INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

28 April 2022 (am)

Subject CM2 - Financial Engineering and Loss Reserving Core Principles

Paper B

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 An investor is seeking to maximise their expected utility by considering the expected return, E, and the standard deviation, σ, of their portfolio. In E–σ space, the investor's lines of constant utility are given by the formula

$$E(R, \sigma) = \frac{1}{2b} - \sqrt{R^2 - \sigma^2}.$$

For this investor, $b = \frac{5}{3}$.

R is a constant set by the investor and could be 0.15, 0.20 or 0.25.

(i) Calculate the expected returns using σ in the range [1%, 10%] with intervals of 0.5% for each of the three possible values of R. [6]

For this investor, the expected utility of any portfolio with expected return E and standard deviation σ is

$$U(E, \sigma) = E(1 - bE) - b\sigma^2.$$

(ii) Verify that each of your (E, σ) pairs produces a constant utility for each value of R. [5]

(iii) Plot the three lines of constant expected utility in E–σ space. [4]

(iv) Explain how your plot in part (iii) shows that the utility functions exhibit non-satiation and risk-aversion. [4]

[Total 19]

2 Claims on a portfolio of insurance policies arrive as a Poisson process with parameter 50. Individual claim amounts are independent and identically distributed and follow a Normal distribution with mean 60 and variance 10^2. The insurer calculates premiums using a loading factor of 25% and has an initial surplus of 150.

[**Hint:** for a Normal distribution with mean μ and variance σ^2, the Moment Generating Function at point t is $e^{ut+\frac{1}{2}\sigma^2 t^2}$ and the first two moments are $m_1 = \mu$ and $m_2 = \mu^2 + \sigma^2$.]

(i) Calculate the expected rate of premium income. [2]

(ii) Calculate the adjustment coefficient. [4]

(iii) Calculate:

(a) an upper bound for the probability of ultimate ruin, $\Psi(150)$.

(b) an estimate of the probability of ruin by time 1, $\Psi_1(150)$.

[8]

(iv) Comment on your results in part (iii). [3]

The insurer has an individual excess of loss reinsurance arrangement with retention level of 75.

(v) Calculate the probability that an individual claim does not exceed the retention level. [3]

(vi) Calculate the probability that, in any given year, there are no claims that exceed the retention level. [3]

(vii) Comment on how the reinsurance arrangement will affect the insurer's probability of ultimate ruin. [5]

[Total 28]

3 An investor has $1,000 available to invest in one of two assets, A or B. You have been provided with the cumulative 1-year distribution of returns for each asset in the 'Q3 data' worksheet.

(i) Calculate for each asset, over a 1-year period:

(a) the expected return.

(b) the variance of the return.

(c) the downside semi-variance of the return.

[14]

(ii) Plot the probability density function of the return on each asset on a chart. [4]

(iii) Explain how you could have determined the relationships between each asset in part (i) using only your chart and no further calculations. [6]

(iv) Suggest, with reasons, what the risk-free rate of return may be. [4]

[Total 28]

4 Consider a European put option with a strike price of $20 and a term of 5 years. The underlying share is currently priced at $25 and has a volatility of 20% p.a. The continuously compounded risk-free rate is 2% p.a.

(i) Calculate, using the Black–Scholes model for the option at time $t = 0$:

(a) the value of the option.

(b) Delta.

(c) Theta.

(d) Rho.

[9]

At time $t = 1$ the share price has fallen to $15 and the continuously compounded risk-free rate has increased to 5% p.a.

(ii) Estimate, using your values from part (i), the new value of the option. [4]

(iii) Calculate, using the Black–Scholes model, the new value of the option. [3]

(iv) Comment on the reasons for the difference between your answers to parts (ii) and (iii). [5]

(v) Suggest how an investor holding the put option could hedge their exposure to movements in the underlying share price. [4]

[Total 25]

END OF PAPER