

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 April 2021 (am)

Subject SP6 - Financial Derivatives Specialist Principles

Time allowed: Three hours and fifteen minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 A stock has a current price of \$70. There is a call option written on this stock that has a strike price of \$80, a time to expiry of 90 days, a trading calendar year of 260 days, a volatility of 32% p.a., a continuously compounded risk-free rate of 2.3% p.a. and no dividends.

(i) Calculate the delta of this option. [2]

(ii) Calculate the number of underlying stocks required to make a delta neutral portfolio comprising four of these options shorted and the underlying stocks. [1]

An investor sells a call option at time 0 receiving a premium of $V(0)$. The underlying stock has price $S(0)$ at time 0. The investor delta hedges this position using the underlying stock at time 0 only. At time t , the investor closes out their position with no cashflows between times 0 and t . Assume no dividends, taxes or transaction costs.

(iii) Determine the value of this portfolio:

(a) at time 0.

(b) at time t .

[3]

The continuously compounded risk-free rate is denoted by r .

(iv) Demonstrate that the profit to the investor at time t is given by

$$\begin{aligned} \text{Profit} &= \Delta(0)[S(t) - S(0)] \\ &\quad - [V(t) - V(0)] \\ &\quad - (e^{rt} - 1)[\Delta(0)S(0) - V(0)] \end{aligned} \quad [3]$$

(v) Describe in words the three terms in the equation in part (iv). [3]

[Total 12]

- 2 (i) Describe, in your own words, the following interest rates:
- (a) Treasury rate
 - (b) London Interbank Offered Rate (LIBOR) rate
 - (c) Overnight Indexed Swap (OIS) rate
 - (d) repo rate.
- [4]
- (ii) Discuss the credit risk inherent in each of these interest rates. [4]

An investment bank derives an OIS curve to value its portfolio of swaps. Where OIS data are not available, the investment bank assumes a fixed LIBOR-OIS spread, consistent with what is available at the longest term.

- (iii) State an alternative approach to deriving the OIS curve where data are not available. [1]
- (iv) Set out why a LIBOR forward curve consistent with OIS discounting must be derived when swaps are being valued using OIS discounting. [2]

The bank's rates desk provides the following interest rate data:

<i>Annual spot rate</i>	<i>1 year (%)</i>	<i>2 year (%)</i>	<i>3 year (%)</i>
OIS	0.90	1.25	n/a
LIBOR	1.25	1.75	2.50

The 2-year and 3-year LIBOR for fixed par swap rates are 1.75% and 2.48%, respectively.

- (v) Calculate, using the data provided, the 1-year forward LIBOR rates at $t = 1$ and $t = 2$, consistent with OIS discounting. [4]
- [Total 15]

3 The Vasicek model for the term-structure of interest rates is an equilibrium model that incorporates mean reversion.

- (i) Describe, in your own words, the key features of an equilibrium model. [2]
- (ii) Explain why mean reversion is a desirable feature of a short-rate model. [1]

An insurer has decided to use the Vasicek model to value the interest rate guarantees embedded in some of its life insurance products. The risk department proposes to calibrate the model by fitting it to historical data.

- (iii) Discuss the considerations that the risk department should make when selecting the historical data set used to fit the model. [4]

The insurer offers a single premium 10-year product that provides a return linked to the value of a portfolio of corporate bonds. Based on its analysis, the risk department has determined the following parameters for the Vasicek model:

$$a = 0.05$$

$$b = 0.03$$

$$\sigma = 0.02.$$

The Vasicek model is governed by the stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

where r_t is the short rate at time t and W_t is standard Brownian motion.

- (iv) Calculate the price of a 10-year zero-coupon bond with a notional value of 100 under the Vasicek model, assuming an initial short rate, r , of 2%. [3]

As part of the product, the insurer also offers a guaranteed return of the invested premium at the end of the life of the product. The insurer must calculate and report the cost of the guarantee as part of its regulatory reporting.

- (v) Explain what changes are needed to the model to enable the insurer to value the guarantee. [2]
- (vi) Comment on the suitability of the Vasicek model for fulfilling regulatory requirements. [2]

[Total 14]

- 4 (i) List the stakeholders in a defined benefit pension scheme who may be concerned with tail risk. [2]

A defined benefit pension scheme has a financial plan to become fully funded in 10 years' time. It currently has a significant exposure to domestic equities. A part of its plan is a strategy to manage equity tail risk.

- (ii) Explain why the pension scheme may be concerned with this risk. [3]

The investment advisors to the pension scheme have suggested hedging against this risk.

- (iii) Describe why purchasing index call options written on the main domestic equity index would NOT be a suitable hedge. [2]

As a basic hedge the pension scheme buys European-style index put options written on the main domestic equity index with a strike price equal to the current spot level and a time to maturity of 10 years.

The notional value of the put options is equal to the current market value of the equity investments of the pension scheme. It can be assumed that there are no cashflows to the equity investments of the scheme over the 10 years.

- (iv) Demonstrate that the potential profit/loss of the hedged equity investments of the scheme in 10 years' time, relative to their current market value, can be written as a call option. State any other assumptions made. [4]

- (v) Evaluate the strategy as a way of hedging the equity tail risks. [6]

[Total 17]

5 Consider an asset with price S_t at time t that follows geometric Brownian motion.

Let B_t be the risk-free bond price at time t satisfying $B_t = B_0 e^{rt}$ with r being the constant risk-free rate. A portfolio (ϕ_t, φ_t) consists of ϕ_t units of S_t and φ_t units of B_t . The value of this portfolio is V_t at time t .

(i) Define a self-replicating strategy both in your own words and algebraically using ϕ_t and φ_t . [3]

(ii) Prove that a strategy is self-financing if, and only if, $d\bar{V}_t = \phi_t d\bar{S}_t$, where $\bar{V}_t = e^{-rt} V_t$ and $\bar{S}_t = e^{-rt} S_t$, stating any assumptions made. [7]

(iii) Justify algebraically that φ_t is determined from V_0, B_t, S_t and ϕ_t so that the strategy is self-replicating. [Hint: consider integrating the equation in part (ii).] [2]

A strategy has been set-up with:

- $\phi_t = V_0/S_0$ for $t \in [0, t_1)$,
- $\phi_t = 3\phi_0$ for $t \in [t_1, t_2)$, and
- $\phi_{t_2} = 0$,

where $0 < t_1 < t_2$ are all fixed at time 0.

(iv) Explain in words what this strategy represents. [2]
[Total 14]

- 6** A stock with price S_t at time t goes ex-dividend at times $0 < t_1 < t_2 < \dots < t_n < T$ with corresponding dividend amounts of D_1, D_2, \dots, D_n at these times.

An American call option is written on this stock with maturity date T .

- (i) State, in your own words, how American derivatives with discrete dividends can be valued. [2]

The current stock price is \$100. The stock is expected to pay dividends of \$1 in 6 months' time and \$2 in 9 months' time. These dates are also the ex-dividend dates. An American call option is written on this stock with a maturity date of 12 months. The call option has a strike price of \$110 and a volatility of 20% p.a., and the continuously compounded risk-free discount rate is 6% p.a.

- (ii) Calculate the current value of the equivalent European call option. [3]
- (iii) Explain whether or not it will be optimal to exercise the American option immediately prior to t_1 (in 6 months' time). [2]

A stock has current price of \$100 and a dividend of \$1 in 6 months' time. A European call option written on this stock has a maturity date of 9 months, a strike price of \$110 and the same volatility and interest rate as above.

- (iv) Calculate the current value of this European call option. [3]
- (v) Demonstrate that the value of the American option with a maturity date of 12 months in part (iii) is approximately \$5.17, stating any assumptions made. [2]

[Total 12]

7 A regulatory capital framework requires insurers to use credit ratings, sourced from approved Credit Rating Agencies (CRAs), in the calculation of their capital requirements.

As part of a review of its regulatory system, the regulatory authorities are investigating alternatives to using credit ratings, sourced from CRAs.

(i) Discuss why the regulator may wish to replace the use of credit ratings in insurers' capital calculations. [4]

One proposal being considered is for insurers to make use of their own internal credit assessments in the calculations.

(ii) Evaluate this proposal from the point of view of an insurer. [3]

An insurer that operates under this regulatory capital framework would like to reduce its regulatory capital requirement by mitigating its credit risk exposure to its non-investment grade corporate bonds.

(iii) Explain how Credit Default Swaps (CDSs) can be used to achieve this insurer's objective. [2]

(iv) List two other derivatives that could be used to achieve this objective. [1]

The insurer is particularly concerned with the credit risk relating to a single non-investment grade corporate bond it owns and wishes to purchase 2-year protection against default.

The insurer's risk team has calculated a conditional default probability of 4.5% p.a. and uses a recovery rate assumption of 50%. Risk-free interest rates are 2% p.a. with continuous compounding.

(v) Calculate the theoretical CDS spread. You should assume that defaults occur mid-way through the year and CDS payments are made annually at the end of each year. [6]

[Total 16]

END OF PAPER