

# INSTITUTE AND FACULTY OF ACTUARIES

## EXAMINATION

15 April 2024 (am)

### **Subject CM2 – Economic Modelling Core Principles**

#### **Paper A**

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

- 1
- (i) Explain whether an investor who believes that semi-strong form Efficient Markets Hypothesis (EMH) applies would invest in an actively managed fund. [3]
- (ii) Describe, in your own words, the behavioural heuristic known as ‘overconfidence’ and how this may arise. [3]
- (iii) Describe an example of a situation that illustrates overconfidence in the context of fund management. [2]
- [Total 8]

2 An insurer writes insurance policies that pay out a fixed benefit of \$250 on a claim. The number of claims is assumed to follow a Poisson process with parameter  $\lambda$ .

Premiums are received continuously and a premium loading of 10% is applied.

The insurer is considering entering a reinsurance agreement and is looking at two different types of contract:

- Proportional reinsurance
- Excess of loss reinsurance.

- (i) Describe the differences between these two forms of reinsurance. [2]

The insurer decides to use proportional reinsurance and will retain a proportion  $\alpha$  of each risk. The reinsurer uses a premium loading of 12%.

- (ii) Show that with this reinsurance, the adjustment coefficient  $R$  for the insurer is defined by the following equation:

$$e^{250\alpha R} = (280\alpha - 5)R + 1 \quad [4]$$

- (iii) Find the value of  $\alpha$  that maximises the adjustment coefficient, by differentiating the formula from (ii) with respect to  $\alpha$ .

[Note: You can assume that when you find a turning point it is a maximum without checking the second derivative.] [7]

[Total 13]

**3** Consider two individuals with the following utility functions:

- Individual A:  $U(w) = w + 0.01w^2$ ,  $w > 0$
- Individual B:  $U(w) = \ln(w)$ ,  $w > 0$ .

Each individual has a current wealth of \$300.

- (i) Calculate the current utility of wealth for each individual. [1]
- (ii) Show that individual A has increasing absolute risk aversion and individual B has decreasing absolute risk aversion. [3]

Each individual is offered the chance to gamble for free. The outcomes of the gamble are distributed as follows: \$100 gain (i.e. increase in wealth) with probability 20%, no change in wealth with probability 70%, and \$200 loss with probability 10%.

- (iii) Calculate the expected change in wealth from the gamble. [1]
- (iv) Determine, for each individual, whether they should accept the gamble. [3]
- (v) Discuss the relationship between your answers to parts (iii) and (iv). [3]

The person organising the gamble now wants to charge participants an entry fee.

- (vi) Show that the maximum entry fee that individual A should be willing to pay for the gamble is \$8.68. [3]

[Total 14]

- 4 An investment analyst assumes the return on a fund follows a discrete distribution as set out in the table below:

<i>Return (% p.a.)</i>	<i>Probability (%)</i>
0	10
2	20
4	40
6	25
10	5

- (i) Calculate the expected return and the variance of the return. [3]
- (ii) Calculate the following risk measures:
- (a) The semi-variance of return
  - (b) The shortfall probability relative to a benchmark return of 5%
  - (c) The expected shortfall relative to a benchmark return of 5%.

[3]

A second analyst believes the true distribution of the return on the fund is as set out in the table below:

<i>Return (% p.a.)</i>	<i>Probability (%)</i>
<b>-10</b>	10
2	20
4	40
6	25
<b>30</b>	5

- (iii) Comment on how using this distribution would affect your answers to parts (i) and (ii), without performing any further calculations. [3]

[Total 9]

**5** An individual has a liability of \$1,000 payable in exactly 3 years' time. To determine the present value of the liability, they assume an annual return that follows a log-normal distribution with parameters  $\mu$  and  $\sigma^2$ . The return in each year is independent of the return in any other year.

(i) Derive the formula for:

(a) the expected present value of the liability.

(b) the variance of the present value of the liability.

[4]

The investor has calculated the expected present value as \$862 and the variance as \$2,232.48.

(ii) Determine the values of  $\mu$  and  $\sigma^2$  used by the investor.

[4]

The investor chooses to invest \$900 and uses the distribution in part (ii) to model this investment.

(iii) Calculate the probability that this investment will **not** be sufficient to meet the liability of \$1,000 in 3 years' time.

[2]

[Total 10]

- 6 An analyst working at a bank wishes to model the return on two securities. They have recommended the following single period multifactor model of security returns:

$$R_i = \alpha + \beta_{i1} M_1 + \beta_{i2} M_2 + \xi_i$$

where  $R_i$  = return on security  $i$ ,  $\alpha$  = constant,  $\beta_{ij}$  for  $i, j$  and  $i \neq j$  are fixed parameters specific to each security,  $M_k$  for  $k = 1, 2$  are correlated rates of change, and  $\xi_i$  is the independent random component of return that is also independent of  $M_k$  for  $k = 1, 2$ .

- (i) Derive an expression for the covariance between the returns of the two securities. [5]
- (ii) State how the expression in part (i) would change if  $M_k$  for  $k = 1, 2$  were independent rates of change. [1]

Now suppose that the multifactor model takes the form:

$$R_i = (1 - X)^2 M_1 + M_2$$

where  $X$  is a fixed parameter.

- (iii) (a) Derive an expression for the single period multifactor model of security returns using principal components.

You may assume that  $M_2 = (1 - X)^2 M_1^*$ , where  $M_1^*$  is the first principal component.

- (b) Discuss a conclusion that can be drawn from the result in part (iii)(a).

[6]

[Total 12]

- 7 The table below shows the cost of claims settled per calendar year for a portfolio of car insurance policies in \$000s:

	<i>Development year</i>		
<i>Accident year</i>	<i>0</i>	<i>1</i>	<i>2</i>
2020	1,729	199	57
2021	2,274	318	
2022	2,511		

The number of settled claims in each period is as follows:

	<i>Development year</i>		
<i>Accident year</i>	<i>0</i>	<i>1</i>	<i>2</i>
2020	247	43	19
2021	291	50	
2022	317		

- (i) Calculate the outstanding claims reserve for this portfolio, using the average cost per claim method with grossing up factors. [7]
- (ii) Explain how an insurer may adjust this calculation to allow for inflation. [3]
- [Total 10]

- 8 Consider a process,  $\{N(t)\}_{t>0}$ , denoting the number of claims an insurer experiences over time. Suppose that  $N(1)$  follows a Poisson distribution with parameter  $\lambda$ .

- (i) Derive, from first principles, the mean of  $N(1)$ . [3]

One of the requirements for the process  $N(t)$  to be Poisson is to assume that, when  $s < t$ , the number of claims in the time interval  $(s, t]$  is independent of the number of claims up to time  $s$ .

- (ii) Explain, in your own words, what this assumption implies about the number and/or rate of claims that the insurer experiences. [1]
- (iii) Discuss whether it is reasonable for the insurer to make this assumption. [2]
- [Total 6]

- 9** An insurance company sells inflation-linked policies to customers. The policy includes a guarantee that the rate of return will have a minimum of 1% p.a. and a maximum of 2.5% p.a., applied at maturity. The insurance company would like to delta hedge the position.

The current rate of inflation is 1% p.a.

You may assume that:

- continuously compounded risk-free interest rate,  $r = 2\%$  p.a.
  - implied volatility,  $\sigma = 15\%$ .
  - term of policy,  $t = 8$  years.
  - the assumptions underlying the Black–Scholes formula apply.
- (i) Explain how to construct a portfolio using European options that replicates the required payoff profile. [2]
- (ii) Calculate the policy's delta. [5]
- (iii) Explain how to construct a delta neutral portfolio. [1]

Due to unforeseen events, the insurance company has been unable to rebalance its portfolio for delta hedging purposes. The policy will expire at the end of the day.

Current inflation stands at 10% p.a.

- (iv) State, with reasons, the value for this policy of:
- (a) delta.
  - (b) gamma.
  - (c) vega.
- [2]  
[Total 10]

- 10** Consider a forward contract on a share over a period where no dividends are payable.  $S_0$  is the initial price of the share,  $r$  is the continuously compounded risk-free rate of interest and  $t$  is the time to delivery of the contract in months.

- (i) Show, by constructing two portfolios and assuming no arbitrage, that the price of this forward contract is  $S_0 e^{rt}$ . [3]

We are now told that  $S_0 = \$10$ ,  $t = 20$  months and  $r = 7\%$  p.a. The share will pay a dividend of 3% of the share price every 6 months and the next dividend is due in 1 month's time. You may assume that dividends are immediately reinvested.

- (ii) Determine the fair price for this forward contract. [5]  
[Total 8]

**END OF PAPER**