

# INSTITUTE AND FACULTY OF ACTUARIES

## EXAMINATION

8 April 2024 (am)

### Subject CP2 – Modelling Practice Core Practices

#### Paper One

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

# Exam requirements

## 1 Modelling steps and checks

Read the background document, which describes the scenario that needs to be modelled and documented for this project.

Construct a spreadsheet model that produces the following calculations and charts. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed (and documented in the audit trail) reasonableness checks at each stage of your calculations.

(i) Generate a set of 6,000 Normal (0,1) random numbers using the random numbers provided from a Uniform (0,1) distribution. [3]

(ii) Verify that the random numbers simulated in part (i) come from the Normal (0,1) distribution by:

- calculating appropriate summary statistics.
- checking the overall distribution using a chart.
- checking the overall distribution by completing a chi square test.

[Note: You are not expected to make any changes to the data. You will also be able to continue to complete the model if you have not completed all the verifications requested above.] [8]

(iii) Generate 1,000 simulations of the share price,  $S(t)$ , for each year from 1 January 2025 to 1 January 2031. [3]

(iv) Construct a chart to illustrate the distribution of the share price,  $S(t)$ , simulations as at both 1 January 2028 and 1 January 2031, grouping the simulations into appropriate price bands. [2]

(v) Calculate the following for Senior Managers for each simulation at both 1 January 2028 and 1 January 2031:

- The total notional profit
- The cost of exercising the options
- The percentage profit margin above the cost of exercising the option.

[4]

(vi) Repeat part (v) for Associates. [3]

- (vii) Calculate the following key statistics for both the Senior Managers and Associates as at 1 January 2028 and 1 January 2031:
- The minimum total notional profit
  - The average total notional profit
  - The maximum total notional profit
  - The average cost of exercising the options
  - The average profit margin (%).
- [3]
- (viii) Construct a chart to show the distribution of total notional profits for both Senior Managers and Associates as at 1 January 2028 and 1 January 2031. You should show the number of simulations that fall into appropriately grouped profit bands.
- [3]
- [Sub-total 29]**

## **2 Modelling technique and good practice**

In addition to the requirements above, marks will be allocated for modelling technique and good practice as follows:

- (i) Auto checks on the modelling completed in question 1, parts (iii)–(viii). [2]
- (ii) Demonstration of good modelling technique and practice. [7]
- [Sub-total 9]**

### 3 Audit trail

Prepare an audit trail for your spreadsheet model that includes the following aspects:

- Purpose of the model
- Data (including checks) and assumptions used
- Methodology, i.e. description of how each calculation stage in the model has been produced
- Explanation of any further checks performed.

You should ensure that your audit trail is suitable for both a senior actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, or may be asked to continue to work on it or to use it again for a similar purpose in the future.

#### Marks available for audit trail:

##### Audit approach

- |       |   |     |
|-------|---|-----|
| (i)   | Communication skills (the audit trail provides enough detail to be read as a stand-alone document). | [4] |
| (ii)  | A fellow student can review and check methods used in the model.                                    | [7] |
| (iii) | A senior actuary can scrutinise and understand what has been done.                                  | [7] |
| (iv)  | Written in clear English.   | [3] |
| (v)   | Written in a logical order.   | [3] |

##### Audit content

- |        |  |      |
|--------|--|------|
| (vi)   | All steps clearly explained.           | [7]  |
| (vii)  | Reasonableness checks included.        | [6]  |
| (viii) | Clear signposting included throughout. | [4]  |
| (ix)   | Statement of assumptions made.         | [5]  |
| (x)    | All model steps accurately covered.    | [16] |

**[Sub-total 62]**

**[Total 100]**

## Background

You work for the actuarial department of Banking Inc., a publicly listed company ('the Company'). The Human Resources (HR) department and Chief Financial Officer (CFO) of the Company are considering offering **share options** (terms in bold will be defined in the Glossary of Terms at the end of the paper) as part of a new incentive package to attract and retain key employees.

The HR manager has advised that throughout the 2025 calendar year, each Senior Manager will be awarded three independent tranches of options (totalling 300 shares) while Associates will be awarded two independent tranches (totalling 200 shares). The tranches will have the same **exercise date** but different **exercise prices**. The HR manager has provided the following schedule of the share options to be offered to employees:

<i>Option tranche details</i>	<i>Exercise price per share (\$)</i>	<i>Number of shares offered per employee</i>
Senior Managers tranche 1	70	100
Senior Managers tranche 2	80	100
Senior Managers tranche 3	90	100

<i>Option tranche details</i>	<i>Exercise price per share (\$)</i>	<i>Number of shares offered per employee</i>
Associates tranche 1	90	100
Associates tranche 2	100	100

Employees will be given the opportunity to decide whether to cash in (**to exercise**) the options at the exercise date, or not. The company is currently deciding on whether the exercise date will be 1 January 2028 or 1 January 2031.

There will be no cost to the employees for holding the option to buy the shares, with any such costs being borne by the Company. Employees will need to pay the exercise price per share if they exercise the options.

The HR manager has requested that it is assumed that employees will exercise their share options only where they would make a profit in doing so. The notional profit is calculated as the share price at the exercise date minus the exercise price, with a minimum of \$0.

The HR manager has also requested an illustration of the employees' expected notional profits due to the share options for employees if the exercise date of the options is either 1 January 2028 or 1 January 2031, to help the Company decide which exercise date to use.

The company share price is expected to be \$100 per share at 31 December 2024.

## Modelling

The CFO has provided the actuarial department with the following model to simulate the Company's share price for each year between  $S_{(0)}$  (Company share price at 1 January 2025) to  $S_{(3)}$  (Company share price at the potential exercise date of 1 January 2028) to  $S_{(6)}$  (Company share price at the potential exercise date of 1 January 2031).

$$S_{(t)} = S_{(t-1)} \times \exp\left[\mu - \frac{1}{2}\sigma^2 + \sigma Z_{(t)}\right]$$

where  $\mu = 0.01$ ,  $\sigma = 0.05$  and where  $Z_{(t)}$  = a random number from the Normal (0,1) distribution.

The CFO has also provided a set of 6,000 random numbers from the Uniform (0,1) distribution to enable the actuarial department to generate 6,000 random numbers from a Normal (0,1) distribution to model  $Z_{(t)}$ . The Uniform (0,1) random numbers have been checked and confirmed to be from the correct distribution.

The CFO has also confirmed that 1,000 simulations of the share price for each year from  $S_{(0)}$  to  $S_{(6)}$  should be carried out.

The CFO has requested the following calculations for each of the 1,000 share price simulations:

- The expected total notional profit, cost to purchase the shares and profit margin (i.e. notional profit divided by total exercise cost of the option) of exercising the options for Senior Managers, if the exercise date is 1 January 2028 and if the exercise date is 1 January 2031.
- The expected total notional profit, cost to purchase the shares and profit margin of exercising the options for Associates, if the exercise date is 1 January 2028 and if the exercise date is 1 January 2031.

The CFO also requires summary statistics from the 1,000 simulations, including minimum, maximum and average of the notional total profit, the average cost of exercising the options and the average profit margin on exercising the options for both Senior Managers and Associates.

The CFO and HR manager have requested a chart to illustrate the range of the Senior Managers' and Associates' total notional profits on exercising the options at the two potential option exercise dates, to help them decide which exercise date to use.

Your manager has delegated you the task of preparing the required calculations and charts.

## Additional guidance

The inverse transform method can be used to simulate values from continuous distributions.

First we generate a random number,  $u$ , from the  $U(0,1)$  distribution. We can use this to simulate a random variate  $X$  with probability density function,  $f(x)$ , by using the Cumulative Density Function (CDF),  $F(x)$ .

Let  $u$  be the probability that  $X$  takes on a value less than or equal to  $x$ , i.e.:

$u = P(X \leq x) = F(x)$ , hence  $x$  can be derived as

$$x = F^{-1}(u)$$

Hence the following two-step algorithm is used to generate a random variate,  $x$ , from a continuous distribution with CDF  $F(x)$ :

1. Generate a random number,  $u$ , from  $U(0,1)$ .
2. Return  $x = F^{-1}(u)$ .

The `NORMINV()` function in Excel can be used to calculate the inverse of the normal cumulative distribution for a given probability value generated from a  $U(0,1)$  distribution.

## Glossary of terms

**Share option:** A share option gives the holder of the option the right (but not an obligation) to purchase shares at a specified price at a specified point of time.

**Exercise date:** The exercise date refers to the date on which the holder of an option decides whether to exercise an option, or not.

**Exercise price:** The exercise price is the price at which the holder of an option can purchase the share associated with the option.

**To exercise:** If the holder of an option decides to purchase the share at the exercise price on the exercise date they have chosen to exercise the option.

**END OF PAPER**