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Essay on "Welfare Effects of Developing Reverse Mortgage Market in China"

Subject SA0 Advisors

Bing Zheng Chen
James Orr

Prepared at School of Economics and Management,
Tsinghua University

Abstract

Population ageing and inadequate social security have introduced serious problems for China. As a result, seeking alternative solutions is becoming increasingly important. Some developed countries, such as U.S., Canada, and Australia have developed reverse mortgage markets to relieve related financial pressures. A natural question is whether or not the reverse mortgage system should be implemented in China to deal with ageing and social security problems. From the perspective of welfare economics, it amounts to asking if reverse mortgage markets can help improve both residential and social welfare in China.

Based on parameters from a dataset which reflects the current status of China's population and economy, this dissertation draws on scenarios that help explore the above question. Building on the life-cycle model, we investigate the role of reverse mortgages in improving the welfare of retired elderly residents. Simulation results show that, reverse mortgage can improve the welfare of residents who are house-rich and cash-poor. However, due to high transaction costs, it is not always the case. Timing the entry of a reverse mortgage contract can help residents attain a higher level of welfare improvement. On average, it is better for residents to wait until after the age of 65 to participate in reverse mortgages. Sensitivity analysis indicates that reverse mortgages can mitigate the adverse effects on residential welfare caused by longevity risk; it could exacerbate the negative effects caused by moving risk. This implies that maintaining transaction costs at a reasonable level is necessary for reverse mortgages to retain their welfare-enhancing function in different economic conditions.

Building on the overlapping generation model, we investigate how reverse mortgages influence the equilibrium allocation of social capital. Simulations reveal that reverse mortgages provide substantial income for elderly residents. This in turn, reduces the burden on younger pension taxpayers, which promotes and smoothes the life-cycle consumption of residents. In equilibrium, this reduces the optimal pension tax and improves social welfare. We further demonstrate that the intensity of the motive to bequest determines whether residential welfare improves under equilibrium. Sensitivity analysis indicates that the welfare-enhancing role of reverse mortgages remain significant as the severity of the ageing problem increases.

Finally, drawing on theoretical analysis and our international experience, we provide suggestions on policies that would assist in developing an effective reverse mortgage market in China as well as open topics for future research.

Key words: reverse mortgage; social security; welfare analysis; life-cycle; overlapping generation

Contents

Chapter 1 Introduction

1.1 Background

Since 1999 China has entered an era where society is ageing. According to the report "Forecasting China's Ageing Problem" from the China Ageing Committee, the ageing elderly[①] will soon total 248 million by 2020, accounting for 17.17% of the whole population where 30.67 million (or 12.37% of ageing elderly) will be over 80 years-old; by 2050, the elderly population will exceed 400 million, over 30% of the whole population, with 94.48 million (or 21.78%) being over 80 years-old. China's social security system is not equipped for such rapid and massive ageing challenge, potentially causing the national treasury to go into heavy debt. Currently, the deficit reaches 1.2 trillion RMB in 2013, and the social security system can only provide limited support for the elderly people. China's ageing problem has imposed great pressure on its social security system and other sources of annuity income need to be explored in order to increase the living standards of elderly people.

To tackle the ageing problem, some countries such as the U.S., Canada and Australia, have developed a solution based on "security through house", also known as the Reverse Mortgage (RM) market. The basic mechanism of RM is that elderly people sign a contract with a financial institution to use their home equity as a mortgage in exchange for a financial lump sum. Unlike regular mortgage contracts, the borrower will be allowed to live in the house until his or her death; the financial institute will receive the house after then. Since the RM contract does not require the debtor to pay for their debt when he or her is still living in the house, this product can also be used to finance many other long-term projects such as education or help relieve unemployment burdens. RM contracts are particularly suitable for elderly people mainly because: (1) they usually own home equity; (2) they usually have a short expected future lifetime, so

[①] Several developed countries categorizes people over 65 as elderly (for example visit www.un.org/esa/population/publications/worldageing19502050), the China National Committee on Aging standard, which is 60 (see http://old.cncaprc.gov.cn:8080/en/info/1897.html) is used as the criteria of elderly person.

that annuity payouts can be high; (3) they usually prefer to live in their own house as they age; (4) many elderly people are starting to open their minds to new security ideas for the ageing. All these reasons increase the attractiveness of RM[①].

The United States is one of these countries with the most mature and well-developed RM market. The first RM contract in the U.S. appeared in 1961. In 1987, the Department of Housing and Urban Development started the RM market, and the government has been providing a guarantee for Home Equity Conversion Mortgages (HECM). The market scale has increased dramatically in the U.S., from 1 billion USD in 1999 to 32.4 billion USD in 2009. Annual applications have increased from 7,900 to 114,000 during the same time period. Home Equity Conversion Mortgages account for 90% of all reverse mortgages originated in the U.S. As of May 2010, there were 493,815 active HECM loans[②]. There are three basic categories of RM products in the U.S. market: HECM, the Home Keeper Plan, and private products. HECM was the original RM product, and so far the most mature with a market share of over 90%.

The Canadian Home Income Plan (CHIP) is the first, as well as the only, private RM business operator in Canada. It started its business in 1986 and has issued CAD contracts worth over 2 billion. The RM market in Canada is entirely market-oriented without the need of any financial support from the government. Reverse Annuity Mortgages, Line of Credit Reverse Mortgages, and Fixed Term Reverse Mortgages are the three main categories in the Canadian market.

The Australian RM market began to operate at the start of 2005. It was established by several RM business operators and supervised by the Senior Australian Equity Release Association of Lenders (SEQUAL). In Australia, the RM market was chiefly led by the major financial institutions, without much intervention or financial support from the government. The Deloitte report, commissioned by SEQUAL, shows that as of 31 December 2010 the Australian reverse mortgage market comprised of more than 41,000

① The RM product of each country briefly described below is tabulated in more detail in Appendix A.
② See Home Equity Conversion Mortgages Monthly Report (May 2010), http://www.hud.gov/offices/hsg/comp/rpts/hecm/hecmmenu.cfm

loans with total outstanding fund of $3bn. The total represents 11% growth in the market from 31 December 2009, and the average loan size has also increased to $72,500, up from $51,148 in 2005[①].

All of these countries have tailored their RM business to elderly people who own home equity. Currently, 90% of families in China have their own home equity, of which 60% were provided by government and corporations as forms of subsidies and 40% were purchased from public housing markets[②]. This translates to huge potential for the RM development in China. In 2013, China's central government plans to launch a pilot program that will offer reverse mortgages to the elderly in the year after.

1.2 Motivation

Given that RM has been proven to be a valid complement of the traditional social security system in foreign developed countries, it is important to ask whether RM can help solve China's ageing problem, taking into account China's economic and demographic characteristics, such as "getting old before becoming rich", "housing price boom", and "one-child policy". From the perspective of welfare economics, it is important for the government and practitioners to know whether developing RM is welfare enhancing.

In the literature of mainstream economics[③], welfare refers to the economic well-being of an individual, group, or economy. Individual welfare is commonly measured by the utility gained through the achievement of economic activities (e.g., labor supply, income and asset allocation), and the social welfare is commonly measured by the total utility gained through aggregating the utilities of all participants. Here, utility is measured using a utility function, which describes individual preference based on an axiomatic foundation[④]. In this thesis, parameterized utility functions were used to determine

① See more at: http://www.reversemortgagewatch.com.au/reverse-mortgage-news-2/reverse-mortgage-market-hits-3-billion/#sthash.Tzfu5KyA.dpuf

② The data were provided by the office of the Department of Housing and Urban and Rural Construction on March 9th, 2013. Additionally, according to the "China Household Finance Report" issued by Southwestern University of Finance and Economics and China's People's Bank, on May 13th, 2013, 90% of Chinese families own home equity.

③ See Samuelson P A, Nordhaus W D. Economics, McGraw-Hill, 2004.

④ See Von Neumann J, Morgenstern O, Theory of Games and Economic Behavior, Princeton, NJ, Princeton University Press, 1944, second ed. 1947, third ed. 1953.

individual and social welfare. A detailed formula will be presented in the following chapters. At this juncture, welfare can be broadly understood as well being or utility gained.

This paper attempts to explore the above question based on China's social parameters from both individual welfare and social welfare perspectives, using both theoretical and empirical methods. The detailed contents of the study include:

(1) The impact of the RM market on retired residents' welfare.

At the very beginning of the RM market development, the major participants of the RM program will be elderly people. So RM will only affect residents' asset allocation after their retirement. We will study the category of residents who will benefit from the RM program, and to what extent retirees' welfare will improve. Using China's demographic data, our empirical study will provide useful insights into this question.

(2) The impact of RM on the life cycle and welfare of residents.

When the RM market matures, residents will anticipate the changes that RM products will have on their asset allocation as they age, and will plan for such changes when they are young as well. This allows us to study the optimal life-cycle asset allocation for an individual in a matured RM market. In fact, the impacts of RM on young residents' asset allocation are mixed. Since RM will increase the income of elderly people, it reduces the younger generation's burden of taking care of their parents and promotes consumption in their present time. However, RM also reduces their parents' bequest, it encourages young residents to consume less and save more. Which impact is more dominant depends on specific economic conditions. The empirical study will give us a clear picture of how RM will influence Chinese residents' life cycle consumption profile, and whether such changes can improve their life cycle welfare.

(3) The impact of RM on social welfare.

As will be shown in the results addressing the question above, when the RM market matures, the residents' life-cycle consumption and asset allocation would change. We will further study how such changes affect the optimal tax rate in the social security program. More precisely, we will explore whether the optimal tax rate can be reduced and whether the social welfare improves.

In summary, Question (1) investigates the welfare effects of RM from an individualistic and short-term perspective. Question (2) discusses the welfare effects of RM from an individualistic and long-term perspective. While Question (3) focuses on the welfare effect of RM from a social and long-term perspective.

1.3 Related Literature

1.3.1 Industry Practices

RM as a subject has attracted a great deal of attention from academics. One of the most important and basic questions most scholars are concerned with is whether the RM market deserves financial support and encouragement from the government. Mayer and Simons (1994) investigated people who are: (1) older than 62；(2) own home equity；(3) have relatively low current income; (4) have no bequeath motivation (no children); (5) live alone; (6) have no debt; (7) have been living in their current home for more than 10 years (no strong moving intention). Their study shows that, through the RM product, six million U.S. residents could get a 20% increase in their annual cash income, and 1.4 million of them can get out of poverty[①]. Of five million residents from the category, the mortgage income paid in a lump sum is twice as much as their total liquid assets. Merrill, Finkeland and Kutty (1994) stated that 18% of the elderly people who used to live below the poverty line can get enough cash income to keep them above poverty. Kutty (1998) showed that RM in the U.S. can shift the proportion of population below the poverty line by 29%. Rasmussen, Megbolugbe and Morgan (1995), and Mitchell and Moore (1998) presented further evidence for the positive effects of the RM market on the social security system.

With the nature of RM contracts, and only the RM recipient’s house equity as collateral, the institute issuing RM faces multiple risk types causing the contracts to be expensive. The risks associated to RM mainly come from interest rate risk, housing price risk, adverse selection, longevity risks, and moral hazards.

① According to the official report from the United States Census Bureau (http://www.census.gov/hhes/www/poverty/data/threshld/index.html), the poverty threshold, or poverty line, is the minimum level of income deemed adequate in a particular country.

Boehm and Ehrhardt (1994) believe that RM is a long term contract, where small fluctuations in the interest rates affect the issuing institute drastically. Chinloy and Megbolugbe (1994) found that pricing of a RM contract is mainly on the value of house equity, which may cause the issuing institute to suffer losses when the housing market fluctuates, this risk is hard to diversify through derivatives. Chen, Chang, Lin and Shyu (2010) had used market data from 1986 till 2008 proved that the volatility of house price affects the pricing of RM. Davidoff and Welke (2004) says that RM consumers get more money from the contract if they live longer, this will naturally attract people with better health conditions, and who tend to have a longer expected lifetime. Longevity risk is evident when considering that RM contracts are long term making it time sensitive, and would usually encounter an adverse selection where more people in better health are attracted, while the contract enables an improvement in living standard would increases their lifespan, pricing a RM contract for unforeseen increasing payments becomes very complex. Miceli and Sirmans (1994), Shiller and Weiss (2000) mentions that moral hazard exists when a participant signing a RM contract foregoes the incentive to upkeep the residence after the property right has been transferred, causing acceleration in the depreciation of the house value. However, institutes may reasonably increase the price of RM to offset the risk inherent to a RM contract.

Many other researchers also noted the high expense of RM contracts. Caplin (2000) states that transaction costs is about 15% of the total RM value. Leviton (2002) states that the typical cost for a median HECM loan of $105,000 USD is estimated at $5,400 USD. High expense is also affected by model risk, such as China being a developing country where not much data is available, with the effects of urbanization and industrialization baring unknown consequence. All of these points act as a driving factor for the high RM expense.

1.3.2 Relation to Life-cycle model

In welfare economics, the life-cycle model is often used for studying consumers' decisions throughout their lives or in some specific periods. The life-cycle model was first presented by Modigliani and Brumberg (1954) for studying consumers' savings behavior in different periods. Since consumers' decisions among different periods are not independent, the life-cycle model is quite suitable for studying income distribution, consumption and savings, investment, housing, security, etc. The basic idea behind the life-cycle model is that the asset allocation of investors will affect their continuing life. So, the life-cycle model is not only used to analyze consumers' lifetime decisions, but also for studying asset allocation within specific periods, such as used by Michelangeli (2007).

The life-cycle model is the standard analytical framework to study household finance. Research on household finance mainly focuses on finding ways for people to optimize their consumption choices with various financial products over the whole lifetime. Cocco (2005) confirms the crowding out effect of house price risk on stock investment, and further found that this effect is more significant or younger and poorer investors. Yao and Zhang (2005) found that residents with home equity will have a higher proportion of risky-assets to liquid assets. Li and Yao (2007) documented that the fluctuation in home equity values would have significant influence on residents' life-cycle consumption and welfare. Many other studies are currently being conducted on household asset optimization in China. For example, Yang and Chen (2006) studied the optimal savings decision problem using a dynamic simulation model. Zhao and Zeng (2006) studied the urban residents' optimal investment portfolio. Huang and Liu (2007) modelled inter-temporal optimal decision-making on house leasing and purchasing. These researches mainly studied optimal decisions from the perspective of consumption-investment allocation, while leaving out RM products and the resulting welfare effects.

Michelangeli (2007) first brought RM into the study of residents' consumption-investment decisions based on the life-cycle framework. He made careful investigations into the welfare change of retired people who participated in the RM market. He used data from the U.S. Health and Retirement Study and Consumption and Activities Mail Survey Database (2000-2005). The targeted group for the RM products was single elderly people (older than 62) who have retired, owned home equity, and had a complete consumption and investment record. The model was built on the life-cycle of 64 to 95 years old and has the following characteristics: (1) residents' preferences regarding housing consumption and non-housing consumption is described by the

Cobb-Douglas utility function; (2) it takes the maintenance cost, transaction cost, and moving cost into account when determining the residents' budget constraint; and (3) residents' welfare gain is defined as the equivalent increase of initial non-housing asset brought about by the RM contract. Michelangeli (2007) stated that RM can possibly reduce the welfare of some market participants. However, his research has two shortcomings. The first one is the author assumes elderly people only participate in the RM program at the beginning of their retirement and does not consider their ability to choose the timing of participation. Second, the author only focuses on the consumption and asset allocation of elderly people, and does not address the influences of RM on young people. As stated above, when the RM market becomes mature, it will also influence residents' asset allocation during their younger years if they anticipate that they will participate in RM when older. This research overcomes these shortcomings by allowing people to choose the best time to participate in the RM program and extending the life cycle to include the younger periods.

1.3.3 Relation to OLG model

The OLG model is used for studying allocation efficiency and welfare results for consumers in different generations. This model was presented by Samuelson (1958) for studying consumption and economic growth. Then, Diamond (1965) introduced government debt and capital accumulation into this model to discuss the reasonable size of government liability. Similar to the life-cycle model, the OLG model also became one of the important welfare models for studying social resource allocation. The OLG model is also used in social security tax rate studies. Longevity risk and population structure were introduced into the model to make a more realistic study of the tax rate design. Fehr (2013) provides a detailed exploration of this topic. Feldstein (1985) first used the OLG model in a study of the optimal social security tax rate and welfare changes.

As the RM market will change people's consumption profile during their life-cycle, it will naturally result in changes in resource allocation and welfare gain of each generation. From the social perspective, Coleman (2010) built an overlapping generation (OLG) model to study the welfare influence of the RM market with longevity risk. A randomly selected resident was assumed to live for four time periods, 0-25 years old, 25-40 years old, 40-65 years old, and 65-90 years old. Residents have to decide on housing investment between 25 years old and 65 years old. They will possess home equity before the age of 65 if they chose to purchase the house. Housing quality can be separated into two categories, high and low, which are both related to housing price. Young residents are divided into two groups, depending on whether they will receive bequest from their parents. Elderly residents are also divided into two categories, according to their bequest motive. Housing prices are determined using the balance in supply and demand of production and housing. Applying market data from New Zealand, Coleman found that the main influence of the RM market is that it makes the residents in every generation invest more in high-quality housing instead of leasing or purchasing low-quality housing. There is an 11% increase in investment in high-quality housing for the residents who have an expected remaining lifetime of 20 years. The influence of the RM market on the working ages depends on their type. For those who will not receive a bequest from their parents, participating in the RM market encourages their consumption in the earlier period of life because RM reduces precautionary savings. However, for those who will receive bequest from their parents, the RM market may decrease their consumption in their earlier part of life because their parents may use RM to increase their living standards and as a result reduce the bequest. The empirical results based on the market data from New Zealand shows that the young between 20 years old and 25 years old will reduce their investment in high-quality housing to increase their consumption, whereas the older people between 40 years old and 65 years old will increase their investment in high-quality housing to prepare for the RM program.

Coleman (2010) made a good attempt at investigating how the RM market will influence the residents' consumption profile in each generation. His paper focused on analyzing the influence of the RM market on residents' housing choices, but not on welfare changes. The study was based on New Zealand, and the interpretation of results cannot be straightforwardly translated for China's usage. This paper will study the impacts of RM on individual welfare and social welfare based on the demographic and economic characteristics of China.

In essence, the basic idea of these models is to simulate the rational behavior in financial market, and find the optimal solution based on budget constraint. The model setup in this research follows this direction.

The remainder of this paper is organized as follows. In Chapter 2, we study the effects of RM on the welfare of retired residents, where we allow retired residents to choose the best time to participate in the RM program. Chapter 3 extends the life-cycle model to include the younger periods, which studies how RM affects residents’ consumption at different ages, the resulting life cycle welfare changes, and formulate an OLG model to illustrate how RM affects the optimal tax rate and social welfare. We conclude this paper in Chapter 4.

Chapter 2 The Welfare Effects of Reverse Mortgage on Retired Residents

In this chapter, we extend the framework of Michelangeli (2007) and Cocco (2005) by formulating a life-cycle asset allocation model to study how reverse mortgage influences the welfare of residents and the ideal time of entering a contract. We will incorporate Chinese demographic and economic data into the model, and perform an in-depth empirical study.

Throughout this chapter, we measure the welfare of retired residents via the sum of their discounted inter-temporal consumption utility and the bequest utility, as illustrated in Equation (2.13) below.

2.1 Model Setup

The consumption-asset allocation process of retired residents in our life-cycle framework is illustrated in Figure 2.1.

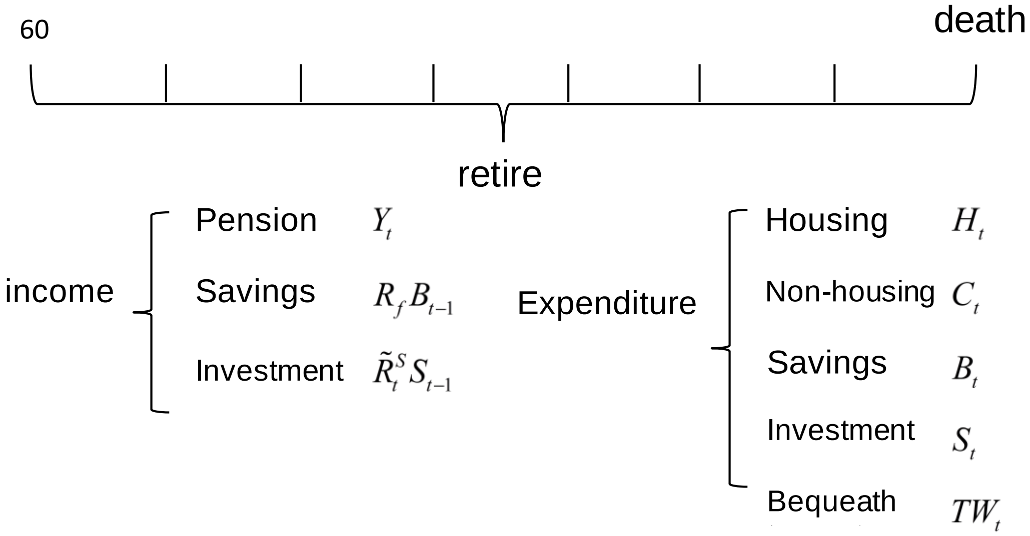

Figure 2.1 The consumption-asset allocation process of retired residents in a life-cycle framework

The utility of a retired resident is determined by housing consumption, non-housing consumption, savings, investment, and bequest. A retiree may receive income from the pension, financial assets and housing assets. Retired residents have to make consumption decisions and investment decisions. They strive to maximize their expected utility by optimizing their asset allocations. Residents will participate in the reverse mortgage market only if it maximizes their individual welfare. Key variables in our model of this chapter are shown in Table 2.1.

Table 2.1 Key variables in the life-cycle model

Variables	Meaning
Y_t	Income of retired residents in time t
H_t	Average housing size of retired residents in time t
I_t^o	Indicator of retired residents ownership of housing in time t
I_t^m	Indicator of retired residents forced to move in time t
C_t	Non-housing consumption of retired residents in time t
S_t	Risky assets of retired residents in time t
B_t	Non-risky assets of retired residents in time t
I_t^r	Indicator of retired residents' participation in reverse mortgage in time t
D_t^r	Lump-sum payment of a reverse mortgage to retired residents in time t
W_t	Initial wealth of retired residents in time t
G_t	Cumulative value of reverse mortgage loans of retired residents in time t

Throughout this chapter, we make the following assumptions.

Assumption 1: Retirement starts at 60, and the life-cycle of a retiree is 60 years old to 94 years old. Each cycle contains 7 time periods, and each period lasts for 5 years. t represent the period: $t=1$ denotes the time period when the retiree is 60 to 64 years old, $t=2$ denotes the time period when the retiree is 65 to 69 years old, and so on.

Assumption 2: Retirees live no more than 7 periods, and the probability of their remaining life is defined by the survival function:

$$N(t) = \prod_{j=1}^{t} n_j \tag{2.1}$$

where n_j is the conditional probability of the retiree who is alive in period $j-1$ will be still alive in period j （$j = 1, \dots, T$）.

Assumption 3: The income of the retired residents in each period is a fixed proportion to their income of the last period Y.

$$Y_t = \lambda Y, \qquad \lambda \in (0,1) \tag{2.2}$$

Assumption 4: There are two kinds of housing consumption. One is living in a rented house, and the other is buying a house. The housing consumption level is measured by the living space H_t (square meters), and there is a minimum living space requirement for each resident, $H_t \geq H_{\min}$.

Assumption 5: For the same living space, the utility from owning home equity will be higher than renting the house. This assumption will be in force by fixing the preference parameter on owning a house to be higher than renting, i.e. $\theta_{own} > \theta_{rent}$. Residents' housing consumption in period t will be denoted by an indicator function I_t^o. $I_t^o = 1$ if residents own home equity, while $I_t^o = 0$ if residents rent the house. The maintenance cost of the purchased house will be φ_{own} per square meter, while the rental cost is φ_{rent} per square meter.

Assumption 6: Residents face an exogenous moving shock in period t, denoted by a random dummy I_t^m ($I_t^m = 1$ when residents have to move in period t, and $I_t^m = 0$ when they don't) ①. The probability of moving is related to the residents' age, and is denoted by q_t^m. Also, residents can sell their home equity to increase their expected utility, which will be denoted by I_t^s ($I_t^s = 1$ when residents sell the home equity in

① Motives of moving include the need for help, lack of cash, willingness to change environment, allergy to the climate change, serious illness and willingness to live with their children.

period t, and $I_t^s = 0$ when residents stay). When a resident has to move in period t, the house will be sold, and a transaction cost of $\psi_{own} P_t^H H_t$ will be incurred.

Assumption 7: The rate of return of house prices $\tilde{R}_t^H = P_t^H / P_{t-1}^H$ is denoted by:

$$\ln\left(\tilde{R}_t^H\right) = \mu_H + \tilde{\varepsilon}_H \tag{2.3}$$

where μ_H is the expected rate of return on house prices, $\tilde{\varepsilon}_H$ is the residual term that follows a normal distribution of $N(0, \sigma_H^2)$

Assumption 8: Residents have a minimum non-housing consumption constraint such that the ratio of non-housing consumption over the income has to be larger than a lower bound $C_t / Y_t \geq c_{\min}$.

Assumption 9: There are only two assets available in the financial market. One is risk free and the other is risky. The rate of return of the risk free asset is a constant, denoted by B_t, while the rate of return of the risky asset is random, described by

$$\ln\left(\tilde{R}_t^s\right) = \mu_s + \tilde{\varepsilon}_s \tag{2.4}$$

where the expected rate of return is μ_s, and $\tilde{\varepsilon}_s$ is the residual randomness, which follows a normal distribution $N\left(0, \sigma_s^2\right)$. The correlation between the rate of return of risky asset and that of the growth rate of house price is ρ_{HS}.

Assumption 10: Only residents with fully owned home equity can participate in the reverse mortgage program. We used a dummy variable to characterize residents' RM decision. $I_t^r = 1$ if the resident participates in a reverse mortgage program in period t, and $I_t^r = 0$ otherwise.

Assumption 11: The reverse mortgage contract will pay the resident a lump sum amount[①] of D_t^r, and the transaction cost will be $\psi_{rev} D_t^r$. The interest rate of the RM product is $R_L - 1$, and the value of the contract in period $t+j$ is $G_{t+j} = D_t^r \cdot R_L^j$.

Assumption 12: In each period t, residents have a non-housing consumption C_t, and

① Annuity payments of reverse mortgage loan is another common type. However, since we assume that the retired residents are fully rational and they can annuitize the payment in a utility-maximizing way, our qualitative insights do not depend on whether the payment is annuitized.

a housing consumption H_t. The residents' utility in period t is characterized by a Cobb-Douglas function:

$$u(C_t, H_t) = \frac{\left(C_t^{1-\theta} H_t^{\theta}\right)^{1-\gamma}}{1-\gamma} \tag{2.5}$$

where γ is the relative risk aversion parameter, and θ is residents' relative preference on housing consumption. The larger the θ is, the more the resident places importance on housing consumption.

Assumption 13: If the resident passes away at the end of period t, the bequest is $B(TW_t)$. Following Li and Yao (2007), the bequest utility function is described by

$$B(TW_t) = L\frac{\left[TW_t\left(\theta / \varphi_{rent} P_t^H\right)^{\theta}(1-\theta)^{1-\theta}\right]^{1-\gamma}}{1-\gamma} \tag{2.6}$$

In this equation, L measures the relative importance of the bequest. The larger the L is, the more a bequest is preferred.

Under Assumptions 1 to 13, the wealth at the beginning of time period t is

$$\begin{aligned} W_t = {} & I_{t-1}^o\left[I_{t-1}^r \max\left(0, P_t^H H_{t-1}(1-\psi_{own}) - G_t\right) + \left(1-I_{t-1}^r\right) P_t^H H_{t-1}(1-\psi_{own})\right] \\ & + R_f B_{t-1} + \tilde{R}_t^s S_{t-1} \end{aligned} \tag{2.7}$$

It is seen from this equation that, if residents participating in the RM program move out, they have to either repay the debt of the RM or give the house to the related financial institution. The net cash value the retiree can receive if they move out is $\max\left(0, P_t^H H_{t-1}(1-\psi_{own}) - G_t\right)$. The total disposable wealth in period t is:

$$Q_t = W_t + Y_t \tag{2.8}$$

where Y_t is the income, and we assume that the real income of retired residents will be the same in each period. The budget constraint in period t is:

$$\begin{aligned} Q_t = {} & C_t + B_t + S_t + \left(1-I_{t-1}^o\right) P_t^H H_t\left[I_t^o(1+\varphi_{own}) + \left(1-I_t^o\right)\varphi_{rent}\right] \\ & + I_{t-1}^o\left[I_t^m + \left(1-I_t^m\right) I_t^s\right] P_t^H H_t\left[I_t^o(1+\varphi_{own}) + \left(1-I_t^o\right)\varphi_{rent}\right] \\ & + I_{t-1}^o\left(1-I_t^m\right)\left(1-I_t^s\right)\left(1+\varphi_{own} + I_t^r m_{t-1} O_{rev} - \psi_{own}\right) P_t^H H_{t-1} \end{aligned} \tag{2.9}$$

where $m_{t-1} = G_t / P_t^H H_{t-1}$, $O_{rev} = \psi_{rev} - 1$ if residents participate in the RM at the current period and $O_{rev} = -1$ if residents participate in the RM in the past period

Within the budget constraint, residents maximize their utility function

$$\begin{aligned} V &= \max_{A_t} E\sum_{t=1}^{T}\left\{\beta^t\left[N(t)u(C_t,H_t)+\left(N(t-1)-N(t)\right)B(Q_t)\right]\right\} \\ A_t &= \left\{C_t,H_t,B_t,S_t,I_t^o,I_t^s,I_t^r\right\} \qquad t=1,\dots T \end{aligned} \tag{2.10}$$

In this equation, E[.] is the expectation operator, V[.] is the expected utility value, A_t is the set of decision variables, β is the utility discount factor, and $N(t)$ is the probability that a resident will be able to survive till period t. Let X_t be the set of state parameters in period *t*, where $X_t=\left\{I_{t-1}^o,I_{t-1}^r,I_t^m,P_t^H,H_{t-1},G_{t-1},Q_t\right\}$. Then the Bellman equation for the optimization problem (2.10) becomes

$$V_t(X_t)=\max_{A_t}\left\{n_t u(C_t,H_t)+(1-n_t)B(Q_t)+n_t\beta E_t\left[V_{t+1}(X_{t+1})\right]\right\} \tag{2.11}$$

In order to reduce the computational burden associated with simulation, we follow Yao and Zhang (2005) to simplify the Bellman equation. We normalize the decision variables and state variables by using the total wealth in each period Q_t. Then we get the non-housing consumption ratio $c_t=C_t/Q_t$, the housing consumption ratio $h_t=P_t^H H_t/Q_t$, the risk free asset ratio $b_t=B_t/Q_t$, and the risky investment ratio $s_t=S_t/Q_t$. With such normalization, the decision and state variables reduce to $a_t=\left\{c_t,h_t,b_t,s_t,\mathrm{I}_t^o,\mathrm{I}_t^s,\mathrm{I}_t^r\right\}$, $x_t=\left\{I_{t-1}^o,I_t^m,\mathrm{I}_{t-1}^r,w_t,\bar{h}_{t-1},\bar{m}_{t-1}\right\}$, where $w_t=W_t/Y_t$ is the ratio of assets to income, $\bar{h}_{t-1}=P_t^H H_{t-1}/W_t$ is the ratio of residents' housing value to assets, and $\bar{m}_{t-1}=G_{t-1}R_L/P_t^H H_{t-1}$ is the ratio of initial reverse mortgage to housing value. We define the ratio of income to total wealth as $y_t=Y_t/Q_t$, and the ratio of current reverse mortgage to house value as $m_t=G_t/P_t^H H_t$. Then the residents' total wealth growth rate will be $q_{t+1}=Q_{t+1}/Q_t$.

With the above notation, the budget constraints (2.9) can be rewritten as

$$\begin{aligned} 1 &= c_t+b_t+s_t+h_t\left(1-I_{t-1}^o\right)\left[I_t^o\left(1+\varphi_{own}\right)+\left(1-I_t^o\right)\varphi_{rent}\right] \\ &\quad +h_t I_{t-1}^o\left[I_t^m+\left(1-I_t^m\right)I_t^s\right]\left[I_t^o\left(1+\varphi_{own}\right)+\left(1-I_t^o\right)\varphi_{rent}\right] \\ &\quad +\bar{h}_{t-1}(1-y_t)I_{t-1}^o\left(1-I_t^m\right)\left(1-I_t^s\right)\left(1+\varphi_{own}+I_t^r\bar{m}_{t-1}O_{rev}-\psi_{own}\right) \end{aligned} \tag{2.12}$$

We rewrite the objective function as $v_t(x_t)=V_t(X_t)/\left[\left(Q_t/\left(P_t^H\right)^\theta\right)^{1-\gamma}\right]$. Then the residents' optimization program is transformed to be

$$v_t(x_t)=\max_{a_t}\left\{n_t\left[u(c_t,h_t)+\beta E_t\left[v_{t+1}(x_{t+1})q_{t+1}^{1-\gamma}\left(\tilde{R}_{t+1}^H\right)^{\theta(\gamma-1)}\right]+(1-n_t)B(1)\right]\right\} \tag{2.13}$$

$$s.t. \qquad c_t \geq y_t c_{\min},\ h_t \geq y_t h_{\min},\ b_t \geq 0,\ \mathrm{s}_t \geq 0 \tag{2.14}$$

where $c_{\min}$ is the minimum non-housing consumption, and $h_{\min} = P_t^H H_{\min} / Y_t$ is the minimum housing consumption.

All the optimizations throughout this paper will have to be solved by simulations. To proceed, we discretize the variable $w_t = W_t / Y_t$ by dividing its value interval [0,10] into ten equal-length grids, discretize $\bar{h}_{t-1} = P_t^H H_{t-1} / W_t$ by dividing its value interval [0,1] into 5 equal-length grids, and discretize $\bar{m}_{t-1} = G_{t-1} R_L / P_t^H H_{t-1}$ by dividing its value interval [0.2,0.8] into four equal-length grids. At the terminal time T, the resident's value function equals to the bequest utility $B(1)$. A cubic spline interpolation will be used to generate the value of the utility function when the decision variables and state variables fall within the corresponding intervals. To calculate the expected value of the utility function, we follow Rust (1997) to generate two continuous random variables according to a two-dimensional normal distribution, with one dimension representing the randomness of house prices, and the other dimension representing the randomness of risky returns. We calculate the expected value of the utility function by averaging the utility levels of different random paths. For each random path, we use backward induction to derive the optimal decisions in each time period.

2.2 Variables and Parameters

The parameters based on China's demographic and economic characteristics to be used in the simulations will be defined in this chapter. The baseline model parameters and settings are summarized in Table 2.2, alternative specifications will be considered later in the robustness checks.

Table 2.2 Baseline parameters

Parameters	**Symbol**	**Value**
Mortality rate	n_j	Appendix B
Relative risk aversion	γ	2
Housing preference for purchaser	θ_{own}	0.55
Housing preference for renter	θ_{rent}	0.5
Utility discount factor	β	$0.99^5 = 0.9510$
Bequest factor	L	1
Mean growth rate of house price	μ_H	$3.89\% * 5 = 19.45\%$
Variance of house price growth	σ_H^2	$(2.87\%)^2 * 5 = 0.411\%$
Correlation of house price to risky assets	ρ_{HS}	0
Minimum living space	$H_{\min}$	$35m^2$
Maintenance cost for house owners	φ_{own}	$1.5\% * 5 = 7.5\%$
Rental cost for house renters	φ_{rent}	$4\% * 5 = 20\%$
Transaction cost of house purchase	ψ_{own}	5%
Reverse mortgage loan commitment	m	50%
Transaction cost of reverse mortgage loan	ψ_{rev}	15%
Reverse mortgage interest rate	R_L	$1.04^5 = 1.2167$
Moving risk	q_t^m	0
Pension replacement rate	λ	60%
Minimum non-housing consumption	$c_{\min}$	20%
Return on Risky assets	μ_s	$4.58\% * 5 = 22.9\%$
Variance of the return on risky assets	σ_s^2	$(25.12\%)^2 * 5 = 31.55\%$
Risk-free rate	R_f	$1.01^5 = 1.051$

(1) Time periods in Life-cycle

We assume that the retired residents' life-cycle consists of seven periods, with each period covering 5 years. The maximum remaining lifespan is 35 years. According to the labor law in China, the official retirement age is 60 for males and 50 for females. However, this policy has not been strictly enforced. For example, starting in Oct 2010, many companies in Shanghai have employed a flexible retirement program. In these companies, the retirement age for males can be extended to 65, and the retirement age for females can be extended to 60. In this paper, we ignore such flexibility.

(2) Mortality Risk n_j

To estimate the mortality rate of residents, we employ the data set provided by "China's Population and Employment Yearbook 2010" (see Appendix B). Our estimates show that the expected remaining lifespan for a 60 year-old is 23.24[①] years.

(3) Relative risk aversion γ

Studies on investment consumption in the life-cycle theory usually define γ to be the relative risk aversion parameter. Michelangeli (2007) gave an estimate of 3.87 for retired residents in the U.S. who are under 64 years old. Some local researchers in China such as Chen, Yang and Fang (2005), studied the 1978-2002 consumption data and estimated this parameter to be 1.2875. Ai and Wang's (2005) in their empirical research on China's urban residents' consumption derives an estimate of 1.9139[②]. Following Ai and Wang (2005), we will have $\gamma = 2$ for this chapter.

(4) Housing preference parameter θ

For the parameter measuring residents' preference for housing, we follow Han (2008) in using $\theta_{own} = 0.55$ and $\theta_{rent} = 0.5$, indicating that Chinese residents usually prefer owning than renting a house.

(5) Utility discount factor β

The discount factor captures the time value preference of consumers. A higher discount factor shows that a consumer places much importance on future consumption. According to Chen, Fu, Ge (2006) and Han (2008), the utility discount factor for Chinese residents normally is $\beta = 0.99$.

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① The mortality rates in the table are the central mortality, Zhu and Chen (2009) gave precise formulas for the calculation.

② Derived from the results of formula (11) and parameters in (12) of Ai and Wang (2005)

(6) Bequest factor L

Following Yao and Zhan (2005), this paper uses a bequest intensity of $L = 1$.

(7) Parameters related to house prices $\mu_H, \sigma_H, d, \rho_{HS}$

According to the housing price index of China from 1998-2011, the mean and standard deviation of house price growth rates are 4.89% and 2.87% respectively. The depreciation rate of a house value is assumed to be $d = 1\%$ per year. The correlation between China's housing price index and the Shanghai Stock Exchange Index is found to be -0.11, which is not highly significant. So, we set the correlation between house price growth and return on risky assets ρ_{HS} to be 0. See Appendix C for more details.

(8) Minimum living space $H_{\min}$

In 2011, the China Academy of Social Sciences announced that the minimum living space for a person shall be 35 square meters. The per capital housing area under affordable housing offered by the government is usually between 28-45 square meters. Hence, we will assume $H_{\min} = 35m^2$.

(9) Parameters related to maintenance and rental cost φ_{own} φ_{rent} ψ_{own} ψ_{rent}

According to Zhu (2010), the maintenance cost φ_{own} is 1.5% of the house value per annum, and the rental cost φ_{rent} is 4% of the house value per annum. The transaction cost related to purchasing a house ψ_{own} is 5% of the house value, and none for leasing $\psi_{rent} = 0$[①].

(10) Loan amount, transaction cost, and interest rate of reverse mortgages m ψ_{rev} R_L

The loan amount of a reverse mortgage is commonly 50% of the house value under American standards, with a transaction cost of 14% of the loan amount (Chaplin, 2001). We assume that the transaction cost ψ_{rev} of reverse mortgage in China almost equals that in U.S. which is 15%. Since the supplier of reverse mortgage contracts would have to take on various long-term risks. The interest rates of reverse mortgage will be higher than the interest of a normal mortgage. We set R_L to 4% as the reverse mortgage interest rate.

① Usually one month's rental will be the cost of transaction for renting, which is insignificant.

(11) Moving risk q_t^m

Owing to that lack of data in China, the moving probability of U.S.[①] was used as a proxy to measure moving risk in China. A survey was made by our school on RM, with a sample of 2000 people between 60 and 80 years old in China. A question within the survey addressed the number of shifts a person makes in their lifetime, and the average number of shifting in a Chinese resident's lifetime is about 3. This figure is about 1/4 the U.S. average, and this is used to scale the average moving probability in U.S. to be more appropriate for China. This annual probabilities are then converted into a 5 year probability. See Appendix D for more details.

(12) Pension replacement rate λ

We assume that pension income after retirement will be 60% of the income in the period just before retirement. The real pension after retirement will stay the same.

(13) Minimum non-housing consumption $C_{\min}$

We set minimum non-housing consumption to be 20% of income, according to the proportion of minimum living standard to disposable income per capita in China. See Appendix E for more details.

(14) Saving and investment parameters for retired residents R_f μ_s σ_s

The average inflation rate from 1998 to 2011 in China is 2.13%, while the average nominal interest rate of a 5-year or longer term time deposit is 2.85%. So, we choose a long-term risk-free rate R_f of 1.0%, which will be used as the rate of return on savings. The average return μ_s and standard deviation σ_s of the Shanghai Stock Exchange Index from 1998-2011 is 4.58% and 25.12%, which will be set as the average return and standard deviation of risky assets. See Appendix C for more details.

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① Source: United States Census Bureau http://www.census.gov/hhes/migration/about/cal-mig-exp.html。

2.3 Simulations for the case without timing decisions

This subsection assumes that residents can only apply for reverse mortgage at the beginning of their retirement. This assumption captures the commonly used "Now-or-Never" decision rule and also helps simplify calculations. A more refined scenario will be that residents can apply for reverse mortgage in any time period. This scenario makes considering time of entry a sensible issue and will be studied in the next subsection. Residents who already own a house at the beginning of their retirement period have the following choices:

① sell the house and rent a new one;

② sell the house and purchase another one;

③ keep the house and don't participate in reverse mortgage;

④ keep the house and participate in reverse mortgage.

Amongst these choices, ④ allows the retired to repurchase their house by repaying the reverse mortgage loan and realizing the appreciation of house price, or receiving longevity risk protection provided by the reverse mortgage. However, choice ④ has a disadvantage in that the retired resident will have to pay the transaction costs and interest gains on the reverse mortgage loan, which are usually high. Moreover, retired residents might have to bear the consequence of moving risk. If such adverse events occur, residents choosing ④ may be paying high cost of the reverse mortgage without enjoying its benefits when compared to other choices.

We now assess the retired residents' utility-maximizing choices. During the retirement period of 60-64 years old, based on different ratios of asset to income $w_t = W_t / Y_t$ and of residents' housing value to asset $\overline{h}_{t\text{-}1}=\mathrm{P}_t^H H_{t-1} / W_t$, retired residents' optimal housing decisions are shown in the following table. Recalling that the ratio of asset to income $w_t = W_t / Y_t$ is higher if the resident's total assets are relatively larger, and the ratio of residents' housing value to asset $\overline{h}_{t\text{-}1}=\mathrm{P}_t^H H_{t-1} / W_t$ is higher if the resident is relatively poor in cash. The results are presented in Table 2.3.

Table 2.3 retired residents' optimal choice at the beginning of the retirement (60-64)

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	①	①	①	①	①
	2	①	①	①	④	④
	3	①	①	①	④	④
	4	①	①	③	④	④
	5	②	②	④	④	④
	6	②	②	④	④	④
	7	②	②	④	④	④
	8	②	②	④	④	④
	9	②	②	④	④	④
	10	②	②	④	④	④

Table 2.3 shows that:

(1) When retired residents have a high asset to income ratio (>=5) and a high house value to asset ratio (>=0.6), they are relatively wealthy in total assets but poor in cash. In this case, the reverse mortgage gives this category of retired residents the highest utility amongst the 4 options, making option ④ the optimal one. Given the low moving risk in China, retired residents with higher house value to income ratio will benefit from participating in the RM program.

(2) When residents have a high asset to income ratio (>=5) and a low house value to asset ratio (<=0.4), they are relatively wealthy in total asset and their cash is not too scarce. In this case, a small increase in cash holding would allow them to achieve the optimal allocation. Because of the high cost of reverse mortgage, this category of residents would prefer to sell the original house and purchase a smaller one. Keeping in mind that the cash they need is not too much, the new house they

purchase needs not to be too small. They thus avoid the cost of reverse mortgage by selling a small part of living space away.

(3) The category of residents who have a low asset to income ratio (<=4) and a lower house value to asset ratio (<=0.6), are poor in assets. In this case, the only feasible choice for these residents is to rent a house.

Table 2.4 welfare effects of reverse mortgage toward retired residents

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	--	--	--	--	--
	2	--	--	--	6.9	23.29
	3	--	--	--	3.29	27.43
	4	--	--	--	13.99	17.44
	5	--	--	7.08	19.1	15.15
	6	--	--	12.18	16.11	30.56
	7	--	--	8.39	35.63	101.32
	8	--	--	8.29	54.6	69.79
	9	--	--	15.96	43.65	25.18
	10	--	--	27.44	52.43	9.37

The values in the table above represents the % increase from the initial wealth, based on welfare equations used in this chapter, it follows the principle of equivalent utility with values converted to a % factor of initial wealth

The welfare gained through participating in reverse mortgage is calculated in Table 2.4. From this table, it is seen that retirees with mediocre asset to income ratio and a high house value to asset ratio receives the most benefit from a reverse mortgage. The welfare gain decreases as the house value to asset ratio decreases, or in other words, RM is less necessary as the retiree becomes richer in cash.

As a check of robustness, we now consider how the increase in lifetime expectation (longevity), the increase in moving risk, and the decrease in transaction cost influences the welfare function of the reverse mortgage. In the baseline scenario, we use the mortality rates as in Appendix B. To study the effect of increasing lifespans, we reduce the mortality rates as in Appendix B by 50% in our alternative scenario. Originally, the expected lifespan of a 60 year old retiree is another 23.24 years, with the increase in lifespan, the expected remaining lifetime of 60 year-olds is now 28.15 years. The retired residents' optimal choice is shown in Table 2.5, and the corresponding improvement in welfare is shown in Table 2.6.

Table 2.5 retired residents' optimal choice when expected lifespan increases

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	①	①	①	①	①
	2	①	①	①	①	④
	3	①	①	①	④	④
	4	②	②	④	④	④
	5	②	②	④	④	④
	6	①	①	④	④	④
	7	①	①	④	④	④
	8	①	①	④	④	④
	9	①	④	④	④	④
	10	①	④	④	④	④

Table 2.6 welfare effects of reverse mortgage toward retired residents with increase in lifespan

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	--	--	--	--	--
	2	--	--	--	--	1.62
	3	--	--	--	5.37	11.01
	4	--	--	0.53	11.55	53.32
	5	--	--	6.8	44.86	46.92
	6	--	--	13.09	40.95	38.28
	7	--	--	20.08	34.18	39.07
	8	--	--	24.79	43.51	38.92
	9	--	1.42	21.58	68.69	71.09
	10	--	6.32	20.54	114.84	101.02

The values in the table above represents the % increase from the initial wealth, based on welfare equations used in this chapter, it follows the principle of equivalent utility with values converted to a % factor of initial wealth

Relative to the baseline scenario, Tables 2.5 and 2.6 shows that as expected lifespan increases, retirees would have higher welfare from participating in reverse mortgage, and hence they are more likely to participate. Welfare improvement is more significant for retirees with a higher house value to asset ratio.

In the baseline scenario, we assume that there is no moving risk (moving probability=0) after retirement. To study the effects of increased moving risk, we use the moving risk probability from Appendix D for this alternative scenario. The retired residents' optimal choice is shown in Table 2.7, and the corresponding improvement in welfare is shown in Table 2.8.

Table 2.7 retired residents' optimal choice when moving risk increases

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	①	①	①	①	①
	2	①	①	①	①	④
	3	①	①	①	④	④
	4	①	①	③	④	④
	5	②	②	④	④	④
	6	②	②	④	④	④
	7	②	②	④	④	②
	8	②	②	④	④	②
	9	②	②	④	④	②
	10	②	②	④	④	②

Table 2.8 welfare effects of reverse mortgage toward retired residents when moving risk increases

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	--	--	--	--	--
	2	--	--	--	--	4.02
	3	--	--	--	1.06	38.25
	4	--	--	--	3.15	43.07
	5	--	--	1.28	23.27	47.39
	6	--	--	2.06	43.55	25.55
	7	--	--	5.26	37.5	--
	8	--	--	6.43	8.63	--
	9	--	--	16.07	12.09	--
	10	--	--	26.87	9.53	--

The values in the table above represents the % increase from the initial wealth, based on welfare equations used in this chapter, it follows the principle of equivalent utility with values converted to a % factor of initial wealth

Relative to the baseline scenario, Table 2.7 shows that as moving risk becomes more significant, retirees have less incentive to participate in reverse mortgage. For those who are low in both house value to asset ratio and asset to income ratio, the optimal choice is to rent a house. Welfare improvement from reverse mortgage is also less when considering moving risk, as shown in Table 2.8.

Finally, we consider the effects toward wealth when we assume that the transaction cost of the reverse mortgage is reduced to zero (that is, the transaction cost is paid by the government). The retired residents' optimal choice is shown in Table 2.9, and the corresponding improvement in welfare is shown in Table 2.10. From Table 2.9, as transaction cost on reverse mortgage decreases, the negative impact of reverse mortgage reduces, making the reverse mortgage more attractive, hence increasing the participation in reverse mortgage. The welfare gain through participating in reverse mortgage also dominates those with a non-zero transaction cost shown in Table 2.4.

Table 2.9 retired residents' optimal choice when the transaction cost is 0

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	①	①	①	①	①
	2	①	①	①	①	④
	3	①	①	①	④	④
	4	①	①	④	④	④
	5	②	②	④	④	④
	6	②	②	④	④	④
	7	②	②	④	④	④
	8	②	②	④	④	④
	9	②	④	④	④	④
	10	②	④	④	④	④

Table 2.10 welfare effects of reverse mortgage toward retired residents when transaction cost is 0

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	--	--	--	--	--
	2	--	--	--	--	2.64
	3	--	--	--	19.56	30.08
	4	--	--	7.43	12.73	28.82
	5	--	--	9.2	18.69	26.14
	6	--	--	13.43	22.71	86.34
	7	--	--	13.97	36.42	164.88
	8	--	--	17.05	50.32	145.64
	9	--	0.38	28.16	61.02	28.36
	10	--	1.29	40.66	42.67	16.92

The values in the table above represents the % increase from the initial wealth, based on welfare equations used in this chapter, it follows the principle of equivalent utility with values converted to a % factor of initial wealth

2.4 Simulations for the case with freedom in timing decisions

This subsection relaxes the assumption that the retiree only can participate in the RM program at the beginning of retirement. This relaxation allows us to test the benefit of varying the time of entry. The simulation results with the most optimal timing decisions are shown in Table 2.11. Compared with the situation in which residents can only apply for reverse mortgage at retirement (Table 2.3), optimal housing choice has higher significance when retirees can apply for reverse mortgage at any time.

Table 2.11 retired residents' optimal choice at the beginning of retirement with relaxed timing

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	①	①	①	①	①
	2	①	①	①	②	④
	3	②	②	②	②	④
	4	②	②	②	④	④
	5	②	②	②	④	④
	6	②	②	④	④	④
	7	②	②	②	④	④
	8	②	②	②	④	②
	9	②	②	②	③	②
	10	②	②	②	③	②

For the residents whose asset to income ratio is high (>=9) and has a house value to asset ratio that is high (≥0.8), the optimal choice is not to participate in reverse mortgage at the beginning of the retirement. They can benefit by postponing their participation of reverse mortgage due to high likelihood of the house appreciating. Table 2.12 shows the periods where different categories of retired residents should participate in reverse mortgage. In this Table, the house rich cash poor residents can improve welfare by participating in reverse mortgage not immediately at retirement, but at a later period.

Table 2.12 optimal periods that retired residents should participate in reverse mortgages

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	--	--	--	--	--
	2	--	--	--	--	60-64
	3	--	--	--	--	60-64
	4	--	--	--	60-64	60-64
	5	--	--	--	60-64	60-64
	6	--	--	60-64	60-64	60-64
	7	--	--	--	60-64	60-64
	8	--	--	--	60-64	65-69
	9	--	--	--	70-74	70-74
	10	--	--	70-74	70-74	70-74

"60-64" indicates that retired residents should participate in reverse mortgage between ages of 60 to 64. "--" is the category of retirees who should not participate in reverse mortgage in any period.

Table 2.13 shows the expected level of welfare gained by retirees who should enter in a reverse mortgage at any period. Compared with the baseline scenario (Table 2.3) where reverse mortgage is only applicable at retirement, with optimally timed participation, more retired residents will receive welfare improvements. Especially, when the asset to income ratio is low and house value to assets ratio is high, the welfare gain is much higher than the former case where freedom to participate is restricted.

Table 2.13 welfare effects of reverse mortgage with relaxed timing

		house value to asset ratio				
		0.2	0.4	0.6	0.8	1
asset to income ratio	1	1.35	1.35	1.35	1.35	1.35
	2	2.48	2.48	2.48	6.9	23.29
	3	4.99	4.99	4.99	4.99	27.43
	4	9.2	9.2	8.05	13.99	17.44
	5	9.89	9.89	8.34	19.1	15.15
	6	10.24	10.24	12.18	16.11	30.56
	7	19.21	19.21	17.31	35.63	101.32
	8	40.94	40.94	38.87	54.6	69.79
	9	37.44	37.44	35.11	44.32	37.44
	10	41.27	41.27	38.41	52.43	41.27

The values in the table above represents the % increase from the initial wealth, based on welfare equations used in this chapter, it follows the principle of equivalent utility with values converted to a % factor of initial wealth

We also considered how the increase in expected lifespan (longevity), the increase in moving risk, and the decrease in transaction cost influences the welfare function of residents with reverse mortgage in the case of relaxed timing as a robustness check. The results with these alternative assumptions are quite similar to those obtained in the last subsection, and are thus omitted.

2.5 Summary

This chapter studied the effect of retirees' optimal choice and welfare improvement when reverse mortgage is available as a complementary instrument to annuitization. Our simulations show that when reverse mortgage is only available at retirement, not all retirees can benefit from it. Especially, when retirees are rich in asset but poor in cash, the optimality of reverse mortgage depends on the extent of cash shortage. When cash shortage is more eminent, reverse mortgage is the optimal choice because it offers cash without reducing the retiree's living space. However, when the cash shortage is not too significant, reverse mortgage is less appealing due to its high costs. In this case, the optimal choice is to liquidate the original house and make a re-allocation of wealth between living space and cash. When cash shortage is not too urgent, the retiree can still enjoy a reasonably large living space.

Through sensitivity analysis we found that the welfare enhancing function of reverse mortgage becomes more substantial when expected lifespan increases, or when moving risk or transaction cost decreases. We also studied the optimal time of RM participation, and results show that house rich cash poor retirees can improve welfare by entering reverse mortgage later in retirement. On average, the best time of entry is about 5 years after retirement, due to a high likelihood of house value appreciation.

Chapter 3 The Welfare Effects of Reverse Mortgage on Individual and Society

Building on Feldstein (1985) and Cocco (2010), this chapter introduces housing and reverse mortgage into the overlapping generation (OLG) model to explore the optimal asset-consumption allocation over the whole lifetime of a resident, the optimal social security tax rate in an economy that is in a steady state, and then evaluate the effects of reverse mortgage on individual and social welfare.

Throughout this chapter, we measure an individual's welfare in by summing up utility in one's young, middle-age and elderly periods of one's lifetime, as well as the utility from bequest, as given in Equation (3.12). Social welfare is defined as the sum of utilities of all generations at a specific point in time, as illustrated in Equation (3.11) below.

3.1 Model Setup

Residents are split into three generations: youths, middle-aged, and elderly. Among them, the youths and middle-aged constitute the working population, and the elderly are the retirees. The life-cycle consumption and savings of each generation is illustrated in Figure 3.1. The youths of period $t-2$ are the elderly in period t, with income and expenditures described in the left column of Figure 3.1. The youths of period $t-1$ are middle-aged in period t, with income and expenditure parameters in the middle column. Youths in period t are presented in the right column. In China, considering the longevity of people, youths would unlikely obtain their parent's bequest until they reach middle age, so we assume that bequest is only bestowed upon during middle age.

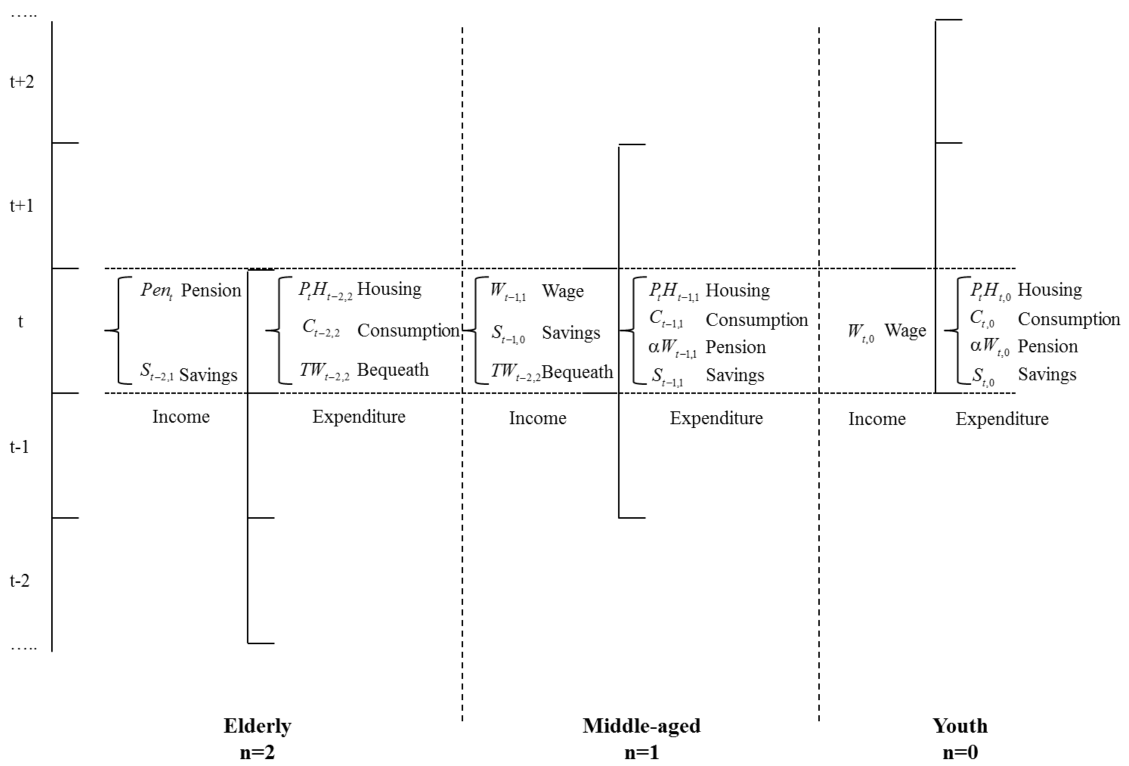

Figure 3.1 asset-consumption allocation of each generation under an overlapping generation framework

We now incorporate China's economic and demographic characteristics, such as “getting old before becoming rich”, “housing price boom”, and “one-child policy” into the OLG model. “Getting old before becoming rich” has two aspects, one, heavy burdens on the younger generation to care for the elderly. Two, a smaller bequest will be left behind by the elderly. “China housing price boom” is an effect that would cause youths to rent a house rather than buy one due to budget constraints. “One-child policy” is characterized by the high proportion of elderly in the population. In this chapter, $W_{s,n}$, $C_{s,n}$, $H_{s,n}$, $S_{s,n}$, $TW_{s,n}$ are variables that represent wage income, non-housing consumption, housing consumption, savings, and bequest for the generation whose birth time is s, and who has already live for n periods. Descriptions of the variables are summarized in Table 3.1.

Table 3.1 Description of variables used in the OLG model

Variable	Description
$C_{t,0}$	Non housing consumption of youths at t
$C_{t-1,1}$	Non housing consumption of middle aged residents at $t-1$
$C_{t-2,2}$	Non housing consumption of the elderly at $t-2$
$H_{t,0}$	Housing consumption of youths at t
$H_{t-1,1}$	Housing consumption of middle aged residents at $t-1$
$H_{t-2,2}$	Housing consumption of elderly at $t-2$
$S_{t,0}$	Savings of youths at t
$S_{t-1,1}$	Savings of middle aged residents at $t-1$
$S_{t-2,2}$	Savings of the elderly at $t-2$
$W_{t,0}$	Wages of youths at t
$W_{t-1,1}$	Wages of middle aged residents at $t-1$
Pen_t	Pension paid by the youth generation and the middle age generation at t
TW_t	Bequest of the elderly at the end of t

We make the following assumptions throughout this chapter.

Assumption 1: Society has three generations: youths, middle-aged, and elderly. The youths and middle-aged generation form the working class, and everyone passes away at the end of their elderly period.

Assumption 2: In the current period t, the proportion of youths, middle-aged, and elderly residents in the overall population are $\lambda_0(t), \lambda_1(t), \lambda_2(t)$ respectively, where $\lambda_0(t)+\lambda_1(t)+\lambda_2(t)=1$, the larger the proportion $\lambda_2(t)$, the more severe the ageing problem.

Assumption 3: In order to simulate an economic steady state, real wages (excluding the impact of inflation) of each period, t, will not change over time.

Assumption 4: Social security tax adopts a pay-as-you-go system. The government imposes a social security tax on working residents, which is paid to retired residents as pension, denoted by Pen_t. For the sake of simplicity, this paper assumes that the social security tax rate is a fixed proportion α of the wage income of working residents. The pension of retired residents will be balanced by the equation of social security budget:

$$\alpha\left(\lambda_0\left(t\right)W_{t,0}+\lambda_1\left(t\right)W_{t-1,1}\right)=\lambda_2\left(t\right)Pen_t \tag{3.1}$$

Assumption 5: The average resident, who is in their youth cannot afford to buy a house and can only rent. Residents in their middle-age can afford to purchase a house, and as they age to become the elderly they will fully own the house equity just before retirement.

Assumption 6: φ_{own} is a percentage of the house price that represents the maintenance cost associated with living in the house. φ_{rent} is a percentage of house price given as rental. $\varphi_{own}<\varphi_{rent}$, the maintenance cost is assumed to be lower than the cost of rental. d is a percentage of house price that accounts for depreciation in each period.

Assumption 7: This paper assumes that house prices remain unchanged in each period, and is denoted by P. Under this assumption, when the overlapping generation model reaches a steady state, the consumption and savings of each generation of residents will maintain at the same rate over time.

Assumption 7 is used simply to capture the key feature of the reverse mortgage, i.e., liquidating illiquid assets rather than betting on the growth of the house price. Note that our qualitative results in this chapter are not dependent on the rising trends of wages and house prices.

Assumption 8: Surplus wealth, net of consumption will be saved and will not be invested in risky assets, the elderly will consume all liquid assets and bequeath the net house worth, i.e., $S_{t-2,2} = 0$.

Investment risk isn't considered in this paper since it bears no direct relation to the welfare of the reverse mortgage market. We had performed several simulations which tests for the situation where residents invest their surplus wealth in savings and risky assets such as stocks. However, there weren't any significant change in the simulation results.

Assumption 9: Only retired residents with housing assets can apply for a reverse mortgage loan, and the ratio of the loan amount to house value is denoted m. Once retired residents have applied for a reverse mortgage, the bequest will be the net house worth minus the reverse mortgage loan amount. It is assumed that the reverse mortgage payout is a one-time compensation.

This chapter does not adopt reverse mortgage payouts in the form of annuities due to the following two reasons. The core function of the reverse mortgage is that the elderly will receive a lump sum cash payment and continue to stay in the house. The annuity in its equivalent form is the current value of future discounted payments, and annuities would further complicate the model. Secondly, under real economic conditions, the demand for commercial pensions is very low. Even in the U.S., with a matured commercial annuity market, overall demand for annuities remains at a low. This makes the real-life annuity purchase hard to explain using utility theory. This phenomenon is known as the famous "Annuity Puzzle" (Huang and Su, 2010).

Assumption 10: The utility of consumption is characterized by a Cobb-Douglas function:

$$u(C,H) = \frac{\left(C^{1-\theta} H^{\theta}\right)^{1-\gamma}}{1-\gamma} \tag{3.2}$$

In the formula above, C represents non-housing consumption, and H represents housing consumption (measured by housing area). Parameter γ is the coefficient of relative risk aversion, and θ measures the relative preference of residents for housing consumption. In order to reflect the utility of owning a house as higher than that of house rentals under the same level of housing consumption, and according to the practice of Yao and Zhang (2005), it is assumed that the housing preference coefficient of house renters θ_{rent} is less than that of house owners θ_{own}.

Assumption 11: The bequest utility function of retired residents takes the form of a power formula:

$$B(TW) = \frac{(TW)^{1-\gamma}}{1-\gamma} \tag{3.3}$$

Residents in each period will choose to save and consume to optimize their lifecycle asset allocation. Based on real situations in China, we further assume that the residents in their youth have relatively little income, so they can only rent a house. In their middle-age, they can afford a reasonably sized house. During their elderly period, they continue to possess the house and leave the residual housing value to the next generation when they pass away, as a form of bequest. With the description above, residents' budget constraints are modeled as follows:

(1) Budget constraint of youths

We assume that youths cannot afford to buy a house (with an average living area of about 90 square meters), but will rent instead. The budget constraint is:

$$Q_{t-2,0} = (1-\alpha)W_{t-2,0} = C_{t-2,0} + \varphi_{rent}PH_{t-2,0} + S_{t-2,0} \tag{3.4}$$

(2) Budget constraint of middle-aged residents

Middle-aged residents will receive the remaining value of housing from their parents. The following are two kinds of situations based on whether elderly residents of the prior generation participate in reverse mortgage:

A. If elderly residents of the prior generation did not participate in reverse mortgage, the house will be bequeathed to the younger generation, hence:

$$\begin{aligned} Q_{t-2,1} &= (1-\alpha)W_{t-2,1} + (1+r)S_{t-2,0} + PH(1-d)^2(1-\psi_{own})\frac{\lambda_2(t)}{\lambda_1(t)} \\ &= C_{t-2,1} + S_{t-2,1} + PH(1+\varphi_{own}) \end{aligned} \tag{3.5}$$

B. If elderly residents of the prior generation participated in reverse mortgage, they can only bequeathed the remaining net worth of the house to their children, hence:

$$\begin{aligned} Q_{t-2,1} &= (1-\alpha)W_{t-2,1} + (1+r)S_{t-2,0} + PH(1-d-m)(1-d)(1-\psi_{own})\frac{\lambda_2(t)}{\lambda_1(t)} \\ &= C_{t-2,1} + S_{t-2,1} + PH(1+\varphi_{own}) \end{aligned} \tag{3.6}$$

(3) Budget constraint of the elderly residents

Elderly residents will continue to possess the house, while savings and pension income will be used for non-housing consumption and housing maintenance. The two types of situations are:

A. Elderly residents did not participate in reverse mortgage:

$$\begin{aligned} Q_{t-2,2} &= Pen_t + (1+r)S_{t-2,1} + PH(1-d)(1-\psi_{own}) \\ &= C_{t-2,2} + PH\left[(1-d)(1-\psi_{own}) + \varphi_{own}\right] \end{aligned} \tag{3.7}$$

Then, the corresponding bequest upon death shall be:

$$TW_{t-2,2} = PH(1-d)^2(1-\psi_{own}) \tag{3.8}$$

B. Elderly residents participated in reverse mortgage:

$$\begin{aligned} Q_{t-2,2} &= Pen_t + (1+r)S_{t-2,1} + PH(1-d)(1-\psi_{own}) \\ &= C_{t-2,2} + PH\left[(1-d)(1-\psi_{own})(1-m) + \varphi_{own}\right] \end{aligned} \tag{3.9}$$

And, the corresponding bequest upon death is:

$$TW_{t-2,2} = PH(1-d-m)(1-d)(1-\psi_{own}) \tag{3.10}$$

The social welfare can be obtained by summing up the welfare of the three generations in the same period with respect to their proportion to the population. In the pay-as-you-go system, the government sets the social security tax rate, requiring a certain amount to be paid by the young and middle-aged residents to support pensions for the elderly. Each generation of residents will optimize their life-cycle welfare through a consumption-asset allocation, given the social security tax rate set by the government. Knowing residents' reaction to the optimal social security tax rate, the government is assumed to set a social security tax rate that maximizes social welfare. Mathematically, given an initial social security tax rate α, individual welfare is given by equation (3.12), and the corresponding social welfare utility is given by (3.11). Situations under various α are simulated, and the α that gives the maximum social welfare is selected

$$Welfare_t = \max_{\alpha} \sum_{i=0}^{2} \lambda_j u\left(C^*_{t-i,i}, H^*_{t-i,i}\right) \tag{3.11}$$

The optimal non-housing consumption and optimal housing consumption $\left(C^*_{t-i,i}, H^*_{t-i,i}\right)$ is a function that gives the maximum aggregate individual welfare:

$$\begin{gathered}\left(C^*_{t-i,i}, H^*_{t-i,i}\right) = \arg\max_{A_t} \left[\sum_{i=0}^{2} \beta^{i-1} u\left(C_{t-i,i}, H_{t-i,i}\right) + \beta^2 B\left(TW_{t-2,2}\right) \right] \\ A_t = \left\{C_{t-i,i}, H_{t-i,i}, I^o_{t-i,i}\right\}, \qquad i = 0,1,2\end{gathered} \tag{3.12}$$

In the optimizations utilizing equations (3.11) and (3.12), the appropriate budget constraints will be applied depending on whether residents participated in the reverse mortgage. Formulas (3.4), (3.5), and (3.7) correspond to residents who do not participate in the reverse mortgage. Formulas (3.4), (3.6), and (3.9) correspond to residents who participate in the reverse mortgage.

3.2 Variables and parameters

A life-cycle is divided into three periods: the first period is from 20 to 40 years old, whom are youths. The second period, are middle-aged residents from 40 to 60 years old. And the third period is from 60 to 94 years old, known as the elderly. The baseline parameter setting is summarized in Table 3.2, and several alternative specifications will be considered for checks of robustness.

Table 3.2 Baseline Parameters

Parameter	**Mathematical Symbol**	**Baseline Value (20 years stage)**
Coefficient of relative risk aversion	γ	2
Preference coefficient of housing buyers	θ_{own}	0.55
Preference coefficient of housing renters	θ_{rent}	0.5
Utility discount factor	β	0.99^{20}=0.8
Housing price	P	$0.96W_{t,0}$
Housing depreciation rate at each stage	d	0.2
Living space	H	90 m^2
The cost of housing maintenance	φ_{own}	0.2
The cost of renting	φ_{rent}	0.5
The transaction cost of buying a house	ψ_{own}	0.1
The commitment of the reverse mortgage loans	m	0.5
The income ratio of the middle aged and youths	$W_{t-1,1}:W_{t,0}$	2.65
The proportion of each generation	$\lambda_0(t):\lambda_1(t):\lambda_2(t)$	0.4:0.4:0.2
Risk-free interest rate	r	$1.01^{20}-1=22.02\%$

(1) The coefficient of relative risk aversion γ, the housing preference for purchasing θ_{own}, the housing preference for renting θ_{rent}, Reverse mortgage loan commitment m, and the utility discount factor β are the same as those in Section 2.

(2) Residents' income and consumption

Assuming that residents' income increases throughout their working years at the rate of 5% per annum, we have $W_{t-1,1}:W_{t,0}=1.05^{20}=2.65$. The residents' average income of each 20 year period is $\frac{1+2.65}{2}W_{t,0}=1.83W_{t,0}$. If savings interest rate is 1% per annum, without the influence of inflation, the cumulative interest from savings in 20 years is $1.01^{20}-1=22.02\%$.

(3) The housing price per square meter P and housing depreciation rate d

The Economic Blue Book issued by Chinese Academy of Social Sciences in 2011 stated that the ratio of housing prices to Chinese urban residents' income is 8.76. In order to reflect rapid house price increase in China following current trends, we set the house price to income ratio to $8.76*120\%=10.51$. Since the average income of a resident in 20 years is about $1.83W_{t,0}$, the average house price will be: $\frac{10.51*1.83}{20}W_{t,0}=0.96W_{t,0}$. We assume a housing depreciation rate per year of 1%.

(4) Living space H

The Economic Blue Book 2011 says that the average living space per person in China is around 30 square meters. To simplify this analysis, we assume that the average family is made up of 3 people, and that each family owns a 90 square meter house.

(5) Parameters for housing and transaction costs φ_{own} φ_{rent} ψ_{own} ψ_{rent}

The cost of housing maintenance per year φ_{own} will be 1% of the house price. The annual cost of rental φ_{rent} is set as 2.5% of the house price. Hence, cumulative cost of maintenance and rental in a 20 year period is 20% and 50% respectively. The transaction cost of housing ψ_{own} is set at 10%, while the transaction cost of rental ψ_{rent} is assumed to be 0.

(6) Population composition $\lambda_0(\mathrm{t}):\lambda_1(\mathrm{t}):\lambda_2(\mathrm{t})$

The Development Trend in China's Population Ageing Forecast Report in 2012 forecasts that by the end of 2020, elderly people will total 248 million, accounting for 17.17% of the total population. To characterize the effects of an ageing population, the proportion of each generation is set to be $\lambda_0(\mathrm{t}):\lambda_1(\mathrm{t}):\lambda_2(\mathrm{t})$=0.4:0.4:0.2.

3.3 Simulation Results

Results with and without reverse mortgage were simulated and compared. A social security tax rate was initialized as α, and then the optimal consumption and savings over the lifecycle of the residents were simulated with their respective budget constraints. We compute the aggregate individual welfare of all generations and sum them up with respect to the proportions of the population to obtain the social welfare utility. Then, we change the social security tax rate α, repeat the above calculations, compute social welfare utilities under different α, and select the optimal tax rate that produces the maximum aggregate social welfare utility. In the computation of optimal asset allocation over the lifecycle of the residents, the backward recursive algorithm is employed. Steady economy arrives when the individual welfare and social welfare becomes Pareto optimal.

In the absence of a reverse mortgage market, the results of residents' consumption-asset allocation in each period are shown in Table 3.3. In this scenario, the optimal social security tax rate is 1%. The corresponding individual welfare over the entire three period life cycle is -4.67E-2, and social welfare is -1.33E-2. The optimal social security tax rate is low, consistent with the results derived by Feldstein (1985) that when individual myopia is fairly little, the tax rate is close to 0. As individual myopia increases, the optimal social security tax rate becomes larger, as will be discussed later.

Table 3.3 Residents' optimal allocation without participation in reverse mortgage

Period	**Non-housing consumption C**	**Housing consumption H (m^2)**	**Savings S**
Youth	4.95E-1 $W_{t,0}$	90 (Renting)	0
Middle-aged	8.8E-1 $W_{t,0}$	90 (Buying)	8.26E-1 $W_{t,0}$
Elderly	8.83E-1 $W_{t,0}$	90 (Homeownership)	0

When the residents participate in reverse mortgage, the results of their consumption-asset allocation in each period are shown in Table 3.4. In this scenario, the optimal social security tax rate is also 1%. The corresponding individual welfare over the entire three period life cycle is -6.02E-2, and the social welfare is -1.31E-2.

Table 3.4 Residents' optimal allocation in the baseline scenario with participation in reverse mortgage

Period	**Non-housing consumption C**	**Housing consumption H (m^2)**	**Savings S**
Youth	4.95E-1 $W_{t,0}$	90 (Renting)	0
Middle-aged	9.57E-1 $W_{t,0}$	90 (Buying)	5.91E-1 $W_{t,0}$
Elderly	9.52E-1 $W_{t,0}$	90 (Homeownership)	0

Compared with the scenario without participation in reverse mortgage, non-housing-consumption and housing-consumption of youths remains unchanged. However, the non-housing consumption of the middle-aged improved from 8.8E-1 $W_{t,0}$ to 9.51E-1 $W_{t,0}$, and non-housing consumption of the elderly increases from 8.83E-1 $W_{t,0}$ to 9.52E-1 $W_{t,0}$. Savings of middle-aged residents significantly decreases from 8.26E-1 $W_{t,0}$ to 5.91E-1 $W_{t,0}$. Without the reverse mortgage market, elderly residents only rely on savings and social security, working residents are unable to consume more because they have savings that would accumulate slowly and a portion of their income is given to the social security fund. Reverse mortgage provides necessary income to the elderly which would reduce dependence on their own savings and pension. This would in turn reduce the social security tax burden of working residents, and help to improve the working and elderly residents' consumption, hence increase welfare benefits.

When the reverse mortgage market is implemented, it would be seen that individual welfare might decrease and social welfare may increase. This is due to the impact of the personal bequest motive. In this chapter, we will assume that individual welfare is composed of consumption utility and bequest utility, and that their level of importance is the same. However, the social welfare value is composed of the aggregate consumption utility of the current living population, whilst bequest utility is of no consequence to the social welfare. Reverse mortgage will increase the consumption of middle-aged and elderly residents and increase the social welfare, however, it reduces residents' individual welfare due to reduced bequest utility. To analyze the impact of residents' bequest utility, we re-simulate the above model under alternate assumptions in which the residents' bequest utility is zero. The results show that individual welfare is -3.25E-2 with the reverse mortgage market, and -3.31E-2 without the reverse mortgage market. So, participating in reverse mortgage should improve individuals' welfare when their bequest motive is not strong.

In the remainder of this subsection, we perform several robustness checks. First, in the baseline scenario, we assume that residents are rational and can plan ahead. However, numerous empirical studies have shown that rationality is usually overestimated in real life. This is particularly true for the youths, since they seldom balance current consumption or savings and future needs for when they grow old (Cremer and Pestieau, 2011). We reduce the annual utility discount factor to 0.9 which gives us a 20-year discount factor of 0.12, to reflect the degree of increased myopia. Under these assumptions, we simulate residents' consumption-asset allocation and the optimal social security tax rate for the elderly, which would give us results on residents' individual and social welfare.

Without reverse mortgage present, the results of the consumption-asset allocation in each period are shown in Table 3.5. In this scenario, the optimal social security tax rate is 12%. The corresponding individual welfare over the entire three period life cycle is -1.91E-2, and the social welfare is -1.35E-2.

Table 3.5 Myopic residents' optimal allocation without participation in reverse mortgage

Period	Non-housing consumption C	Housing consumption H (m^2)	Savings S
Youth	4.4E-1$W_{t,0}$	80 (Renting)	0
Middle-aged	1.4$W_{t,0}$	90 (Buying)	6E-3$W_{t,0}$
Elderly	6.86E-1$W_{t,0}$	90 (Homeownership)	0

When residents participate in reverse mortgage, the results of their consumption-asset allocation in each period are shown in Table 3.6. In this scenario, the optimal social security tax rate is 7%. The corresponding individual welfare over the entire three period life cycle is -1.83E-2, and the social welfare is -1.32E-2.

Table 3.6 Myopic residents' optimal allocation with participation in reverse mortgage

Period	Non-housing consumption C	Housing consumption H (m^2)	Savings S
Youth	4.62E-1$W_{t,0}$	84 (Renting)	6E-3$W_{t,0}$
Middle-aged	1.39$W_{t,0}$	90 (Buying)	1E-2$W_{t,0}$
Elderly	6.82E-1$W_{t,0}$	90 (Homeownership)	0

Compared with the situation without reverse mortgage, the introduction of reverse mortgage causes individual welfare to improve. Due to the high degree of residents' myopia, the role of the bequest utility is reduced. The non-housing consumption and housing consumption of youths are increased, while non-housing consumption of middle-aged and elderly residents are decreased. However, the magnitude of decrease is not large, and the reverse mortgage can smooth the consumption of the residents across their entire lifespan, such that individual welfare increases.

In order to analyze the impact of an ageing population, we assume that the proportion of each generation is now $\lambda_0(t):\lambda_1(t):\lambda_2(t)=0.3:0.4:0.3$, and retain the assumptions of myopic residents. In the absence of reverse mortgage, results of consumption-asset allocation for residents in each period are shown in Table 3.7. In this scenario, the optimal social security tax rate is 23%. The corresponding individual welfare over the entire three period life cycle is -2.16E-2, and the social welfare is -1.36E-2.

Table 3.7 Optimal allocation of myopic residents' in an ageing population without participation in reverse mortgage

Period	**Non-housing consumption C**	**Housing consumption H (m^2)**	**Savings S**
Youth	3.85E-1 $W_{t,0}$	70 (Renting)	0
Middle-aged	1.25 $W_{t,0}$	90 (Buying)	2E-3 $W_{t,0}$
Elderly	8.48E-1 $W_{t,0}$	90 (Homeownership)	0

When the residents participate in reverse mortgage, results of consumption-asset allocation for residents in each period are shown in Table 3.8. In this scenario, the optimal social security tax rate is 14%. The corresponding individual welfare over the entire three period life cycle is -1.96E-2, and the social welfare is -1.31E-2.

Table 3.8 Optimal allocation of myopic residents' in an ageing population with participation in reverse mortgage

Period	Non-housing consumption C	Housing consumption H (m^2)	Savings S
Youth	4.29E-1 $W_{t,0}$	78 (Renting)	2E-3 $W_{t,0}$
Middle-aged	1.25 $W_{t,0}$	90 (Buying)	5E-3 $W_{t,0}$
Elderly	8E-1 $W_{t,0}$	90 (Homeownership)	0

Compared to the scenario without reverse mortgage, the introduction of the reverse mortgage can improve individual welfare. Reverse mortgage can cause the consumption in the residents' lifespan to be smoother, and so individual welfare becomes higher. The introduction of reverse mortgage can also cause the non-housing and housing consumption of youths to increase, while the non-housing consumption of middle-aged residents remains unchanged. The non-housing-consumption of retired residents declines slightly. All of these effects improve social welfare. Reverse mortgage reduces the optimal social security tax rate and allows working residents to save more. Compared with the baseline scenario where ageing problem is not that severe, the welfare gain of participating in reverse mortgage for both individuals and society increases. This implies that the welfare enhancing function of reverse mortgage is more significant when the ageing problem becomes more severe.

Lastly, we analyze the welfare results of reverse mortgage when house prices rise or decrease relative to income, versus the baseline scenario. To simulate house price rise, we run an alternative where house price to income ratio is 12.61 $W_{t,0}$ (i.e., the housing value of 90 square meters is 1.15 $W_{t,0}$). In the absence of reverse mortgage, the results of consumption-asset allocation for residents in each period are shown in Table 3.9. In this scenario, the optimal social security tax rate is 6%. The corresponding individual welfare over the entire three period life cycle is -4.85E-2, and the social welfare is -1.74E-2.

Table 3.9 Optimal allocation of residents' in a high house price environment without participation in reverse mortgage

Period	Non-housing consumption C	Housing consumption H (m^2)	Savings S
Youth	4.4E-1 $W_{t,0}$	46 (Renting)	0
Middle-aged	6.49E-1 $W_{t,0}$	90 (Buying)	9.6E-2 $W_{t,0}$
Elderly	6.51E-1 $W_{t,0}$	90 (Homeownership)	0

When residents participate in reverse mortgage, the results of consumption-asset allocation for residents in each period are shown in Table 3.10. In this scenario, the optimal social security tax rate is 3%. The corresponding individual welfare over the entire three period life cycle is -5.76E-2, and the social welfare is -1.67E-2.

Table 3.10 Optimal allocation of residents' in a high house price environment with participation in reverse mortgage

Period	Non-housing consumption C	Housing consumption H (m^2)	Savings S
Youth	4.73E-1 $W_{t,0}$	50 (Renting)	4E-3 $W_{t,0}$
Middle-aged	6.49E-1 $W_{t,0}$	90 (Buying)	1.2E-2 $W_{t,0}$
Elderly	6.54E-1 $W_{t,0}$	90 (Homeownership)	0

Compared with the scenario without participation in reverse mortgage, reverse mortgage significantly reduces the amount elderly residents' bequests, which reduces the bequest utility. Although, in each period, the residents' consumption and consumption utility is increased with reverse mortgage, the combined effects still is a decrease in overall individual welfare. Compared with the baseline scenario where the house price is not that high, high house prices leads to a decrease in non-housing consumption for all generations. No matter if there is reverse mortgage or not, high house prices will decrease the social welfare. With reverse mortgage and when compared with the baseline scenario, the magnitude of decrease in social welfare is smaller than when there is no reverse mortgage. Hence, in a high house price environment, reverse mortgage can act as a cushion in times of financial burden for the residents.

When the house price to income ratio drops to 8.41 $W_{t,0}$ (i.e., the housing value of 90 square meters is $0.77W_{t,0}$), the results of consumption-asset allocation for residents in each period in the absence of the reverse mortgage market are shown in Table 3.11. In this scenario, the optimal social security tax rate is 1%. The corresponding individual welfare over the entire three period lifecycle is -4.2E-2, and the social welfare is -1.23E-2.

Table 3.11 Optimal allocation of residents' in a low house price environment without participation in reverse mortgage

Period	**Non-housing consumption C**	**Housing consumption H (m^2)**	**Savings S**
Youth	4.95E-1 $W_{t,0}$	111 (Renting)	0
Middle-aged	1 $W_{t,0}$	90 (Buying)	8.82E-1 $W_{t,0}$
Elderly	9.89E-1 $W_{t,0}$	90 (Homeownership)	0

When the residents participate in reverse mortgage, the results of consumption-asset allocation for residents in each period are shown in Table 3.12. In this scenario, the optimal social security tax rate is 1%. The corresponding results for individual welfare over the entire three period life cycle is -6.44E-2, and the social welfare is -1.21E-2.

Table 3.12 Optimal allocation of residents' in a low house price environment with participation in reverse mortgage

Period	**Non-housing consumption C**	**Housing consumption H (m^2)**	**Savings S**
Youth	4.95E-1 $W_{t,0}$	111 (Renting)	0
Middle-aged	1.06 $W_{t,0}$	90 (Buying)	6.99E-1 $W_{t,0}$
Elderly	1.05 $W_{t,0}$	90 (Homeownership)	0

Compared with the situation without participation in reverse mortgage, individual welfare is less because reverse mortgage causes the elderly to bequeath much less. Although residents' consumption and their utility have increase with reverse mortgage, the combined effect is that individual welfare will decrease. The total consumption of youths remains unchanged, the non-housing consumption of middle-aged and elderly residents increases, and the social welfare improves. In addition, the savings of middle-aged residents reduces significantly, this is because the consumable assets of retired residents are increased under the reverse mortgage, and the savings requirement of working residents is lower, hence, there is no burden on working residents to save. Compared with the baseline scenario where the house price is not that low, it doesn't matter if there is reverse mortgage, low house prices will increase the social welfare. With reverse mortgage, the magnitude of increase in social welfare is larger than when there is no reverse mortgage.

3.4 Summary

Our empirical analysis based on the OLG model that incorporates Chinese demographical characteristics is summarized in the following.

Firstly, reverse mortgages provide an important source of income for retired elderlies. These retirees could reduce their dependence on pensions, which can result in a reduction of the social security tax rates. It follows that, the burden on youths to care for the elderly would also reduce and youths would enjoy a smoother consumption throughout their life. This increases the overall welfare of the society.

Secondly, when residents attach too much importance to bequest utility, it is difficult to improve individual welfare over the residents' entire lifespan using reverse mortgage. However, when the bequest motivation of residents weakens or myopia increases, reverse mortgage could improve individual welfare by smoothing out consumption over the residents' lifespan. This indicates that educating people to treat bequest more objectively and more fairly would increase their willingness of participating in the RM program[①].

Lastly, the welfare enhancing function of reverse mortgage is also significant when the ageing problem becomes more severe. Hence, developing a reverse mortgage market can overcome the decline of the social welfare caused by an ageing population effectively.

[①] It is interesting to note that the Dilnot Commission in the UK recommended a cap on individuals' contribution to their social care costs at £35k, because of the same issue. For more details on Dilnot Commission, see http://www.publications.parliament.uk/pa/cm201012/cmselect/cmhealth/1583/158308.htm

Chapter 4 Concluding Remarks

4.1 Main results

Global experience shows that developing a RM program is a feasible solution for the ageing population problem. However, there is no strong consensus about whether RM can improve retired residents' individual and social welfare, especially when considering real economic and demographic characteristics. In this paper, we have conducted welfare analyses from an individual and social welfare perspective to determine if China should develop a RM market. We used life-cycle and OLG models to study the welfare effects from both the individual and social perspectives, and obtained the following results:

First, reverse mortgage is welfare enhancing for many retirees, but not necessarily for all. Especially, when retirees are rich in assets but poor in cash, the optimality of reverse mortgage depends on the extent of the retiree's cash shortage. When cash shortage is more evident, reverse mortgage is the optimal choice because it offers cash without reducing the retiree's living space. However, when there is no shortage of cash, reverse mortgage is less appealing due to its high cost. In the case of cash shortage, the optimal choice is to sell the original house and make a re-allocation between living space and cash holding. When there isn't much cash shortage, the retiree can still enjoy a reasonable living space.

Second, timing the participation of the reverse mortgage is an important determinant of the welfare gain. Our simulations show that most house rich cash poor retirees can improve welfare by entering reverse mortgage later during the retirement period than just at the beginning of their retirement. On average, the best time of entry is about 5 years after retirement, due to the China housing price boom.

Third, we investigated the influence of the RM market on social welfare under the pay-as-you-go system. Building on the OLG model developed by Feldstein (1985), we find that reverse mortgages provide an important source of income for retired elderlies. These retirees could reduce their dependence on pensions, which can result in a reduction of the optimal social security tax rates. It follows that, the burden on youths to care for the elderly would also reduce and youths would enjoy a smoother consumption throughout their life, while increasing the overall welfare of the society.

Fourth, in the OLG model, although reverse mortgage increases social welfare, it does not necessarily increase the residents' individual welfare. This is because bequest utility is not included in social welfare, but plays an important role in individual's life-cycle welfare. If residents attach too much importance to bequest utility, it is difficult for reverse mortgage to improve individual welfare over the residents' entire lifespan. This indicates that educating people to treat bequest more fairly and more objectively would increase people's willingness to join in the RM program.

Lastly, several robustness tests confirm that the welfare enhancing function of reverse mortgage still remains significant when the ageing problem becomes more severe and when the moving risk becomes larger.

In summary, the analysis in this paper quantifies the welfare gain of introducing RM in China through simulations, and in the meantime provides a thorough investigation on the comparative statics of various underlying parameters. This research provides a sound basis in support of the plan of China's central government to launch a pilot program that offers reverse mortgages to the elderly people. Nevertheless, we would highlight that in practice, the timing ability and the bequest intensity would affect people's willingness to participate in the RM program. Education on related issues can help promote a healthy development of RM market.

4.2 Potential Topics for Future Research

We excluded the health risks of residents and the impact from medical care in the models. In the real world, health risks, especially from major diseases, can have huge influence on retired residents' consumption and asset allocation. The RM market can bring cash income for retired residents, which could then be used for both consumption and health care. This factor could be explored in future studies.

Secondly, some psychological and cultural factors should be considered in future studies. For example, the traditional “family security” and “home equity heritage” concepts are so entrenched in Chinese culture that it may hamper the development of RM; the opaqueness and complication of the RM contract may also lead elderly people astray. How we deal with the psychological and cultural factors is an interesting topic for both theorists and practitioners.

Thirdly, we have shown that high contract costs can reduce the welfare of retired residents. The slow development of China’s housing market, and the lack of government support for the RM market may largely restrict China’s RM market development. The study of legal institutions and rules for helping control RM contract costs will be necessary in future studies.

Finally, the data set that we use in this paper is far from complete. Data with a higher quality can help us to obtain more reliable and applicable research results.

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Appendix A RM product in major markets

Although the basic idea of developing RM products is similar in many countries, there are differences worth noting.

1. United States of America

There are three main products: the HECM Program, the Home Keeper Program, and the Financial Freedom Plan (FFP). Currently, HECM has the largest market share (over 90%) and is the leading product in the U.S. RM market. The government guarantee was one of the most important reasons why HECM was able to develop so quickly. A table summarizing the characteristics of these products follows:

Summary of RM products in the U.S.

	HECM	**Home Keeper**	**FFP**
Institution	Federal Housing Authority (FHA) authorized financial institutions	Fannie Mae	Financial Freedom Senior Funding Corporation (FFSFC)
Sponsor	Government	Public company	Private company
Start time	1989	1995	1999
Mortgage Payout	\$161,760 - \$290,319	US\$333,700	US\$700,000
Payment Frequency	Lump sum, installments, annuity	Monthly payment or credit payment	Lump sum, annuity
Fees	2% of house value	2% of house value	Less than or equal to 2% of house value
Interest Rate	Floating: 2%-5%	Floating rate	
Eligibility	Elderly people with low housing assets	Elderly people with medium housing assets	Elderly people with high housing assets
Markets	49 States in the U.S.	49 States in the U.S.	12 States in the U.S. and D.C.
Guarantee	FHA guaranteed	no	no

2. Canada:

In late 1980s, the Canadian Home Income Plan (CHIP), borrowing from successful experiences in the U.S. and G.B. RM markets, developed its own RM products. Canadian RM products include: Reverse Annuity Mortgages (RAM), Line of Credit Reverse Mortgages (LCRM), and Fixed Term Reverse Mortgages (FTRM). RAM has the largest market share in the Canadian RM market. The Canadian RM market has no government regulation or legal protection. However, supervision from national financial authorities nurtured a healthy development of the market.

Summary of RM products in Canada

	RAM	LCRM	FTRM
Institution		CHIP	
Sponsor		Private companies	
Mortgage Payout		Supervised by national financial authorities	
Payment Frequency	Monthly payment	Variable payment	Lump sum in fixed period (5 or 10 years)
Fees	high	low	low
Eligibility	People with stable income	Urgent demand	Short term demand
Market share	large	small	small
Guarantee	no	no	no

3. Australia

Australia began considering RM in the 1980s. At the beginning of 2005, without government participation, several financial institutions founded the RM market, which was regulated by the Senior Australians Equity Release Association of Lenders. In Australia, annuity RMs was the major products.

Summary of RM products in Australia

	Reverse Mortgage, RM	Home Reversion, HR
Number of Products	More than 10	
Institutions	12	
Regulator	Senior Australians Equity Release Association of Lenders (SEQUAL)	
Sponsor	SEQUAL and Private companies	
Interest Rate	Floating, usually 1% higher than the commercial mortgage interest rates	
Property Rights	Belongs to resident	Belongs to issuer
Eligibility	60 years and above	60 years and above
Guarantee	Type 1: No negative equity guarantee, NNEG, which means that the loan amount will not increase the equity value Type 2: Equity Protection Option, which means the borrower can keep a pre-determined amount with only small amount of premiums	

Appendix B Mortality Rates

Central mortality rates for urban residents in 2009

Age	Mortality (%)	Age	Mortality (%)	Age	Mortality (%)
0—4	0.89	35—39	0.44	70—74	17.11
5—9	--	40—44	1.12	75—79	32.81
10—14	--	45—49	1.73	80—84	50.76
15—19	--	50—54	2.77	85—89	65.46
20—24	0.22	55—59	3.90	90+	151.96
25—29	0.08	60—64	6.41		
30—34	0.43	65—69	11.20		

Central mortality rates for urban residents at age x who died before age $x+n$ is defined as ${}_nq_x$ and the probability of population at age x who are dead before age $x+n$ is defined as ${}_nq_x$. The relationship between ${}_nq_x$ and ${}_nq_x$ follows:

$$ {}_nq_x = \frac{n \times {}_nm_x}{1+n\left(1-{}_nf_x\right){}_nm_x} $$

where ${}_nf_x = 0.5$ is the separation factor that deaths occurred on average at the midpoint of the age interval. From the above equation, we could get the urban population mortality rate for each period, see the following table.

Mortality rate for urban population on every age period, 2009

Age	Mortality (%)	Age	Mortality (%)	Age	Mortality (%)
0—4	4.44	35—39	2.20	70—74	82.04
5—9	--	40—44	5.58	75—79	151.61
10—14	--	45—49	8.61	80—84	225.22
15—19	--	50—54	13.75	85—89	281.27
20—24	1.10	55—59	19.31	90+	550.62
25—29	0.40	60—64	31.54		
30—34	2.15	65—69	54.47		

Appendix C Housing Price Growth and Return on Risky Assets

	Stock Index	Housing Price Growth
1998	11.36%	1.40%
1999	8.05%	0.00%
2000	30.72%	1.10%
2001	7.15%	2.20%
2002	-20.55%	3.70%
2003	-6.26%	4.80%
2004	-6.29%	9.70%
2005	-14.62%	7.60%
2006	44.60%	5.50%
2007	68.88%	7.60%
2008	-17.46%	6.50%
2009	-27.62%	5.00%
2010	10.27%	6.30%
2011	-24.07%	7.10%
Mean	**4.58%**	**4.89%**
St. Deviation	**25.12%**	**2.87%**
Correlation	**-0.11**	

The correlation between the risky assets return and housing price growth is -0.11, which is consistent with the assumption of 0 correlation.

Appendix D Moving Probability

Average Moving Probability in the US, 2009

Age	Average Moving Prob.	Age	Average Moving Prob.
25-29	29.75%	60-64	6.78%
30-34	20.97%	65-69	6.04%
35-39	16.11%	70-74	5.62%
40-44	12.47%	75-79	6.27%
45-49	10.06%	80-84	7.46%
50-54	8.68%	85-89	8.89%
55-59	7.72%	90^{+}	10.67%

The moving probability for China is:

$$\text{Average moving probability in China for 5 years}$$
$$= 1 - \left(1 - \frac{\text{Average moving probability in US for 1 year}}{4}\right)^{5}$$

So we got:

Average Moving Probability for China

Age	Average Moving Prob.	Age	Average Moving Prob.
25-29	32.05%	60-64	8.19%
30-34	23.60%	65-69	7.33%
35-39	18.58%	70-74	6.83%
40-44	14.65%	75-79	7.60%
45-49	11.96%	80-84	8.98%
50-54	10.39%	85-89	10.63%
55-59	9.28%	90+	12.64%

Appendix E Minimum Non-housing Consumption

The average disposable income for urban residents in 2011 is shown in the following table: (from the statistical data of Chinese government)

Groups	Percentage	Disposable Income
Lowest	10%	5253.23
Low	10%	8162.07
Medium-low	20%	11243.55
Medium	20%	15399.92
Medium-high	20%	21017.95
High	10%	28386.47
Highest	10%	46826.05

We combined the lowest and low groups into a new low group; the medium-low, medium and medium-high groups into a new medium group; the high and highest groups into a new high group.

Groups	Percentage	Disposable Income
Low	20%	6707.6
Medium	60%	15887.14
High	20%	37606.26

The average minimum living standard in 2011 is RMB 3209.6, which is about 20.20% of medium group’s average income, so we set the minimum non-housing consumption to be 20% of income.