INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

16 April 2024 (am)

CM2 – Economic Modelling
Core Principles

Paper B

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 An investor has \$20,000 in cash, and plans to use this to purchase shares on a non-dividend paying security. The return on the shares over 1 year follows a log-normal distribution with $\mu = 8\%$ and $\sigma^2 = 3\%$.

The investor uses the log-normal distribution to generate 100 simulations of the 1-year returns on the shares over the next 3 years. These are shown in the 'Q1 Data' worksheet.

(i) Calculate, using the simulations supplied, the following values:

(a) The expected value of the investment at the end of year 3

(b) The standard deviation of the value of the investment at the end of year 3.

[5]

The investor also needs to repay a loan in 3 years' time. The current value of the loan is \$25,000 and it carries a continuously compounded interest rate of 6% p.a. The loan will be repaid in a single lump sum at maturity including interest.

(ii) Calculate, using the simulations supplied, the following values at the end of year 3 for the difference between the value of the shareholding and the loan repayment:

(a) 95% value at risk

(b) 95% tail value at risk.

[6]

(iii) Calculate, using the probability distribution, the following values:

(a) The expected value of the share investment at the end of year 3

(b) The standard deviation of the value of the share investment at the end of year 3.

[4]

(iv) Calculate, using the probability distribution, the probability that the value of the shares will be sufficient to cover the repayment of the loan including interest at the end of year 3. [3]

(v) Calculate, using the probability distribution, the following risk measures at the end of year 3 applied to the difference between the value of the shareholding and the loan repayment:

(a) 95% value at risk

(b) 95% tail value at risk.

[8]

(vi) Discuss how your results from part (v) compare to those from part (ii). [2]

The investor now wishes to assess their net wealth (i.e. assets minus loan) using the following utility function:

$$U(w) = w - 0.00003w^2$$

(vii) Calculate:

(a) the investor's current utility of net wealth at time $t = 0$.

(b) the investor's expected utility of net wealth at time $t = 3$.

[6]

The investor appeals to the bank who wrote the loan to reduce the interest rate to a more affordable level.

(viii) Calculate the interest rate on the loan that would give equal utility of net wealth at $t = 0$ and $t = 3$. [4]

(ix) Comment on the suitability of the investor's utility function for assessing their net wealth. [6]

[Total 44]

2 The Ornstein–Uhlenbeck process is given by the equation:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t$$

This has the Normally distributed solution:

$$X_t \sim N\left(X_0 e^{-\theta t} + \mu\left(1 - e^{-\theta t}\right), \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta t}\right)\right)$$

In this question, $\theta = 1$, $X_0 = 0$, $\mu = 1$, $\sigma = 1$. You have also been given a column of *iid* $U(0, 1)$ variables for part (ii) and ten columns of *iid* $U(0, 1)$ variables for part (iii) on the 'Q2 Data' worksheet.

(i) Calculate the mean and variance of X_1. [2]

(ii) Calculate simulated values of X_1 assuming a single time step between times 0 and 1. [3]

(iii) Calculate simulated values of X_1 assuming ten time steps between times 0 and 1. [5]

(iv) Calculate the mean and variance of the simulated X_1 from parts (ii) and (iii). [2]

(v) Comment on your results from part (iv). [5]

(vi) Plot a suitable chart of the first five simulations of the path between X_0 and X_1 from part (iii). [3]

(vii) Discuss, without producing any further charts, how your plot may change if each of the following happened:

(a) $\theta = 10$ and no other changes

(b) $X_0 = 1$ and no other changes

(c) $\mu = 5$ and no other changes.

[5]

[Total 25]

3 An investor holds a single non-dividend paying share, which has a current price of \$7 and volatility of 20%. The investor wants to create a delta hedge using European put options on the same underlying share. The put option matures in 3 years and has a strike price of \$6. The continuously compounded risk-free rate is 2% p.a.

(i) Calculate, using the Black–Scholes model, the price of the put option. [3]

(ii) Calculate how many put options the investor needs to hold to create the delta hedge (assuming it is possible to hold fractions of options). [3]

(iii) Demonstrate, by considering an equivalent call option, that put–call parity holds. [4]

(iv) Plot on a suitable chart, for a current share price range of \$1 to \$15:

- the intrinsic value of the put option.
- the price of the put option.

[8]

(v) Discuss how and why each of the following changes, in isolation, would impact the chart you produced in part (iv):

(a) The put option is an American option. [3]

(b) The volatility of the underlying share increases to 30%. [2]

(c) The strike price reduces to \$5. [3]

(d) The underlying share starts paying dividends. [2]

(e) The time to maturity increases to 10 years. [3]

[Total 31]

END OF PAPER