INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

13 September 2023 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 (i) State, with reasons, whether each of the following scenarios are examples of evidence for or against the Efficient Markets Hypothesis (EMH):

(a) A stock price is still moving in response to a public earnings announcement that was made 3 months ago.

(b) The performance of an actively managed index fund is the same as a passive fund tracking the same index.

(c) The volatility of a share is greater than that implied by a discounted cashflow model.

(d) Two equity analysts at different firms have determined target prices for the stock of a specific company, and their target prices are different. Both have access to the same information.

[4]

(ii) Give an example of a scenario that demonstrates semi-strong form EMH. Your example should be different to those in part (i). [1]

(iii) Explain how your scenario given in part (ii) demonstrates semi-strong form EMH. [2]

[Total 7]

2 Consider four assets that deliver a return of r% over a fixed period, T, with defined probabilities as set out in the table below:

	$P(r = -2\%)$ (%)	$P(r = -1\%)$ (%)	$P(r = 0\%)$ (%)	$P(r = 1\%)$ (%)	$P(r = 2\%)$ (%)
Asset A	10	30	20	30	10
Asset B	30	10	20	10	30
Asset C	25	25	0	25	25
Asset D	40	10	10	20	20

Determine which pairs of assets demonstrate:

(a) first-order stochastic dominance.

(b) second-order stochastic dominance.

[9]

3 The price at time t of a portfolio of stocks, S_t, follows a lognormal distribution with parameters $\mu = 0.09t$ and $\sigma^2 = 0.04t$, where t is the time from now measured in years and $S_0 = 1$. An investor needs to repay a loan of \$150,000 exactly 5 years from now.

(i) Determine the amount the investor would need to invest in the portfolio at time $t = 0$ to give an 80% probability of having at least enough to repay the loan at time $t = 5$. [4]

The investor has \$100,000 available, which they invest in the portfolio today.

(ii) Calculate the following risk measures applied to the difference at time $t = 5$ between the value of the investor's portfolio and the amount required to repay the loan:

(a) 95% value at risk

(b) Expected shortfall or surplus.

[5]

(iii) Discuss what conclusions and actions the investor may draw from your answers to parts (i) and (ii). [2]

[Total 11]

4 An insurance company sells car insurance policies that each pay out a fixed amount of \$25,000 if a car is damaged. Let X be a random variable representing the number of insured cars damaged. X is assumed to follow a binomial distribution with parameters n = number of policies and $p = 1\%$. Let the random variable L_P represent the loss per policy, i.e. $L_P = \$25{,}000X/n$. Each policy is for one car.

(i) Calculate the mean and standard deviation of L_P for the following scenarios:

(a) $n = 1$

(b) $n = 20$

(c) $n = 200$.

[4]

(ii) Explain which insurance concept is demonstrated by your results from part (i). [2]

The government of Country Z, where this insurance is sold, has set the premium for this type of policy at \$260.

(iii) Discuss, with reference to your answer to part (i), whether individuals are likely to buy this policy and whether insurers are likely to offer it. [3]

(iv) Comment on how adverse selection and moral hazard could impact this policy. [3]

[Total 12]

5 An analyst at a bank wants to model interest rates and is considering using either the Vasicek or Cox–Ingersoll–Ross models.

(i) State two similarities and two differences between these models. [2]

The analyst is looking to calibrate their model to the yield curves of a single country. In the period modelled, the short-term interest rates in this country were consistently negative over a period of multiple years.

(ii) Explain why the Cox–Ingersoll–Ross model will not be appropriate for the analyst. [2]

(iii) Explain how the analyst could adjust the Cox–Ingersoll–Ross model to reflect the points you have identified in part (ii). [2]

[Total 6]

6 (i) Define, in your own words, the market price of risk in modern portfolio theory. [1]

A panel of investment experts has provided annual expected investment returns for three asset classes that vary depending on the state of the economy (recession, normal or bubble). These are shown in the table below. You may assume the risk-free annual rate of return is 2%.

	Asset class			*Probability*
	Property	*Stock*	*Bonds*	
Recession	–1%	–2%	6%	0.1
Normal	4%	6%	2%	0.6
Bubble	8%	12%	5%	0.3
Market capitalisation ($ billion)	25	50	50	

(ii) Calculate the market price of risk. [5]

An analyst is looking at a portfolio (P_A) that offers a mean return, $\mu_A = 6\%$, and standard deviation of return, $\sigma_A = 3.15\%$. They are unsure whether a more efficient portfolio (P_B) can exist that offers either of:

- $\mu_B \geq \mu_A$ and $\sigma_A = \sigma_B$
- $\sigma_B \leq \sigma_A$ and $\mu_A = \mu_B$.

(iii) Determine whether a more efficient portfolio, P_B, can be found. [3]

[Total 9]

7 Consider the process A_t defined by the following integral:

$$\int_0^t W_s ds$$

where W_s is a standard Brownian motion.

(i) Explain why this process is not a martingale. [2]

(ii) Show that A_t is Normally distributed with mean 0 and variance $\frac{t^3}{3}$.

[**Hint**: Note that $tW_t = \int_0^t t dW_s$.] [6]

[Total 8]

8 Under certain assumptions, the adjustment coefficient r for an insurer's surplus process is a parameter that is the unique positive root of the following equations:

$$\lambda M_X(r) - \lambda - cr = 0$$

$$c = (1 + \theta)\lambda m_1$$

$M_X(r)$ is the moment generating function of the individual claim amount distribution.

(i) Write down the key assumption that is required to make these equations valid. [1]

(ii) Define the following terms from the equations above:

(a) λ

(b) c

(c) θ

(d) m_1

[2]

The insurer is looking at a product where claims take a value 1 with probability p, and 0 otherwise.

(iii) Show that the approximate relationship $r \approx 2\theta$ holds. You may assume r is small enough that terms of order r^3 and above can be discarded. [4]

[Total 7]

9 An insurance company is considering offering a 10-year investment product. The product will provide returns linked to a stock index. The total return at the end of the term is guaranteed to be no less than 10% and no more than 50%.

You may assume:

- continuously compounded risk-free rate, $r = 2\%$ p.a.
- implied stock price volatility, $\sigma = 10\%$ p.a.

(i) Calculate, using the Black–Scholes model, the price at time $t = 0$ of the investment product. [7]

(ii) Explain why the treatment of σ in the Black–Scholes model in part (i) may be a simplification of the real-world situation. [2]

[Total 9]

10 A stock, S, is currently priced at $S_0 = \$9$.

A derivative contract is written on S. The contract will pay an amount, Y, in exactly 1 year's time, where $Y = S_1^2$.

An analyst has estimated the price volatility of S at 20% p.a. The risk-free rate of interest is 12% p.a.

(i) Calculate the fair price of the derivative contract. You may assume that the assumptions underlying the Black–Scholes model hold. [5]

The analyst wishes to set up a delta hedged portfolio for the derivative contract, consisting of S and a holding in cash.

(ii) Determine the analyst's hedging portfolio at $t = 0$. [4]

[Total 9]

11 A surplus process for an insurer is denoted as $U(t)$ with initial surplus U.

(i) Define the surplus process for this insurer in terms of the initial surplus, premium income and aggregate claims process. All notations used should be defined. [1]

An insurer writes a 1-year pet insurance policy. The policy covers 100 pets and will pay out \$10 on each death of an insured pet during the policy term.

The pet mortality rate is assumed to be 2.5% per month with claims being independent and assumed to follow a Poisson process. Premiums are paid continuously at \$29 per month.

The insurer has an initial surplus of \$50.

(ii) Calculate the probability that the time to the first claim is longer than 1 month. [2]

(iii) Calculate the expected value of $U(t)$ at:

(a) the end of month 1.

(b) the end of the year.

[4]

(iv) Calculate the probability that the initial surplus and the insurer's premium income will be insufficient to cover the total claims in the first year. [3]

The insurer increases the estimate of pet mortality to 3.25% per month.

(v) Comment on the effect that this increase in the mortality estimate will have on the:

(a) probability of ruin in the next year.

(b) ultimate probability of ruin.

[3]
[Total 13]

END OF PAPER