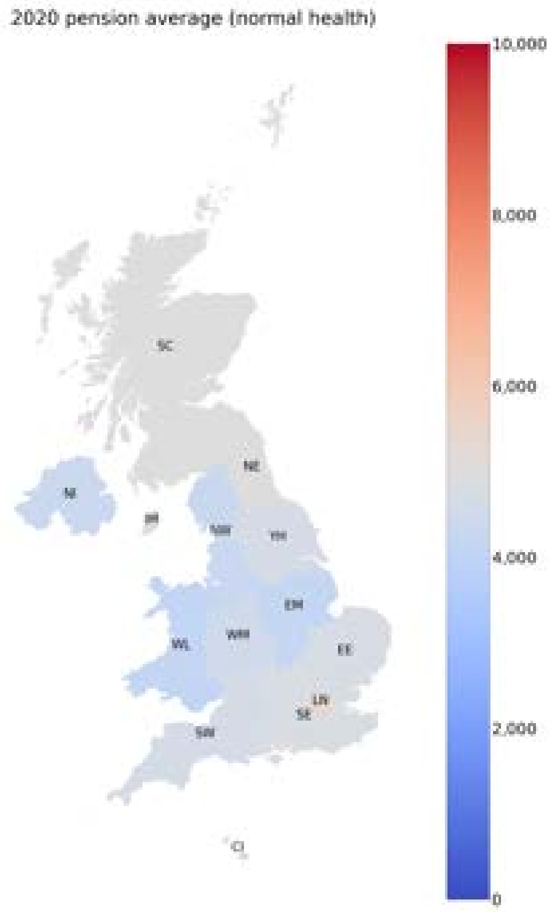
Region label	Region	2020 pension average (male)
IM	Isle of Man	9,043
CI	Channel Islands	6,760
YH	Yorkshire and The Humber	6,273
SE	South East (England)	6,166
SC	Scotland	6,163
LN	London	6,083
SW	South West (England)	5,972
EE	East of England	5,964
	Unknown	5,902
NE	North East (England)	5,860
WM	West Midlands (England)	5,542
WL	Wales	5,526
NW	North West (England)	5,426
EM	East Midlands (England)	5,311
NI	Northern Ireland	4,724

Figure 7.43: Mean pension values (in 2020 terms) of male pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



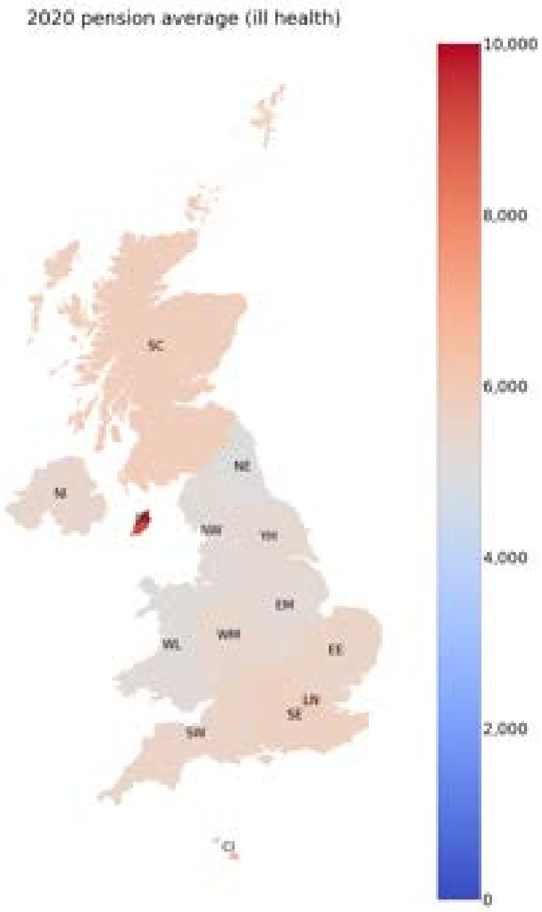
Region label	Region	2020 pension average (female)
LN	London	3,905
NI	Northern Ireland	3,593
CI	Channel Islands	3,540
SC	Scotland	3,517
SE	South East (England)	3,411
	Unknown	3,400
NE	North East (England)	3,277
EE	East of England	3,272
SW	South West (England)	3,147
WM	West Midlands (England)	3,132
YH	Yorkshire and The Humber	3,096
NW	North West (England)	2,910
WL	Wales	2,839
EM	East Midlands (England)	2,833
IM	Isle of Man	2,619

Figure 7.44: Mean pension values (in 2020 terms) of female pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



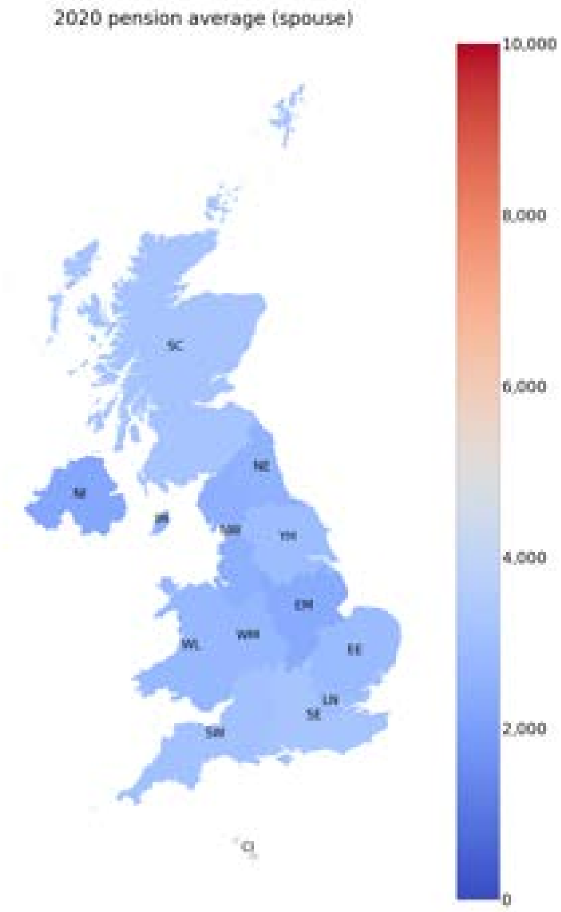
Region label	Region	2020 pension average (normal health)
LN	London	5,336
NE	North East (England)	5,002
SC	Scotland	4,963
IM	Isle of Man	4,937
	Unknown	4,932
SE	South East (England)	4,866
CI	Channel Islands	4,813
EE	East of England	4,770
YH	Yorkshire and The Humber	4,702
SW	South West (England)	4,665
WM	West Midlands (England)	4,456
NI	Northern Ireland	4,300
NW	North West (England)	4,270
EM	East Midlands (England)	4,166
WL	Wales	4,101

Figure 7.45: Mean pension values (in 2020 terms) of normal health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



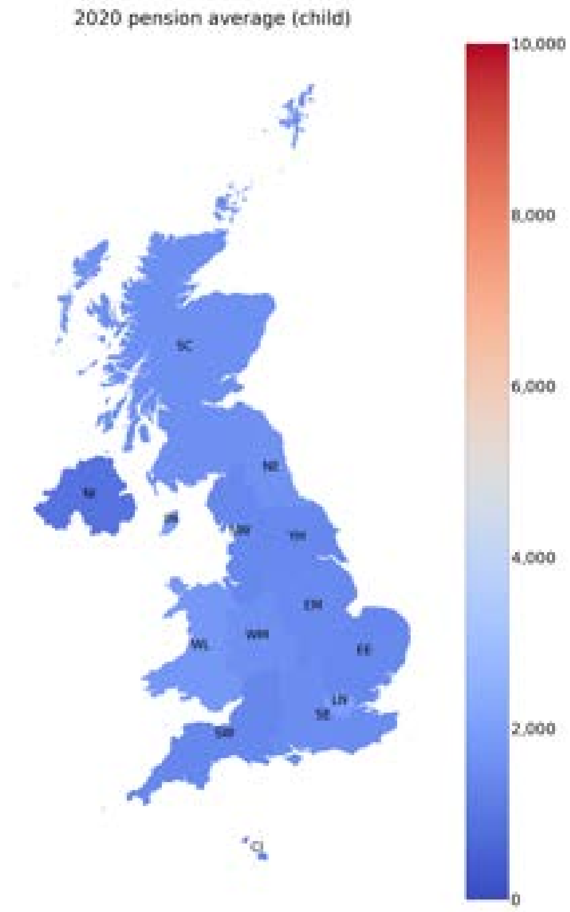
Region label	Region	2020 pension average (ill health)
IM	Isle of Man	9,209
CI	Channel Islands	6,496
LN	London	6,092
SC	Scotland	5,821
	Unknown	5,741
SE	South East (England)	5,728
EE	East of England	5,585
SW	South West (England)	5,582
NI	Northern Ireland	5,451
WM	West Midlands (England)	5,407
YH	Yorkshire and The Humber	5,347
EM	East Midlands (England)	5,293
NW	North West (England)	5,215
WL	Wales	5,179
NE	North East (England)	5,042

Figure 7.46: Mean pension values (in 2020 terms) of ill health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



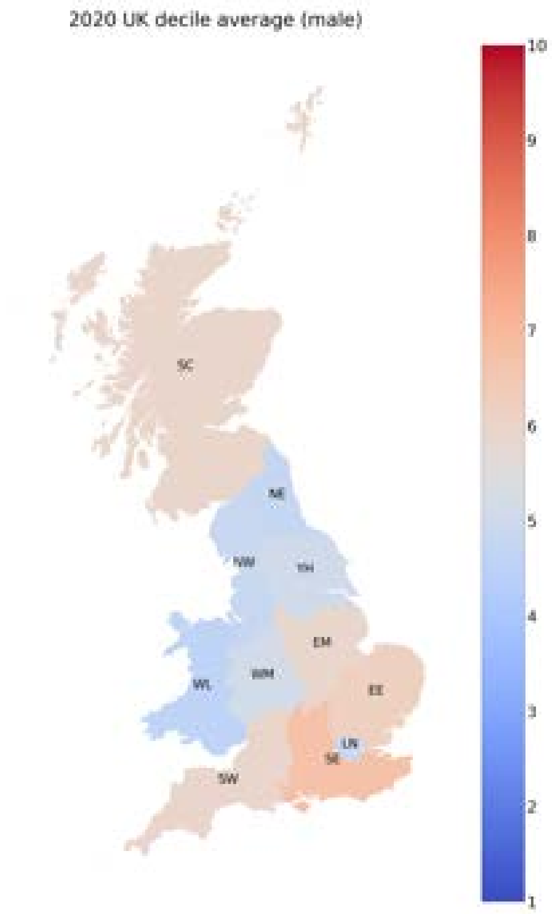
Region label	Region	2020 pension average (spouse)
CI	Channel Islands	5,027
SE	South East (England)	3,233
SC	Scotland	3,219
SW	South West (England)	3,108
LN	London	3,032
YH	Yorkshire and The Humber	2,942
EE	East of England	2,870
	Unknown	2,824
WL	Wales	2,799
WM	West Midlands (England)	2,735
IM	Isle of Man	2,619
NE	North East (England)	2,600
NW	North West (England)	2,512
EM	East Midlands (England)	2,366
NI	Northern Ireland	2,235

Figure 7.47: Mean pension values (in 2020 terms) of dependant spouse scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



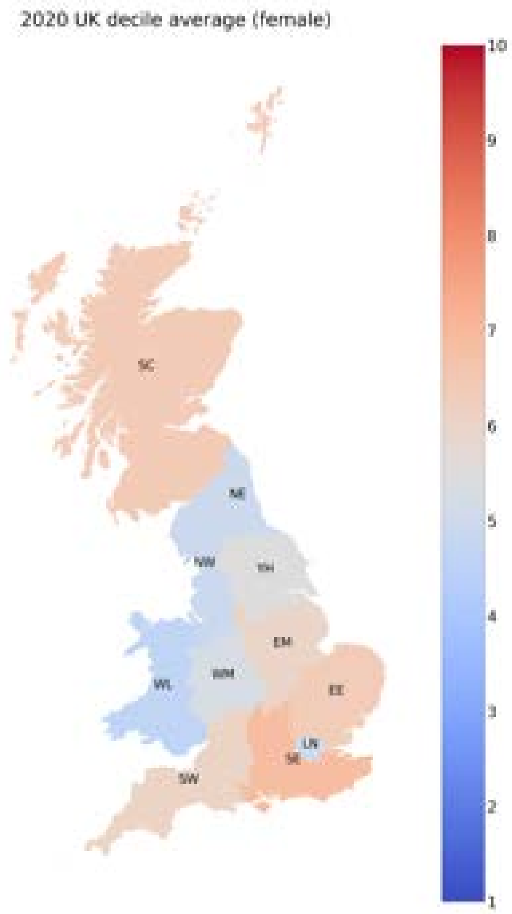
Region label	Region	2020 pension average (child)
IM	Isle of Man	1,956
LN	London	1,944
WL	Wales	1,799
NE	North East (England)	1,643
WM	West Midlands (England)	1,633
SC	Scotland	1,631
SE	South East (England)	1,550
	Unknown	1,492
EM	East Midlands (England)	1,457
EE	East of England	1,442
YH	Yorkshire and The Humber	1,440
NW	North West (England)	1,438
SW	South West (England)	1,375
CI	Channel Islands	1,368
NI	Northern Ireland	904

Figure 7.48: Mean pension values (in 2020 terms) of dependant child scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



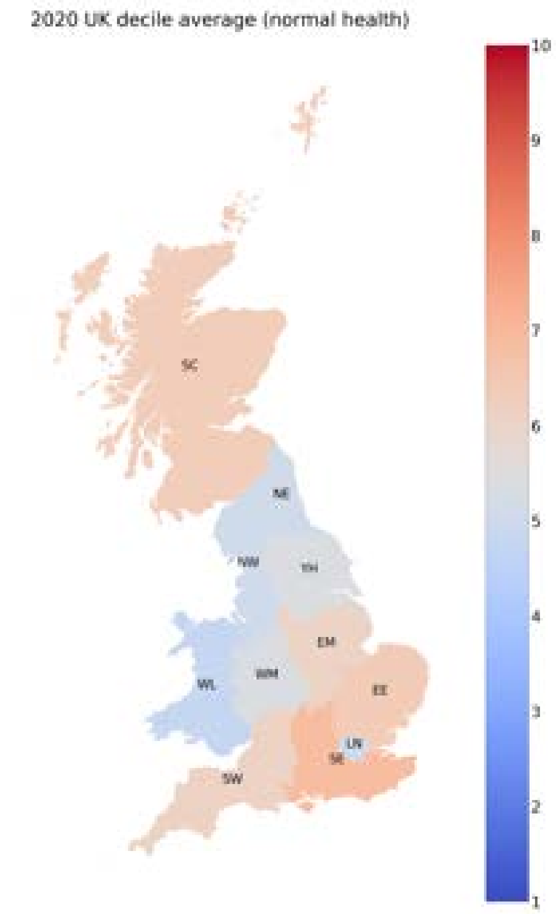
Region label	Region	2020 UK decile average (male)
SE	South East (England)	6.71
EE	East of England	6.20
SC	Scotland	5.95
SW	South West (England)	5.94
EM	East Midlands (England)	5.90
WM	West Midlands (England)	5.13
YH	Yorkshire and The Humber	5.07
LN	London	4.93
NW	North West (England)	4.81
NE	North East (England)	4.76
WL	Wales	4.57

Figure 7.49: Mean UK decile of male pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



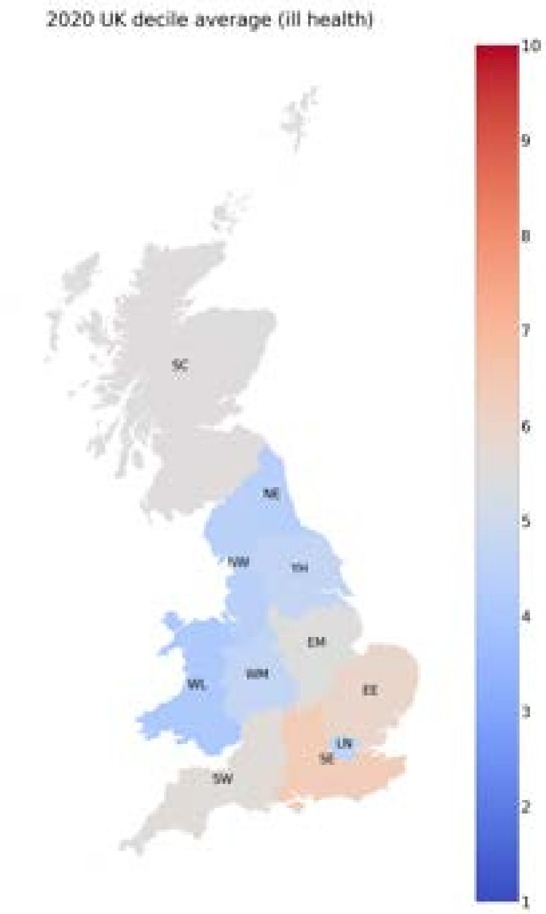
Region label	Region	2020 UK decile average (female)
SE	South East (England)	6.85
SC	Scotland	6.38
EE	East of England	6.37
SW	South West (England)	6.08
EM	East Midlands (England)	6.01
YH	Yorkshire and The Humber	5.52
WM	West Midlands (England)	5.32
LN	London	5.07
NE	North East (England)	4.95
NW	North West (England)	4.94
WL	Wales	4.60

Figure 7.50: Mean UK decile of female pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



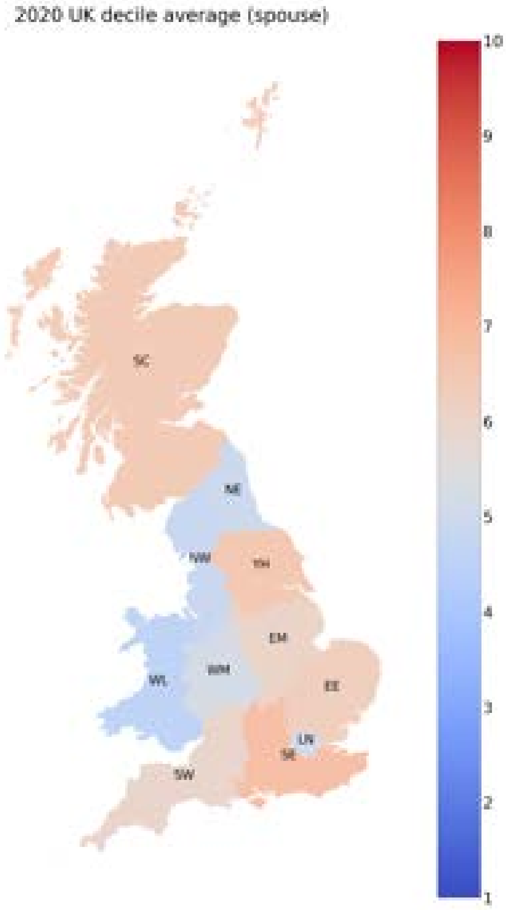
Region label	Region	2020 UK decile average (normal health)
SE	South East (England)	6.89
EE	East of England	6.39
SC	Scotland	6.32
SW	South West (England)	6.12
EM	East Midlands (England)	6.07
YH	Yorkshire and The Humber	5.42
WM	West Midlands (England)	5.39
NW	North West (England)	5.08
LN	London	5.07
NE	North East (England)	5.06
WL	Wales	4.77

Figure 7.51: Mean UK decile of normal health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



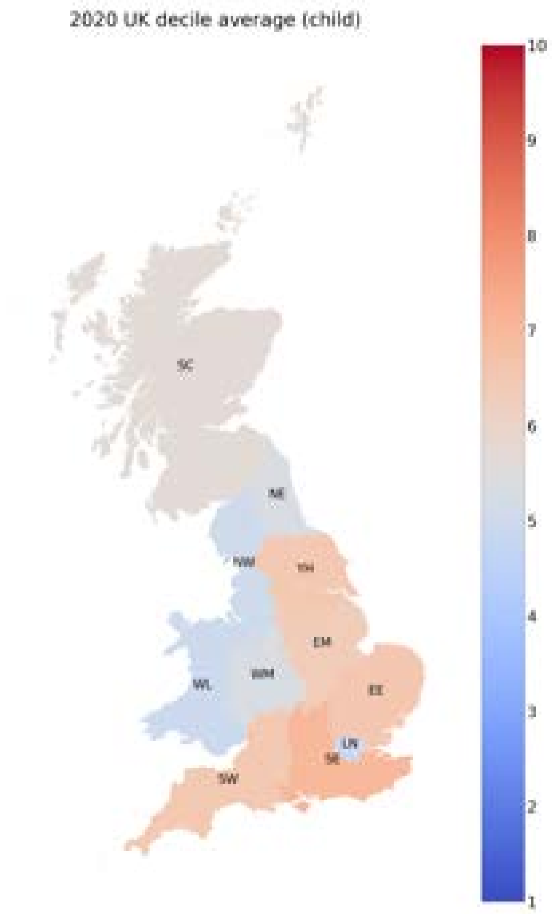
Region label	Region	2020 UK decile average (ill health)
SE	South East (England)	6.31
EE	East of England	5.92
SW	South West (England)	5.62
SC	Scotland	5.55
EM	East Midlands (England)	5.48
WM	West Midlands (England)	4.68
YH	Yorkshire and The Humber	4.66
LN	London	4.56
NW	North West (England)	4.43
NE	North East (England)	4.37
WL	Wales	4.10

Figure 7.52: Mean UK decile of ill health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



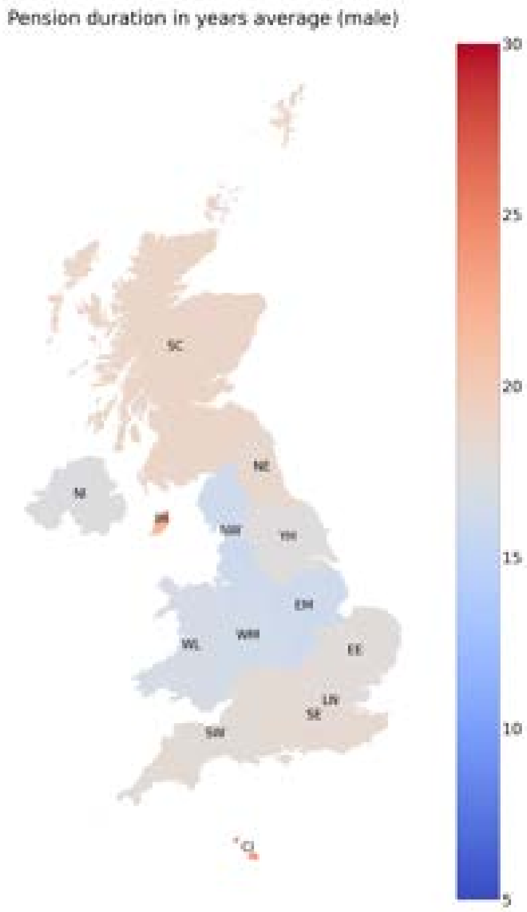
Region label	Region	2020 UK decile average (spouse)
SE	South East (England)	6.77
YH	Yorkshire and The Humber	6.49
SC	Scotland	6.40
EE	East of England	6.21
EM	East Midlands (England)	6.01
SW	South West (England)	5.94
WM	West Midlands (England)	5.25
LN	London	5.14
NE	North East (England)	4.94
NW	North West (England)	4.85
WL	Wales	4.58

Figure 7.53: Mean UK decile of dependant spouse scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



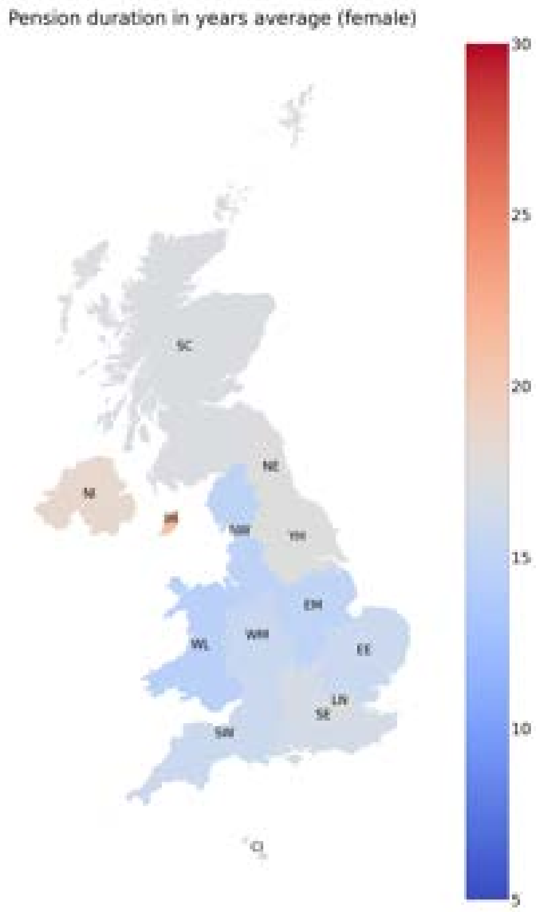
Region label	Region	2020 UK decile average (child)
SE	South East (England)	7.00
EE	East of England	6.55
SW	South West (England)	6.52
YH	Yorkshire and The Humber	6.49
EM	East Midlands (England)	6.37
SC	Scotland	5.70
NE	North East (England)	5.36
WM	West Midlands (England)	5.34
NW	North West (England)	5.03
WL	Wales	4.95
LN	London	4.93

Figure 7.54: Mean UK decile of dependant child scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



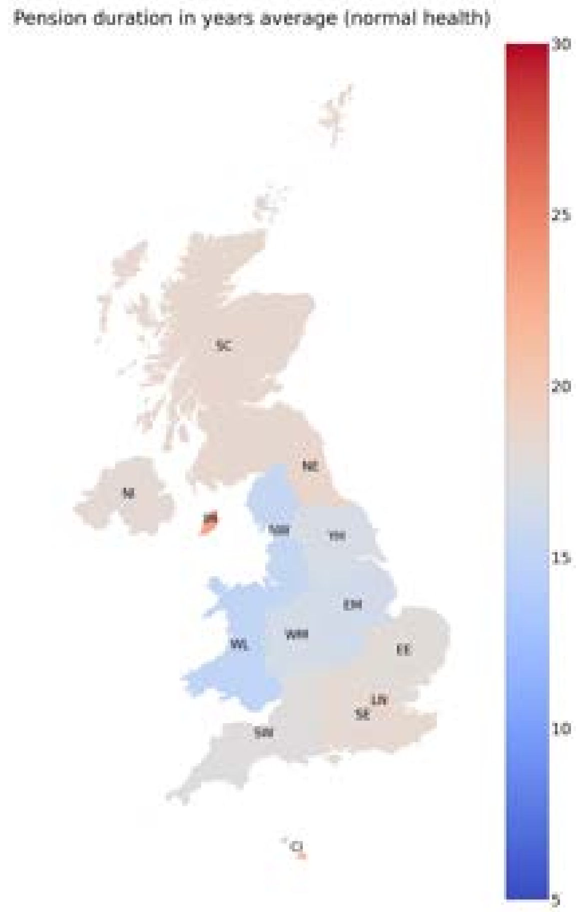
Region label	Region	Pension duration in years average (male)
CI	Channel Islands	23.46
IM	Isle of Man	21.79
SC	Scotland	18.80
NE	North East (England)	18.49
	Unknown	18.38
SE	South East (England)	18.20
LN	London	18.09
SW	South West (England)	18.06
EE	East of England	17.83
YH	Yorkshire and The Humber	17.62
NI	Northern Ireland	17.34
WL	Wales	16.81
WM	West Midlands (England)	16.47
NW	North West (England)	16.21
EM	East Midlands (England)	16.14

Figure 7.55: Mean number of years between pension commencement and ceasement of male pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



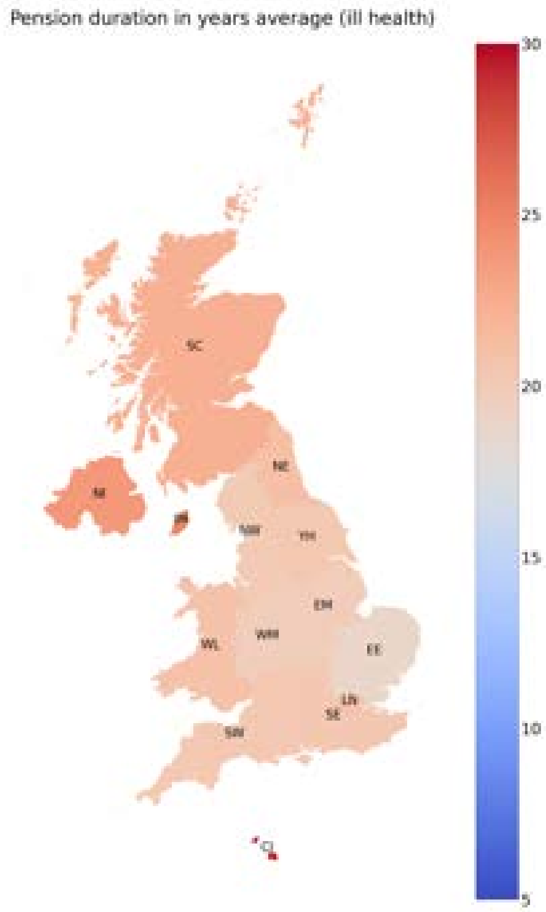
Region label	Region	Pension duration in years average (female)
IM	Isle of Man	21.28
NI	Northern Ireland	18.53
NE	North East (England)	17.58
LN	London	17.52
YH	Yorkshire and The Humber	17.44
CI	Channel Islands	17.24
SC	Scotland	17.22
	Unknown	16.91
SE	South East (England)	16.67
SW	South West (England)	15.90
EE	East of England	15.87
WM	West Midlands (England)	15.69
EM	East Midlands (England)	15.03
NW	North West (England)	14.95
WL	Wales	14.52

Figure 7.56: Mean number of years between pension commencement and ceasement of female pensioner and dependant scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



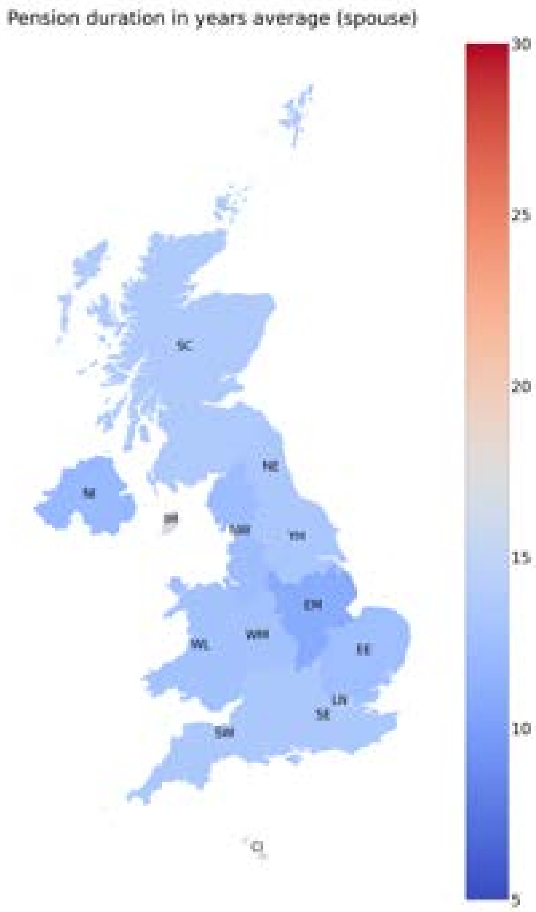
Region label	Region	Pension duration in years average (normal health)
IM	Isle of Man	23.14
CI	Channel Islands	20.63
LN	London	19.03
NE	North East (England)	18.62
SC	Scotland	18.40
SE	South East (England)	18.28
	Unknown	18.23
NI	Northern Ireland	18.09
EE	East of England	17.91
SW	South West (England)	17.65
YH	Yorkshire and The Humber	16.91
WM	West Midlands (England)	16.63
EM	East Midlands (England)	16.39
NW	North West (England)	15.49
WL	Wales	15.04

Figure 7.57: Mean number of years between pension commencement and ceasement of normal health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



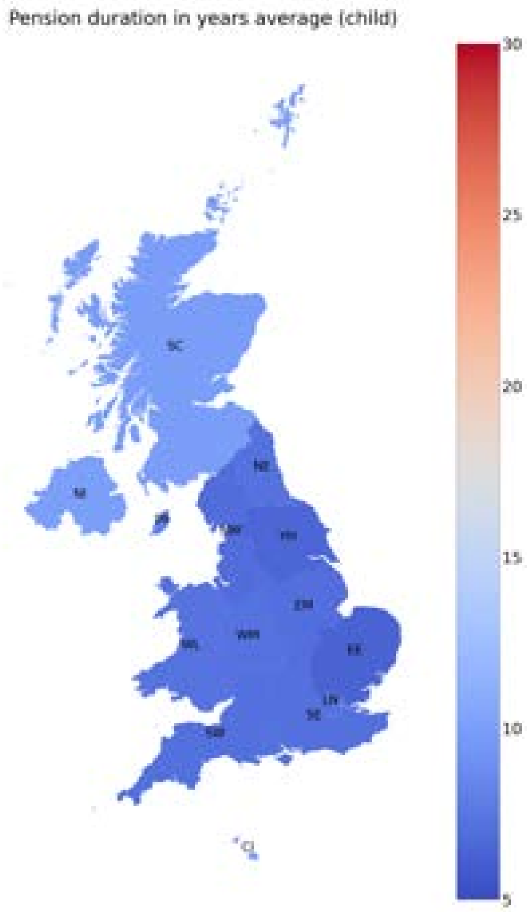
Region label	Region	Pension duration in years average (ill health)
CI	Channel Islands	28.88
IM	Isle of Man	25.09
NI	Northern Ireland	23.79
SC	Scotland	22.27
	Unknown	21.40
NE	North East (England)	21.03
WL	Wales	20.85
LN	London	20.83
SE	South East (England)	20.43
SW	South West (England)	20.32
YH	Yorkshire and The Humber	20.24
NW	North West (England)	20.10
EM	East Midlands (England)	19.78
WM	West Midlands (England)	19.69
EE	East of England	18.84

Figure 7.58: Mean number of years between pension commencement and ceasement of ill health retired pensioner scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



Region label	Region	Pension duration in years average (spouse)
CI	Channel Islands	17.96
IM	Isle of Man	16.53
	Unknown	14.29
LN	London	13.74
SC	Scotland	13.72
SE	South East (England)	13.49
NE	North East (England)	13.43
SW	South West (England)	13.42
YH	Yorkshire and The Humber	13.39
WM	West Midlands (England)	12.89
WL	Wales	12.86
EE	East of England	12.56
NW	North West (England)	12.38
NI	Northern Ireland	12.05
EM	East Midlands (England)	10.97

Figure 7.59: Mean number of years between pension commencement and ceasement of dependant spouse scheme members, exiting between 1st April 2013 and 31st March 2020 by region.



Region label	Region	Pension duration in years average (child)
CI	Channel Islands	10.23
NI	Northern Ireland	10.01
SC	Scotland	9.94
IM	Isle of Man	8.10
WL	Wales	7.30
LN	London	7.23
NE	North East (England)	7.22
WM	West Midlands (England)	7.19
SW	South West (England)	7.09
SE	South East (England)	7.08
	Unknown	7.03
EM	East Midlands (England)	7.02
NW	North West (England)	6.87
YH	Yorkshire and The Humber	6.55
EE	East of England	6.28

Figure 7.60: Mean number of years between pension commencement and ceasement of dependant child scheme members, exiting between 1st April 2013 and 31st March 2020 by region.

7.3 Approximate initial EtR

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
Normal health males	81.09	80.76	80.94	80.80
Normal health females	80.98	81.01	80.55	80.13
Ill health males	75.62	77.26	77.38	77.10
Ill health females	73.46	78.22	78.02	78.13
Spouse males	78.11	78.23	80.03	79.85
Spouse females	87.67	88.38	88.56	88.21

Figure 7.61: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
Normal health males	20,141,173,583	461,626,826	476,365,358	456,253,223	454,187,784
Normal health females	14,613,012,506	211,641,570	214,174,701	206,124,285	205,452,910
Ill health males	3,810,853,309	147,091,489	146,329,646	137,015,262	128,231,600
Ill health females	3,055,886,705	93,699,818	89,726,540	79,477,049	76,282,911
Spouse males	387,609,370	11,188,474	11,179,567	10,555,078	9,719,746
Spouse females	2,573,975,801	139,475,485	142,096,320	135,920,097	125,707,735

Figure 7.62: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.01	1.02
Normal health females	0.99	1.03	1.03
Ill health males	1.01	1.07	1.15
Ill health females	1.04	1.18	1.23
Spouse males	1.00	1.06	1.15
Spouse females	0.98	1.03	1.11

Figure 7.63: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.96	1.02	1.01
Normal health females	0.95	0.94	0.96
Ill health males	1.18	1.15	1.19
Ill health females	1.39	1.25	1.30
Spouse males	0.96	1.40	1.38
Spouse females	0.95	1.09	1.12

Figure 7.64: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to equal total expected and actual exit amounts.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.03	1.01
Normal health females	0.95	0.95	0.97
Ill health males	1.13	1.08	1.12
Ill health females	1.19	1.06	1.05
Spouse males	0.97	1.34	1.33
Spouse females	0.93	1.06	1.12

Figure 7.65: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

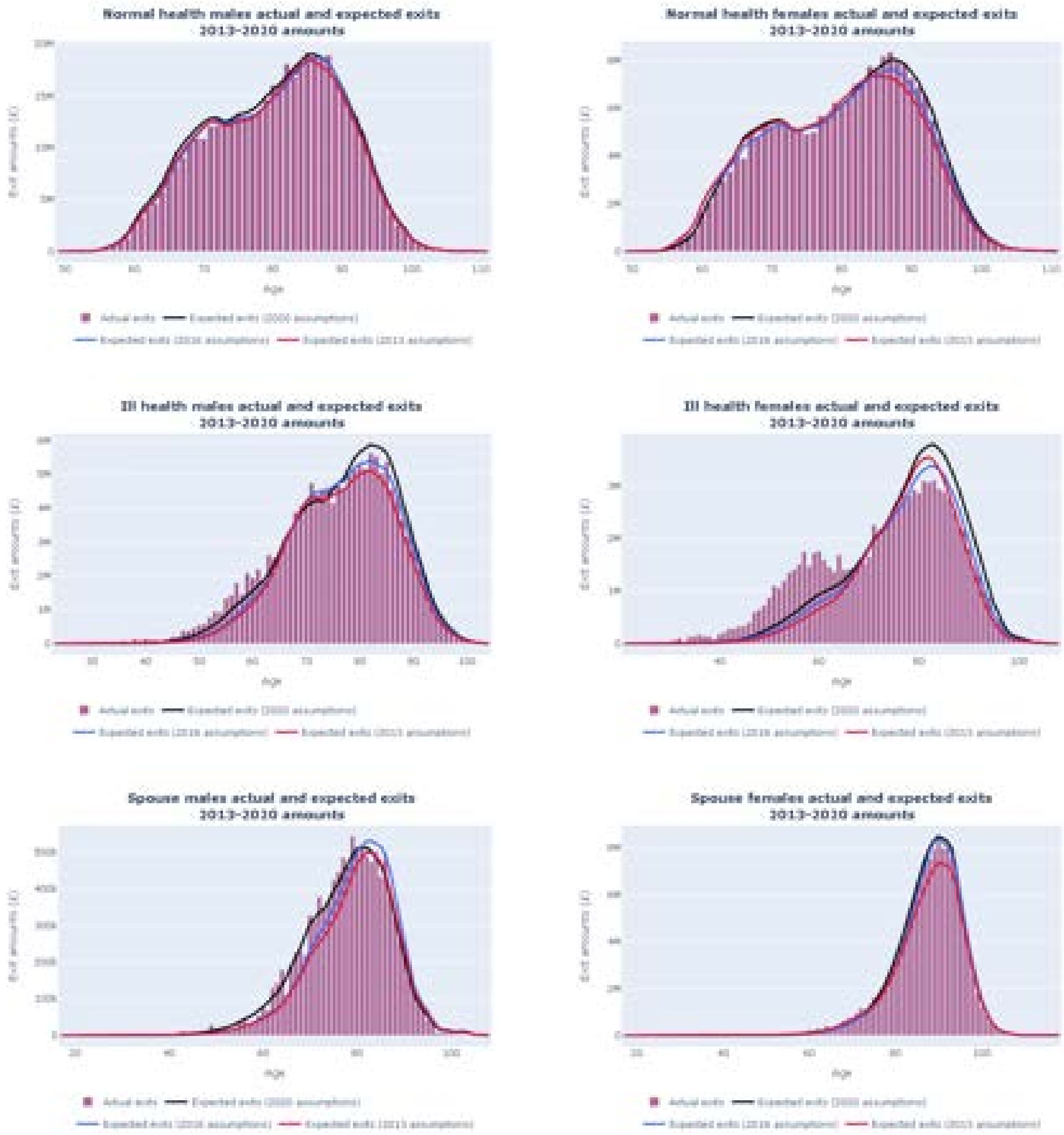


Figure 7.66: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

7.4 Exact central EtR

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
Normal health males	81.10	80.78	80.94	80.79
Normal health females	81.00	81.05	80.55	80.10
Ill health males	75.63	77.28	77.39	77.08
Ill health females	73.47	78.26	78.06	78.12
Spouse males	78.13	78.30	80.08	79.90
Spouse females	87.69	88.46	88.61	88.25

Figure 7.67: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
Normal health males	19,998,589,086	461,626,826	478,091,312	457,141,967	454,776,360
Normal health females	14,584,531,689	211,641,570	215,101,259	206,358,231	205,394,333
Ill health males	3,741,058,297	147,091,489	147,269,569	137,320,541	128,013,251
Ill health females	3,012,088,193	93,699,818	90,639,151	79,673,544	76,293,302
Spouse males	381,578,219	11,188,474	11,197,830	10,636,562	9,751,116
Spouse females	2,504,587,865	139,475,485	143,368,338	136,941,039	125,822,246

Figure 7.68: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.01	1.02
Normal health females	0.98	1.03	1.03
Ill health males	1.00	1.07	1.15
Ill health females	1.03	1.18	1.23
Spouse males	1.00	1.05	1.15
Spouse females	0.97	1.02	1.11

Figure 7.69: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.96	1.02	1.00
Normal health females	0.94	0.94	0.96
Ill health males	1.18	1.15	1.20
Ill health females	1.39	1.25	1.30
Spouse males	0.96	1.40	1.39
Spouse females	0.94	1.08	1.12

Figure 7.70: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to equal total expected and actual exit amounts.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.03	1.01
Normal health females	0.95	0.95	0.96
Ill health males	1.13	1.08	1.12
Ill health females	1.19	1.05	1.05
Spouse males	0.97	1.35	1.34
Spouse females	0.92	1.05	1.12

Figure 7.71: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

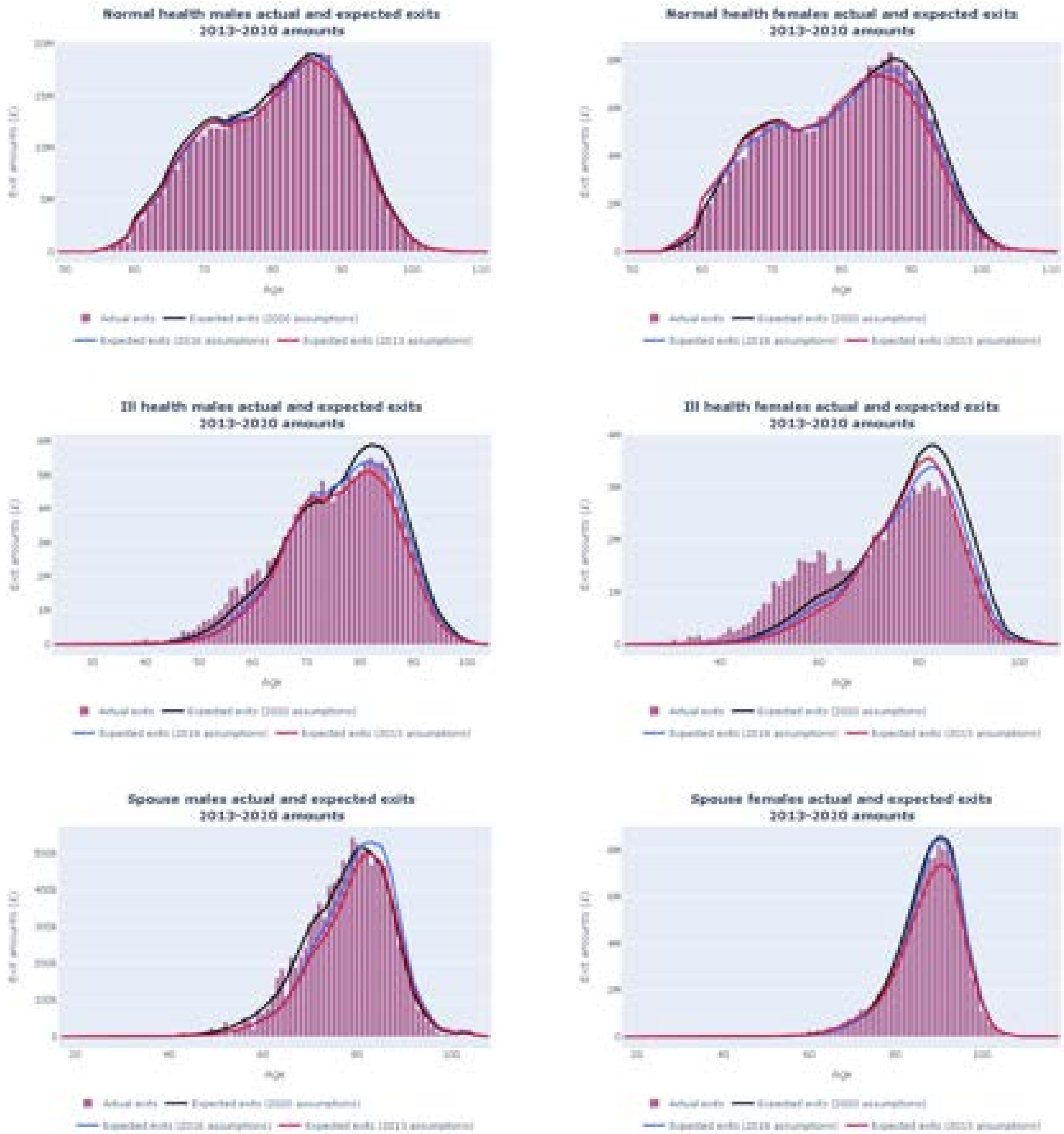


Figure 7.72: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

7.5 Approximate central EtR

Category	Actual average exit age	2020 assumption	2016 assumption	2013 assumption
Normal health males	81.09	80.79	80.95	80.81
Normal health females	80.98	81.07	80.57	80.12
Ill health males	75.62	77.28	77.39	77.08
Ill health females	73.46	78.26	78.05	78.12
Spouse males	78.11	78.29	80.07	79.89
Spouse females	87.67	88.46	88.60	88.24

Figure 7.73: Actual average exit ages, from 1st April 2013 to 31st March 2020 for each category, and expected average exit ages based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	EtR	Actual exits	2020 assumption	2016 assumption	2013 assumption
Normal health males	19,910,360,171	461,626,826	477,390,231	456,495,891	454,131,671
Normal health females	14,507,191,721	211,641,570	214,686,114	205,923,520	204,954,185
Ill health males	3,737,307,564	147,091,489	147,150,146	137,221,059	127,927,927
Ill health females	3,009,036,796	93,699,818	90,527,899	79,583,533	76,217,870
Spouse males	382,015,133	11,188,474	11,203,807	10,640,235	9,754,408
Spouse females	2,504,238,058	139,475,485	143,137,111	136,713,029	125,629,458

Figure 7.74: EtR and actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.01	1.02
Normal health females	0.99	1.03	1.03
Ill health males	1.00	1.07	1.15
Ill health females	1.04	1.18	1.23
Spouse males	1.00	1.05	1.15
Spouse females	0.97	1.02	1.11

Figure 7.75: Actual exit amounts, from 1st April 2013 to 31st March 2020 for each category, divided by expected exits based on assumptions recommended at the 2013, 2016 and 2020 valuations.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.96	1.02	1.01
Normal health females	0.94	0.94	0.96
Ill health males	1.18	1.15	1.20
Ill health females	1.39	1.25	1.31
Spouse males	0.96	1.40	1.39
Spouse females	0.94	1.09	1.12

Figure 7.76: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to equal total expected and actual exit amounts.

Category	2020 assumption	2016 assumption	2013 assumption
Normal health males	0.97	1.03	1.01
Normal health females	0.95	0.95	0.97
Ill health males	1.13	1.09	1.12
Ill health females	1.19	1.06	1.05
Spouse males	0.97	1.35	1.34
Spouse females	0.93	1.06	1.12

Figure 7.77: Proposed optimal scaling factors applicable to unscaled mortality base and improvement tables used for the 2013, 2016 and 2020 valuation assumptions for each category, to minimise the least square residuals of actual and expected exit amounts across each age.

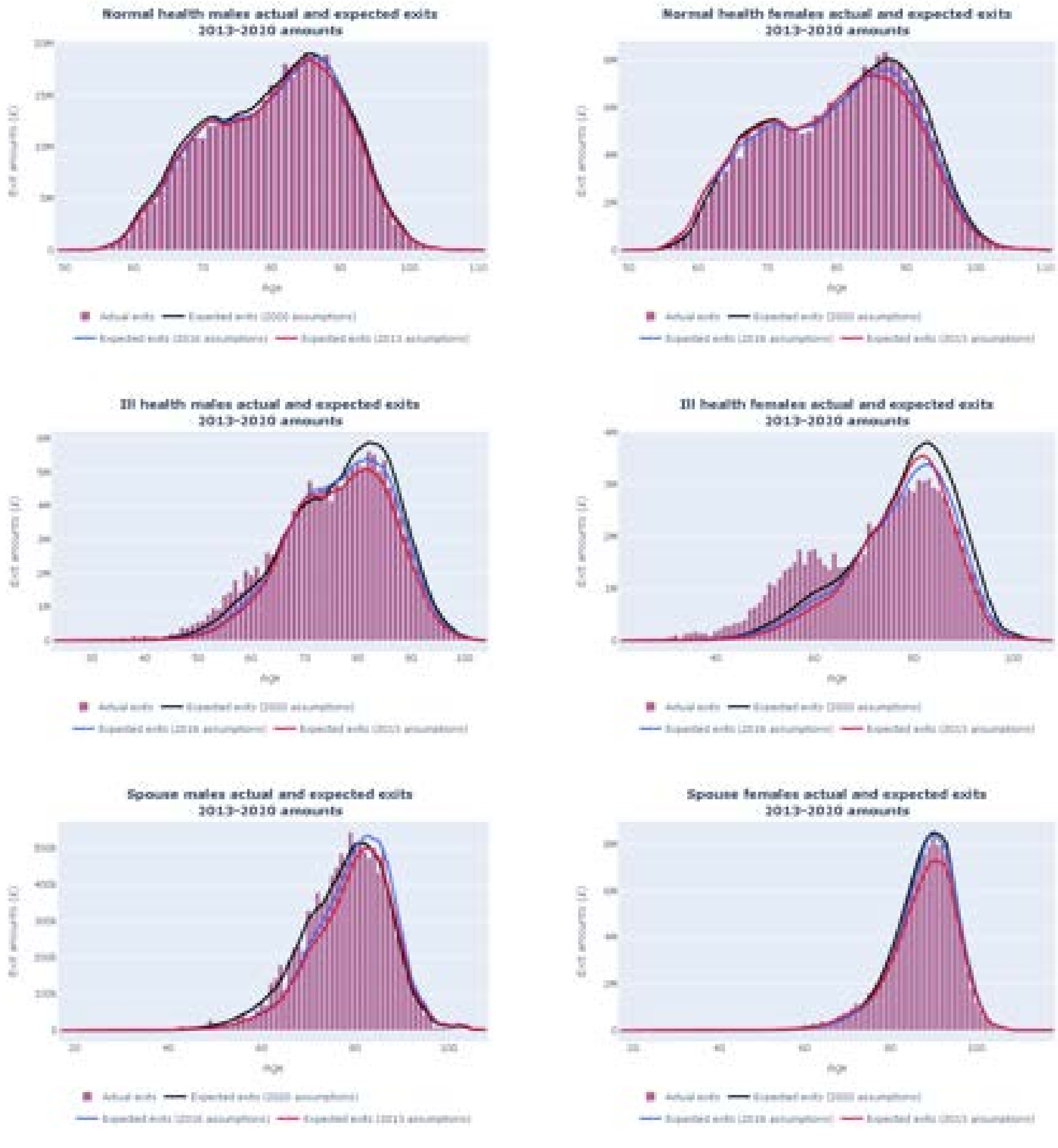


Figure 7.78: Actual exit amounts, from 1st April 2013 to 31st March 2020 by age for each category, and expected exit amounts based on assumptions, as recommended by GAD at the 2013, 2016 and 2020 valuations.

7.6 Gompertz graduation statistics

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	10	2	3	4	5	6	7	8
AIC	83.79	148.79	326.92	72.89	73.78	75.78	77.78	79.78
AICc	92.59	149.15	327.67	74.19	75.78	78.68	81.78	85.12
BIC	99.62	151.96	331.67	79.23	81.70	85.29	88.87	92.45
Deviance	63.79	144.79	320.92	64.89	63.78	63.78	63.78	63.78
Chi squared	0.00005	0.00000	0.00000	0.00046	0.00045	0.00030	0.00020	0.00012
R squared	0.99307	0.98602	0.98410	0.99304	0.99307	0.99307	0.99307	0.99307
VIF	2.45	2.45	2.45	2.45	2.45	2.45	2.45	2.45
QBIC	61.84	66.19	141.56	40.79	43.92	47.50	51.08	54.67
Signs test	0.40503	0.50000	0.50000	0.38228	0.40503	0.40503	0.40503	0.40503
Runs test	0.49884	0.01342	0.04389	0.40410	0.49884	0.49884	0.49884	0.49884
Correlation test	0.78174	0.00857	0.00002	0.79140	0.78145	0.78152	0.78131	0.78147
a1								
a2								
a3								
a4								
a5								
a6								
b1	1.18e+01	-1.34e+01	-1.60e+01	2.43e+00	1.19e+01	1.19e+01	1.19e+01	1.19e+01
b2	-9.32e-01	1.28e-01	1.88e-01	-4.69e-01	-9.36e-01	-9.35e-01	-9.38e-01	-9.35e-01
b3	1.59e-02		-3.36e-04	7.43e-03	1.60e-02	1.60e-02	1.60e-02	1.60e-02
b4	-9.93e-05			-3.05e-05	-1.00e-04	-9.98e-05	-1.00e-04	-9.99e-05
b5	2.08e-07				2.10e-07	2.10e-07	2.11e-07	2.10e-07
b6	1.00e-20					1.00e-20	1.00e-20	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.79: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the normal health male category.

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	10	2	3	4	5	6	7	8
AIC	161.08	391.34	2,997.33	185.70	151.08	169.52	154.83	156.72
AICc	169.88	391.71	2,998.08	186.99	153.08	172.42	158.83	162.05
BIC	176.91	394.51	3,002.08	192.03	159.00	179.02	165.92	169.38
Deviance	141.08	387.34	2,991.33	177.70	141.08	157.52	140.83	140.72
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	0.98305	0.93515	0.79071	0.97986	0.98305	0.98363	0.98307	0.98308
VIF	5.43	5.43	5.43	5.43	5.43	5.43	5.43	5.43
QBIC	61.84	78.55	562.03	47.08	43.92	50.53	51.04	54.60
Signs test	0.50000	0.13250	0.50000	0.50000	0.50000	0.38228	0.50000	0.50000
Runs test	0.30464	0.00000	0.00245	0.30464	0.30464	0.07162	0.30464	0.30464
Correlation test	0.00797	0.00000	0.00000	0.00110	0.00797	0.00867	0.00809	0.00814
a1								
a2								
a3								
a4								
a5								
a6								
b1	1.14e+01	-1.40e+01	-2.26e+01	3.10e+01	1.14e+01	-1.33e+03	1.12e+01	1.12e+01
b2	-5.96e-01	1.32e-01	3.28e-01	-1.57e+00	-5.96e-01	8.29e+01	-5.88e-01	-5.85e-01
b3	3.23e-03		-1.11e-03	2.11e-02	3.23e-03	-2.07e+00	3.10e-03	3.04e-03
b4	5.89e-05			-8.65e-05	5.89e-05	2.57e-02	5.99e-05	6.04e-05
b5	-4.40e-07				-4.40e-07	-1.58e-04	-4.44e-07	-4.45e-07
b6	1.00e-20					3.86e-07	1.00e-20	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.80: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the normal health female category.

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	10	2	3	4	5	6	7	8
AIC	88.12	406.08	1,074.58	74.30	78.09	80.11	82.14	84.12
AICc	96.92	406.45	1,075.33	75.59	80.09	83.01	86.14	89.45
BIC	103.95	409.25	1,079.33	80.63	86.01	89.61	93.22	96.79
Deviance	68.12	402.08	1,068.58	66.30	68.09	68.11	68.14	68.12
Chi squared	0.00001	0.00000	0.00000	0.00025	0.00012	0.00008	0.00005	0.00003
R squared	0.98414	0.90691	0.86268	0.98205	0.98415	0.98414	0.98414	0.98414
VIF	2.62	2.62	2.62	2.62	2.62	2.62	2.62	2.62
QBIC	61.84	160.64	418.63	39.64	43.91	47.50	51.09	54.67
Signs test	0.13206	0.40503	0.50000	0.50000	0.13206	0.13206	0.13206	0.13206
Runs test	0.31077	0.00071	0.00057	0.06587	0.31077	0.31077	0.31077	0.31077
Correlation test	0.02423	0.00000	0.00000	0.00943	0.02429	0.02424	0.02418	0.02422
a1								
a2								
a3								
a4								
a5								
a6								
b1	-1.11e+02	-1.02e+01	-1.65e+01	6.48e+01	-1.11e+02	-1.11e+02	-1.11e+02	-1.11e+02
b2	6.15e+00	9.43e-02	2.41e-01	-2.74e+00	6.15e+00	6.15e+00	6.15e+00	6.15e+00
b3	-1.32e-01		-8.49e-04	3.52e-02	-1.32e-01	-1.32e-01	-1.32e-01	-1.32e-01
b4	1.24e-03			-1.45e-04	1.24e-03	1.24e-03	1.24e-03	1.24e-03
b5	-4.29e-06				-4.29e-06	-4.29e-06	-4.29e-06	-4.29e-06
b6	1.00e-20					1.00e-20	1.00e-20	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.81: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the ill health male category.

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	10	2	3	4	5	6	7	8
AIC	127.72	1,557.69	1,476.18	81.52	117.69	76.06	121.78	123.76
AICc	136.52	1,558.06	1,476.93	82.81	119.69	78.96	125.78	129.09
BIC	143.56	1,560.86	1,480.93	87.86	125.61	85.56	132.86	136.43
Deviance	107.72	1,553.69	1,470.18	73.52	107.69	64.06	107.78	107.76
Chi squared	0.00000	0.00000	0.00000	0.00003	0.00000	0.00018	0.00000	0.00000
R squared	0.95827	0.70626	0.71211	0.96859	0.95828	0.97803	0.95825	0.95826
VIF	4.14	4.14	4.14	4.14	4.14	4.14	4.14	4.14
QBIC	61.84	382.17	365.59	32.08	43.91	36.96	51.10	54.68
Signs test	0.38228	0.38228	0.38228	0.40503	0.38228	0.13206	0.38228	0.38228
Runs test	0.32968	0.00003	0.00003	0.24350	0.32968	0.42722	0.32968	0.32968
Correlation test	0.00052	0.00000	0.00000	0.01288	0.00052	0.08496	0.00051	0.00051
a1								
a2								
a3								
a4								
a5								
a6								
b1	-3.22e+01	-1.18e+01	-1.14e+01	8.68e+01	-3.21e+01	2.02e+03	-3.23e+01	-3.23e+01
b2	2.54e+00	1.10e-01	9.86e-02	-3.51e+00	2.54e+00	-1.29e+02	2.55e+00	2.55e+00
b3	-7.04e-02		6.41e-05	4.38e-02	-7.03e-02	3.28e+00	-7.05e-02	-7.05e-02
b4	7.75e-04			-1.76e-04	7.74e-04	-4.16e-02	7.76e-04	7.75e-04
b5	-2.94e-06				-2.94e-06	2.64e-04	-2.95e-06	-2.95e-06
b6	1.00e-20					-6.66e-07	1.00e-20	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.82: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the ill health female category.

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	50	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46	46
Parameters	10	2	3	4	5	6	7	8
AIC	134.61	127.15	181.03	164.62	183.62	185.53	187.42	189.30
AICc	140.90	127.42	181.60	165.60	185.12	187.69	190.37	193.19
BIC	152.90	130.80	186.51	171.94	192.77	196.50	200.22	203.92
Deviance	114.61	123.15	175.03	156.62	173.62	173.53	173.42	173.30
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	0.98858	0.98471	0.98205	0.98238	0.98663	0.98663	0.98663	0.98663
VIF	3.18	3.18	3.18	3.18	3.18	3.18	3.18	3.18
QBIC	74.29	46.34	66.46	64.51	73.68	77.48	81.27	85.06
Signs test	0.11700	0.34126	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
Runs test	0.50000	0.21055	0.31308	0.17320	0.21055	0.21055	0.21055	0.21055
Correlation test	0.87265	0.85518	0.34170	0.13238	0.48648	0.48731	0.48829	0.48938
a1								
a2								
a3								
a4								
a5								
a6								
b1	6.48e+02	-1.12e+01	-1.37e+01	1.44e+01	-2.25e+02	-2.25e+02	-2.25e+02	-2.25e+02
b2	-4.59e+01	1.05e-01	1.62e-01	-8.90e-01	1.15e+01	1.15e+01	1.15e+01	1.15e+01
b3	1.27e+00		-3.33e-04	1.27e-02	-2.27e-01	-2.26e-01	-2.26e-01	-2.26e-01
b4	-1.74e-02			-5.29e-05	1.97e-03	1.97e-03	1.97e-03	1.97e-03
b5	1.18e-04				-6.36e-06	-6.36e-06	-6.35e-06	-6.35e-06
b6	-3.16e-07					1.00e-20	1.00e-20	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.83: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the male spouse category.

	Saturated	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Category	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F
Formula	G(10)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Low age	50	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46	46
Parameters	10	2	3	4	5	6	7	8
AIC	88.32	376.66	124.10	78.76	78.32	79.21	81.22	84.32
AICc	94.61	376.94	124.68	79.73	79.82	81.37	84.16	88.21
BIC	106.61	380.32	129.59	86.07	87.46	90.19	94.02	98.95
Deviance	68.32	372.66	118.10	70.76	68.32	67.21	67.22	68.32
Chi squared	0.00084	0.00000	0.00000	0.00264	0.00437	0.00465	0.00342	0.00169
R squared	0.99894	0.99816	0.99839	0.99894	0.99894	0.99895	0.99895	0.99894
VIF	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
QBIC	74.29	204.02	73.72	52.60	55.14	58.39	62.22	66.63
Signs test	0.50000	0.50000	0.50000	0.46139	0.50000	0.50000	0.50000	0.50000
Runs test	0.22718	0.03021	0.02546	0.50000	0.22718	0.14904	0.14904	0.22718
Correlation test	0.75553	0.00000	0.01460	0.68336	0.75553	0.78783	0.78778	0.75553
a1								
a2								
a3								
a4								
a5								
a6								
b1	-1.10e+01	-1.31e+01	-7.86e+00	7.83e+00	-1.10e+01	-6.63e+01	-6.62e+01	-1.10e+01
b2	4.12e-01	1.23e-01	2.16e-03	-5.89e-01	4.12e-01	4.15e+00	4.14e+00	4.12e-01
b3	-1.16e-02		6.94e-04	8.02e-03	-1.16e-02	-1.11e-01	-1.11e-01	-1.16e-02
b4	1.39e-04			-3.00e-05	1.39e-04	1.45e-03	1.45e-03	1.39e-04
b5	-5.39e-07				-5.39e-07	-9.11e-06	-9.09e-06	-5.39e-07
b6	1.00e-20					2.22e-08	2.21e-08	1.00e-20
b7	1.00e-24						1.00e-24	1.00e-24
b8	1.00e-28							1.00e-28
b9	1.00e-32							
b10	1.00e-36							

Figure 7.84: Parameters and statistics for graduated mortality rates generated using different Gompertz formulae for the female spouse category.

7.7 Gompertz-Makeham graduation statistics

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	2	3	3	4	4	4	5	5
AIC	19,729.66	293.10	17,753.78	420.69	110.87	1,214.37	153.72	78.49
AICc	19,730.02	293.85	17,754.53	421.98	112.16	1,215.66	155.72	80.49
BIC	19,732.83	297.85	17,758.53	427.02	117.20	1,220.71	161.63	86.41
Deviance	19,725.66	287.10	17,747.78	412.69	102.87	1,206.37	143.72	68.49
Chi squared	1.00000	0.00000	1.00000	0.00000	0.00000	1.00000	0.00000	0.00010
R squared	-8.72870	0.98490	-0.70205	0.97219	0.99080	0.88915	0.98943	0.99282
VIF	2.45	2.45	2.45	2.45	2.45	2.45	2.45	2.45
QBIC	8,047.65	127.78	7,245.02	182.55	56.27	506.07	76.50	45.83
Signs test	0.01254	0.50000	0.47313	0.50000	0.50000	0.26412	0.50000	0.38228
Runs test	0.00000	0.01534	0.00000	0.00057	0.03446	0.00005	0.20490	0.40410
Correlation test	0.00000	0.00002	0.00000	0.00000	0.02949	0.00000	0.06659	0.73750
a1	-4.59e-01	-1.01e-03	1.82e+00	-5.95e-02	3.29e-02	-4.71e+00	-3.05e-01	-1.10e-02
a2	6.82e-03		-5.32e-02		-5.54e-04	2.06e-01		2.31e-04
a3			3.87e-04			-2.99e-03		
a4						1.45e-05		
a5								
a6								
b1		-1.33e+01		4.19e+00	-1.23e+01		-7.24e+00	-2.34e+01
b2		1.27e-01		-2.19e-01	1.18e-01		2.77e-01	3.45e-01
b3				1.73e-03			-4.24e-03	-1.17e-03
b4							2.18e-05	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.85: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the normal health male category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M	NH_M
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36
Parameters	5	5	6	6	6	6	6
AIC	112.87	100.39	81.38	76.80	74.70	78.48	77.31
AICc	114.87	102.39	84.28	79.70	77.59	81.37	80.21
BIC	120.79	108.31	90.88	86.30	84.20	87.98	86.81
Deviance	102.87	90.39	69.38	64.80	62.70	66.48	65.31
Chi squared	0.00000	0.00000	0.00006	0.00021	0.00042	0.00014	0.00020
R squared	0.99080	0.99141	0.99253	0.99303	0.99312	0.99276	0.99285
VIF	2.45	2.45	2.45	2.45	2.45	2.45	2.45
QBIC	59.85	54.76	49.78	47.91	47.06	48.60	48.12
Signs test	0.50000	0.50000	0.50000	0.38228	0.50000	0.50000	0.50000
Runs test	0.03446	0.30464	0.44177	0.40410	0.43488	0.44177	0.44177
Correlation test	0.02949	0.08768	0.42667	0.78620	0.75749	0.53917	0.59562
a1	3.29e-02	7.79e+00	-2.55e+00	-1.32e-03	1.04e-01	2.60e+00	-5.47e-01
a2	-5.55e-04	-4.55e-01		4.47e-05	-3.51e-03	-1.50e-01	9.60e-02
a3	2.21e-09	1.00e-02			3.07e-05	2.63e-03	-4.48e-03
a4		-9.81e-05				-2.07e-05	9.11e-05
a5		3.63e-07					-8.65e-07
a6							3.17e-09
b1	-1.23e+01		3.20e+00	-5.59e+00	-3.06e+01	-2.17e+00	
b2	1.18e-01		-1.34e-01	-2.16e-01	4.84e-01	4.15e-02	
b3			3.00e-03	4.74e-03	-1.86e-03		
b4			-2.99e-05	-2.09e-05			
b5			1.13e-07				
b6							
b7							
b8							
b9							
b10							

Figure 7.86: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the normal health male category.

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	2	3	3	4	4	4	5	5
AIC	24,316.24	2,198.94	16,510.17	2,449.81	1,028.18	1,128.89	971.23	87.89
AICc	24,316.60	2,199.69	16,510.92	2,451.10	1,029.47	1,130.18	973.23	89.89
BIC	24,319.41	2,203.69	16,514.92	2,456.15	1,034.52	1,135.22	979.14	95.81
Deviance	24,312.24	2,192.94	16,504.17	2,441.81	1,020.18	1,120.89	961.23	77.89
Chi squared	1.00000	0.00000	1.00000	0.00000	0.00000	1.00000	0.00000	0.00001
R squared	-26.06720	0.86830	-3.68177	0.75586	0.89264	0.83382	0.93200	0.98918
VIF	5.43	5.43	5.43	5.43	5.43	5.43	5.43	5.43
QBIC	4,487.74	414.89	3,052.35	464.34	202.35	220.91	195.06	32.27
Signs test	0.01254	0.50000	0.28893	0.13206	0.50000	0.26412	0.50000	0.13206
Runs test	0.00000	0.00000	0.00000	0.00000	0.00498	0.00005	0.00173	0.44177
Correlation test	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.54161
a1	-3.51e-01	-1.93e-03	1.45e+00	-6.05e-02	6.18e-02	-3.57e+00	-8.43e-01	-2.28e-02
a2	5.20e-03		-4.21e-02		-1.05e-03	1.57e-01		4.17e-04
a3			3.05e-04			-2.29e-03		
a4						1.11e-05		
a5								
a6								
b1		-1.37e+01		4.57e+00	-1.16e+01		-3.37e+00	-4.34e+01
b2		1.29e-01		-2.25e-01	1.08e-01		1.42e-01	7.72e-01
b3				1.72e-03			-2.10e-03	-3.49e-03
b4							1.04e-05	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.87: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the normal health female category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F	NH_F
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36
Parameters	5	5	6	6	6	6	6
AIC	1,030.20	266.84	271.88	157.44	481.88	325.66	133.51
AICc	1,032.20	268.84	274.78	160.34	484.77	328.56	136.40
BIC	1,038.11	274.76	281.38	166.94	491.38	335.16	143.01
Deviance	1,020.20	256.84	259.88	145.44	469.88	313.66	121.51
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	0.89264	0.97217	0.97192	0.98386	0.95789	0.96798	0.98633
VIF	5.43	5.43	5.43	5.43	5.43	5.43	5.43
QBIC	205.93	65.25	69.40	48.31	108.10	79.31	43.89
Signs test	0.50000	0.50000	0.50000	0.40503	0.50000	0.50000	0.38228
Runs test	0.00498	0.00180	0.00180	0.08224	0.00173	0.00180	0.13002
Correlation test	0.00000	0.00053	0.00050	0.01539	0.00007	0.00025	0.12155
a1	6.18e-02	1.27e+00	-6.15e+01	-3.23e-02	-2.14e+00	-3.74e+00	2.52e+01
a2	-1.05e-03	-9.95e-02		5.69e-04	4.35e-02	-9.56e-02	-1.68e+00
a3	3.10e-11	2.75e-03			-5.66e-04	1.76e-05	4.43e-02
a4		-3.25e-05				-1.56e-05	-5.76e-04
a5		1.41e-07					3.67e-06
a6							-9.10e-09
b1	-1.16e+01		4.14e+00	-1.10e+02	-9.20e-05	1.31e+00	
b2	1.08e-01		-1.57e-03	2.96e+00	-1.47e-03	2.06e-02	
b3			4.37e-05	-2.75e-02	1.49e-04		
b4			-5.20e-07	8.79e-05			
b5			2.26e-09				
b6							
b7							
b8							
b9							
b10							

Figure 7.88: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the normal health female category.

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	2	3	3	4	4	4	5	5
AIC	3,487.75	525.78	561.23	226.09	159.27	111.72	114.31	116.12
AICc	3,488.11	526.53	561.98	227.38	160.56	113.01	116.31	118.12
BIC	3,490.92	530.53	565.98	232.42	165.61	118.06	122.22	124.04
Deviance	3,483.75	519.78	555.23	218.09	151.27	103.72	104.31	106.12
Chi squared	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	-1.84565	0.89825	0.86290	0.94041	0.95709	0.96920	0.96904	0.97637
VIF	2.62	2.62	2.62	2.62	2.62	2.62	2.62	2.62
QBIC	1,336.94	209.16	222.69	97.58	72.08	53.93	57.73	58.42
Signs test	0.20049	0.13206	0.50000	0.50000	0.13206	0.13206	0.13206	0.50000
Runs test	0.00000	0.00050	0.00000	0.00000	0.00013	0.00180	0.00180	0.03029
Correlation test	0.00000	0.00000	0.00000	0.00000	0.00001	0.00019	0.00018	0.00085
a1	-4.61e-01	-2.60e-03	1.54e+00	-5.91e-01	4.94e-01	-4.91e-01	-6.81e+01	-4.33e-02
a2	7.09e-03		-4.55e-02		-1.30e-02	3.49e-02		9.85e-04
a3			3.39e-04			-7.09e-04		
a4						4.51e-06		
a5								
a6								
b1		-9.94e+00		1.42e+00	-3.21e+00		4.21e+00	-6.43e+01
b2		9.20e-02		-5.82e-02	3.41e-02		5.01e-04	1.28e+00
b3				4.41e-04			-1.03e-05	-6.53e-03
b4							6.55e-08	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.89: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the ill health male category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M	IH_M
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36
Parameters	5	5	6	6	6	6	6
AIC	161.27	178.86	181.12	48.51	179.67	115.73	68.86
AICc	163.27	180.86	184.01	51.41	182.56	118.63	71.75
BIC	169.19	186.78	190.62	58.01	189.17	125.23	78.36
Deviance	151.27	168.86	169.12	36.51	167.67	103.73	56.86
Chi squared	0.00000	0.00000	0.00000	0.18216	0.00000	0.00000	0.00172
R squared	0.95709	0.95952	0.95945	0.99007	0.96024	0.96920	0.98590
VIF	2.62	2.62	2.62	2.62	2.62	2.62	2.62
QBIC	75.66	82.37	86.05	35.44	85.50	61.10	43.20
Signs test	0.13206	0.50000	0.50000	0.13206	0.13206	0.13206	0.13206
Runs test	0.00013	0.00003	0.00003	0.19432	0.00050	0.00180	0.44177
Correlation test	0.00001	0.00005	0.00005	0.73681	0.00005	0.00019	0.13040
a1	4.94e-01	-1.44e+01	-7.33e+00	-1.06e-01	-9.38e+00	-1.45e+00	1.37e+02
a2	-1.30e-02	7.71e-01		1.76e-03	3.25e-01	3.24e-02	-9.24e+00
a3	7.84e-14	-1.52e-02			-6.44e-03	-7.12e-04	2.48e-01
a4		1.30e-04				4.50e-06	-3.31e-03
a5		-4.05e-07					2.19e-05
a6							-5.76e-08
b1	-3.21e+00		4.82e-03	1.58e+02	-1.46e-05	-3.77e-02	
b2	3.41e-02		1.06e-01	-6.26e+00	5.11e-02	2.58e-03	
b3			-2.10e-03	7.86e-02	-1.38e-04		
b4			1.80e-05	-3.20e-04			
b5			-5.63e-08				
b6							
b7							
b8							
b9							
b10							

Figure 7.90: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the ill health male category.

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	60	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36	36
Parameters	2	3	3	4	4	4	5	5
AIC	5,367.73	714.75	1,269.60	140.78	105.88	129.55	130.91	207.12
AICc	5,368.09	715.50	1,270.35	142.07	107.17	130.84	132.91	209.12
BIC	5,370.89	719.50	1,274.35	147.12	112.21	135.88	138.82	215.04
Deviance	5,363.73	708.75	1,263.60	132.78	97.88	121.55	120.91	197.12
Chi squared	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	-5.36595	0.78820	0.58145	0.94134	0.95572	0.94877	0.94899	0.92650
VIF	4.14	4.14	4.14	4.14	4.14	4.14	4.14	4.14
QBIC	1,301.76	181.81	315.73	46.38	37.96	43.67	47.10	65.49
Signs test	0.04277	0.13250	0.13206	0.50000	0.50000	0.13206	0.13206	0.40503
Runs test	0.00000	0.00008	0.00000	0.00180	0.03129	0.03129	0.03129	0.00698
Correlation test	0.00000	0.00000	0.00000	0.00009	0.00113	0.00032	0.00033	0.00001
a1	-3.59e-01	5.67e-03	1.57e+00	-1.80e-01	2.90e-01	-1.68e+00	-6.27e+01	2.83e-02
a2	5.47e-03		-4.52e-02		-5.27e-03	8.35e-02		-1.84e-04
a3			3.27e-04			-1.35e-03		
a4						7.22e-06		
a5								
a6								
b1		-1.27e+01		3.31e+00	-6.79e+00		4.11e+00	-4.98e+01
b2		1.18e-01		-1.46e-01	6.31e-02		1.32e-03	9.42e-01
b3				1.08e-03			-2.14e-05	-4.57e-03
b4							1.15e-07	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.91: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the ill health female category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F	IH_F
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	60	60	60	60	60	60	60
High age	95	95	95	95	95	95	95
Observations	36	36	36	36	36	36	36
Parameters	5	5	6	6	6	6	6
AIC	107.88	136.78	129.69	89.24	133.60	133.47	123.10
AICc	109.88	138.78	132.59	92.14	136.50	136.37	126.00
BIC	115.80	144.70	139.19	98.75	143.10	142.97	132.60
Deviance	97.88	126.78	117.69	77.24	121.60	121.47	111.10
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	0.95572	0.94681	0.95008	0.97288	0.94876	0.94879	0.95892
VIF	4.14	4.14	4.14	4.14	4.14	4.14	4.14
QBIC	41.54	48.52	49.91	40.14	50.85	50.82	48.32
Signs test	0.50000	0.38228	0.13206	0.13206	0.38228	0.13206	0.38228
Runs test	0.03129	0.03446	0.03129	0.44177	0.03446	0.03129	0.07162
Correlation test	0.00113	0.00024	0.00040	0.01942	0.00032	0.00032	0.00135
a1	2.90e-01	-2.01e+00	-9.97e+00	-6.27e-02	-4.67e+00	-2.74e+00	1.04e+02
a2	-5.27e-03	1.01e-01		1.05e-03	1.04e-04	7.64e-02	-6.90e+00
a3	1.38e-12	-1.69e-03			-1.85e-03	-1.37e-03	1.82e-01
a4		1.01e-05				7.14e-06	-2.39e-03
a5		-9.42e-09					1.56e-05
a6							-4.03e-08
b1	-6.79e+00		2.14e+00	1.73e+02	1.15e+00	6.12e-02	
b2	6.31e-02		7.97e-03	-6.75e+00	2.33e-02	6.54e-03	
b3			-1.30e-04	8.36e-02	-3.14e-05		
b4			6.97e-07	-3.36e-04			
b5			3.83e-12				
b6							
b7							
b8							
b9							
b10							

Figure 7.92: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the ill health female category.

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	50	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46	46
Parameters	2	3	3	4	4	4	5	5
AIC	3,055.83	129.56	1,523.28	174.61	150.12	214.99	161.29	158.11
AICc	3,056.11	130.13	1,523.85	175.58	151.10	215.96	162.79	159.61
BIC	3,059.48	135.05	1,528.77	181.92	157.44	222.30	170.43	167.25
Deviance	3,051.83	123.56	1,517.28	166.61	142.12	206.99	151.29	148.11
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03069	0.01007
R squared	-0.23019	0.98468	0.81467	0.98043	0.98340	0.97522	0.98177	0.98408
VIF	3.18	3.18	3.18	3.18	3.18	3.18	3.18	3.18
QBIC	966.26	50.30	488.08	67.65	59.96	80.33	66.66	65.67
Signs test	0.02882	0.34126	0.25588	0.34126	0.50000	0.11960	0.50000	0.44852
Runs test	0.00000	0.21055	0.00000	0.15941	0.50000	0.02382	0.11353	0.20354
Correlation test	0.00000	0.85159	0.00000	0.10242	0.33136	0.00780	0.15944	0.29859
a1	-2.88e-01	-3.25e-05	8.51e-01	-5.27e-02	2.85e-02	-1.72e+00	-7.44e-01	-5.31e-02
a2	4.82e-03		-2.77e-02		-5.42e-04	8.34e-02		1.05e-03
a3			2.24e-04			-1.34e-03		
a4						7.19e-06		
a5								
a6								
b1		-1.12e+01		7.06e-01	-1.02e+01		-1.93e+00	-5.69e+01
b2		1.05e-01		-1.28e-01	9.48e-02		8.02e-02	1.09e+00
b3				1.15e-03			-1.31e-03	-5.36e-03
b4							7.18e-06	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.93: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the male spouse category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M	SP_M
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46
Parameters	5	5	6	6	6	6	6
AIC	152.12	128.90	131.16	135.06	132.12	136.85	146.54
AICc	153.62	130.40	133.31	137.21	134.27	139.01	148.69
BIC	161.27	138.04	142.13	146.03	143.09	147.82	157.51
Deviance	142.12	118.90	119.16	123.06	120.12	124.85	134.54
Chi squared	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
R squared	0.98340	0.98614	0.98612	0.98685	0.98605	0.98575	0.98409
VIF	3.18	3.18	3.18	3.18	3.18	3.18	3.18
QBIC	63.79	56.49	60.40	61.63	60.70	62.19	65.23
Signs test	0.50000	0.34126	0.34126	0.50000	0.34126	0.34126	0.50000
Runs test	0.50000	0.45987	0.45987	0.32371	0.45987	0.45987	0.50000
Correlation test	0.33134	0.87948	0.87699	0.80852	0.86779	0.82282	0.59839
a1	2.85e-02	1.17e+00	-6.51e+01	-7.09e-02	-1.79e+02	-4.75e+00	1.61e+01
a2	-5.42e-04	-8.37e-02		1.29e-03	6.61e-01	-1.32e-01	-1.16e+00
a3	2.41e-16	2.22e-03			-5.88e-03	5.92e-06	3.30e-02
a4		-2.60e-05				-1.59e-05	-4.61e-04
a5		1.15e-07					3.15e-06
a6							-8.36e-09
b1	-1.02e+01		4.19e+00	8.98e+01	5.20e+00	1.65e+00	
b2	9.48e-02		-1.26e-03	-3.94e+00	-4.11e-03	1.89e-02	
b3			3.36e-05	5.21e-02	3.65e-05		
b4			-3.95e-07	-2.18e-04			
b5			1.74e-09				
b6							
b7							
b8							
b9							
b10							

Figure 7.94: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the male spouse category.

Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Category	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F
Formula	GM(2,0)	GM(1,2)	GM(3,0)	GM(1,3)	GM(2,2)	GM(4,0)	GM(1,4)	GM(2,3)
Low age	50	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46	46
Parameters	2	3	3	4	4	4	5	5
AIC	10,591.93	77.44	2,907.34	121.19	79.09	531.42	77.92	76.94
AICc	10,592.21	78.02	2,907.92	122.17	80.06	532.39	79.42	78.44
BIC	10,595.59	82.93	2,912.83	128.51	86.40	538.73	87.06	86.08
Deviance	10,587.93	71.44	2,901.34	113.19	71.09	523.42	67.92	66.94
Chi squared	0.00000	0.00309	0.00000	0.00000	0.00262	0.00000	0.00493	0.00676
R squared	0.57612	0.99887	0.91277	0.99826	0.99891	0.99100	0.99895	0.99895
VIF	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
QBIC	5,586.63	49.13	1,540.26	74.96	52.77	291.11	54.93	54.41
Signs test	0.00599	0.30200	0.08953	0.34126	0.30200	0.34841	0.34126	0.50000
Runs test	0.00000	0.24994	0.00000	0.01347	0.24994	0.00000	0.50000	0.22718
Correlation test	0.00000	0.65801	0.00000	0.01917	0.66686	0.00000	0.76689	0.79548
a1	-2.26e-01	3.65e-03	7.92e-01	-1.75e-02	6.94e-03	-1.98e+00	2.86e-03	-4.11e-03
a2	3.76e-03		-2.53e-02		-5.76e-05	9.45e-02		1.61e-04
a3			2.00e-04			-1.49e-03		
a4						7.75e-06		
a5								
a6								
b1		-1.38e+01		1.35e+00	-1.36e+01		-1.03e-01	-2.07e+01
b2		1.30e-01		-1.84e-01	1.29e-01		-3.56e-01	2.77e-01
b3				1.64e-03			5.74e-03	-7.82e-04
b4							-2.25e-05	
b5								
b6								
b7								
b8								
b9								
b10								

Figure 7.95: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the female spouse category.

Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Category	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F	SP_F
Formula	GM(3,2)	GM(5,0)	GM(1,5)	GM(2,4)	GM(3,3)	GM(4,2)	GM(6,0)
Low age	50	50	50	50	50	50	50
High age	95	95	95	95	95	95	95
Observations	46	46	46	46	46	46	46
Parameters	5	5	6	6	6	6	6
AIC	81.09	108.20	79.10	78.99	78.86	78.82	81.60
AICc	82.59	109.70	81.26	81.14	81.02	80.97	83.75
BIC	90.23	117.34	90.07	89.96	89.83	89.79	92.57
Deviance	71.09	98.20	67.10	66.99	66.86	66.82	69.60
Chi squared	0.00192	0.00000	0.00494	0.00496	0.00519	0.00525	0.00268
R squared	0.99891	0.99846	0.99894	0.99895	0.99895	0.99895	0.99893
VIF	1.90	1.90	1.90	1.90	1.90	1.90	1.90
QBIC	56.60	70.89	58.33	58.27	58.20	58.18	59.64
Signs test	0.30200	0.11700	0.34126	0.11700	0.50000	0.50000	0.50000
Runs test	0.24994	0.44460	0.30867	0.22298	0.22718	0.22718	0.14549
Correlation test	0.66686	0.07845	0.79099	0.79407	0.79764	0.79883	0.71211
a1	6.94e-03	3.50e+00	-3.91e-01	-3.57e-03	-8.20e-03	1.17e-01	-4.26e+00
a2	-5.76e-05	-2.22e-01		1.50e-04	3.11e-04	-6.83e-03	3.39e-01
a3	6.86e-12	5.26e-03			-1.39e-06	1.38e-04	-1.08e-02
a4		-5.52e-05				-9.53e-07	1.71e-04
a5		2.17e-07					-1.36e-06
a6							4.35e-09
b1	-1.36e+01		2.47e+00	-1.84e+01	-2.01e+01	-1.11e+01	
b2	1.29e-01		-2.27e-01	2.01e-01	2.65e-01	1.05e-01	
b3			5.68e-03	4.18e-05	-7.20e-04		
b4			-6.30e-05	-2.98e-06			
b5			2.63e-07				
b6							
b7							
b8							
b9							
b10							

Figure 7.96: Parameters and statistics for graduated mortality rates generated using different Gompertz-Makeham formulae for the female spouse category.

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