

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

28 April 2022 (am)

Subject SP6 - Financial Derivatives Specialist Principles

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the
Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on
T. 0044 (0) 1865 268 873.

1 A trader in exotic derivatives working for an investment bank has sold a binary cash-or-nothing call option on stock S to an insurance company.

- (i) (a) State two examples of other exotic derivatives.
(b) Describe why each of these may be useful to an insurance company. [3]
- (ii) (a) Suggest two possible reasons why some investment banks may engage in exotic options business.
(b) Suggest two possible reasons why some investment banks may choose not to do so. [4]

The trader observes the following market information in relation to derivatives on stock S.

	<i>Delta</i>	<i>Gamma</i>
Binary cash-or-nothing call at strike K	1.7104	-0.0070
Call option at strike K	0.6248	0.0178
Future	1.0000	0.0000

- (iii) Calculate, showing all workings, the number of futures and call options the trader would need to enter into in order to hedge their short position in the binary cash-or-nothing call option. [3]
- (iv) Comment on the hedge effectiveness using futures and call options to hedge this binary option, compared to using futures on stock S only. [3]
- (v) State the range of possible values for the delta and gamma of a binary cash-or-nothing call option. [2]

Call options and futures on stock S are traded and cleared on an exchange, whereas the exotic binary option is traded Over-The-Counter (OTC) with bilateral clearing. The differences in collateral arrangements between these cause collateral inefficiencies for the investment bank.

- (vi) Describe key differences in relation to collateral arrangements for derivatives that are exchange traded and cleared versus derivatives that are OTC traded with bilateral clearing. [5]
- (vii) Suggest possible actions that the bank could take to reduce the impact of these collateral inefficiencies. [3]

[Total 23]

- 2** Consider a Credit Default Swap (CDS) with a term of 5 years, assuming a 1% p.a. risk-free rate and that defaults and payments only occur at the end of each year. Spread payments are only made if there are no defaults.

Credit risk characteristics:

- recovery rate: 60%
- implied probability of default: 3%.

- (i) Calculate, showing all workings, the CDS spread at which the CDS has zero value at outset. [4]

Consider a 5-year first-to-default swap on a basket of 10 corporate issuers that have the same credit risk characteristics as shown above and assume that their credit risk is uncorrelated.

- (ii) Calculate, showing all workings, the probability of the first-to-default CDS triggering over its lifetime. [3]

- (iii) Explain the impact of the corporate issuers having a higher credit risk correlation than assumed on the first-to-default swap valuation. [2]
- [Total 9]

3 In a developed country, all 10-year fixed interest government bonds are being issued with a negative bond yield.

(i) Describe the term ‘bond yield’. [2]

(ii) Explain what it means to a bondholder for a bond yield to be negative. [1]

Additionally, in the country the interest rates are also negative at some durations.

(iii) Discuss the impact of negative interest rates on derivatives. [4]

Negative interest rates and negative bond yields have caused problems with using Black’s model for valuing some interest rate derivatives. One solution proposed by an academic is to consider modelling forward prices (F) under a risk-neutral measure using

$$dF = \sigma_s(F + s)dB_t.$$

Here, s and σ_s are fixed constant values and B_t is Brownian motion.

(iv) Derive the value of a vanilla call option with forward price F using this stochastic model. The model with $s = 0$ can be used as a starting point. [5]

(v) Explain why the model with $s = 0$ cannot be used for modelling negative forward rates. [1]

(vi) Assess the suitability of using this model for valuing interest rate derivatives with negative forward rates. [4]

In this country, some financial services firms choose NOT to model the possibility of negative interest rates.

(vii) Discuss the implications of NOT being able to model negative interest rates. [3]

[Total 20]

- 4 (i) Describe how a DB or DC pension scheme may use Value at Risk (VaR) to manage financial risk. [4]
- (ii) Describe some of the limitations that a pension scheme may face in using VaR to manage financial risk. [3]

A defined benefit pension scheme has a foreign currency exposure. To manage the risk of currency fluctuations, it has a portfolio of options relating to the exchange rate between the local currency and the foreign currency. The current exchange rate is 3 (that is, one unit of the local currency is equal to three units of the foreign currency). The current delta of the option portfolio is 1,350 and the daily return volatility of the exchange rate is 0.5%.

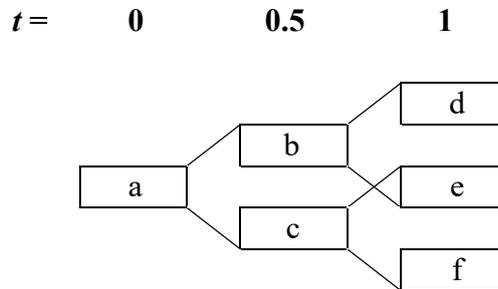
Let the daily change in the value of the portfolio of the options be δP , the daily change in the exchange rate be δS and Δ be the delta of the portfolio. It is assumed that $\delta P = \Delta \delta S$ and that the daily returns of the exchange rate have a normal distribution with mean 0.

- (iii) Demonstrate that the 1-day 95% VaR of this portfolio is 33.308. [4]
- (iv) Select and then communicate a method that a defined benefit pension scheme might use for calculating VaR. [2]
- [Total 13]

- 5** (i) Describe, in your own words, how the control variate technique can be used to improve the pricing of American options. [3]

A stock, S , is currently valued at 130. It has a dividend yield of 2% p.a. and volatility of 40% p.a. The risk-free rate is 3% p.a.

A two-step binomial model has the following structure:



- (ii) Calculate, showing all workings, the value of a 12-month American put option on stock S with a strike price of 135, using the control variate technique and the node naming conventions given in the two-step binomial model. [8]

An investment bank trades a range of derivatives. It uses Monte Carlo simulation to value Asian options and binomial trees to value American options.

- (iii) Explain why the investment bank may opt to use different approaches to value Asian and American options. [3]
- (iv) State an approach that could enable the investment bank to use Monte Carlo simulation to value American options. [1]
- [Total 15]

- 6** (i) Describe how Black–Scholes pricing formulae for non-dividend paying stocks are adjusted to value currency options. [2]

A UK-based retailer is considering ordering Bluetooth headphones from a US-based manufacturer. The Finance Director of the retailer proposes using currency options to ensure that sales of the headphones generate a profit of at least £100,000.

The Finance Director has asked you to prepare the profit projections for discussion at the next Board meeting. Their team send you the following data:

- The unit cost is \$50.
- The order needs to be placed 6 months in advance of delivery and payment.

Market data:

- USD/GBP spot rate: 0.65, i.e. 1 US Dollar = 0.65 GB Pounds.
- GBP risk-free interest rate: 2% p.a. at all durations.
- USD risk-free interest rate: 3% p.a. at all durations.
- Volatility of USD/GBP exchange rate: 20% p.a.

The Sales Manager has also provided you with the following sales scenario estimates:

- If the retail price is £40, the retailer expects to sell 20,000 units.
- If the retail price is £35, the retailer expects to sell 70,000 units.

You should assume that the profit is calculated as the gross profit less the cost of the options.

- (ii) Demonstrate, showing all calculations, that the minimum profit can be achieved by purchasing call options at the following strike prices:
- (a) USD/GBP 0.67 if headphones are sold at £40
- (b) USD/GBP 0.62 if headphones are sold at £35. [7]
- (iii) Recommend, with reasons, whether the Finance Director should propose to order 20,000 or 70,000 units, assuming that the sales forecasts provided are correct. [3]

The Finance Director receives a research note that suggests that USD/GBP 0.70 is expected to be the maximum level that could be reached over the next 12 months and asks whether this research would have any impact on your recommendations.

- (iv) Explain how a bull call spread could be used to alter the profit profile from the headphones sales. [2]
- (v) Calculate, showing all workings, the revised option premiums for both scenarios if bull call spreads are implemented with the following strike prices:
- (a) USD/GBP 0.67 and USD/GBP 0.70 if headphones are sold at £40
- (b) USD/GBP 0.62 and USD/GBP 0.70 if headphones are sold at £35. [3]
- (vi) Comment on your recommendation given in part (iii), in light of your calculations in part (v). [3]
- [Total 20]

END OF PAPER