# PENSIONS, FUNDING AND RISK

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ABSTRACT

In this paper, we identify the economic system in which occupational pension schemes operate. This system includes all parties to the pension arrangement, particularly the members of the scheme and the shareholders of the sponsoring company. This requires us to model the pension scheme not as a self-contained fund, but simultaneously alongside the company, and to recognise its interaction with other parties with financial interests in the scheme. Under this method, the stakes of the various parties aggregate to 100% of the assets of the company including the assets of the pension scheme. The power of this approach is that decisions taken in relation to the management of the pension scheme can be assessed by the effect they have on the way in which the overall economic value of the company is divided between stakeholders.

KEYWORDS

Actuarial; Pensions; Funding; Risk; Financial Economics; Deflators

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## 1. Introduction

### 1.1 *A Value-Based Approach to Strategic Pension Decisions*

1.1.1 This paper describes a value-based approach to strategic decision making in pensions. The practical application of the approach is in quantifying, in value terms, who gains and who loses from decisions in relation to funding levels and investment policy. At the heart of the approach is a link between pension schemes and the concept of shareholder value.

1.1.2 For many companies, pension considerations are material in

strategic and reporting terms. We reflect this by linking the cash flows for the pension scheme and the sponsoring firm, which enables us to value the interests of all parties to pension scheme transactions. This approach contrasts with the traditional actuarial approach, which is 'scheme-centric', i.e. focussed solely on the finances on the pension scheme. Some of the insights provided by a value-based approach confirm traditional actuarial conclusions, but some do not.

1.1.3 Companies are in business to create value for their shareholders, and many do successfully and simultaneously create value for their shareholders, employees and pensioners; but some companies fail, and one outcome of our approach is that it provides insights into the interaction of the solvency of companies and the solvency of pension schemes. It may, therefore, prove a useful tool in investigating the effects of minimum funding requirement (MFR) changes.

1.1.4 The paper concentrates on the financial implications of the strategic decisions taken in the course of running a pension scheme. This focus on the financial aspects is deliberate for two major reasons:

(1) the decisions taken in the running of a pension scheme are primarily about the transfer of financial stakes; and
(2) the understanding of financial risks offers the best opportunity for improved risk management in the provision of pensions.

1.2 *Basis of Modelling*

1.2.1 Our starting point is to identify a closed economic system that includes all the key parties who have a financial interest in the pension arrangement. We refer to these parties as stakeholders. We have assessed decisions taken in relation to the pension scheme by how they affect each stakeholder's share of the overall 'cake', in particular by identifying who wins and who loses.

1.2.2 The model in this paper uses deflators (as discussed in Section 4.3), and has been calibrated to market conditions. The deflator approach ensures that the model is economically coherent and, in particular, that asset values reconcile to those observed in the market. This paper demonstrates the insights that models incorporating deflators can provide in a pensions context.

1.2.3 We recognise that there is a wide range of projection models, and that different models may produce different valuation results. However, we would expect all economically coherent models, that are calibrated to the same initial market conditions and based on the same system of stakeholders, to produce broadly the same strategic insights, even though the precise valuation figures differ.

1.2.4 As well as exploring these value-based concepts using their own models, readers may wish to follow up potential research topics arising from the conclusions. Particular areas are: solvency issues for weaker companies;

and funding and investment policies for pension schemes that are material in relation to their sponsor. In addition, if the system of stakeholders is extended, the inclusion of new or re-defined stakeholders will result in changes to the stakes of the original stakeholders.

1.3 *Structure of the Paper*

At the core of this paper are the results demonstrating how strategic decisions within a pension scheme affect the value of the different stakeholders' interests. Sections 2 and 3 set the scene, and Section 4 outlines the main principles of the method. Details of the stakeholder model are given in Appendix A. The results are introduced in Section 5, and discussed in detail in Section 6. Section 7 includes some remarks about disclosure, and summarises our conclusions.

## 2. Current Situation

2.1 *Current Pension Provision*

2.1.1 Measured by value of assets, most occupational pension provision in the United Kingdom is defined benefit in nature. Typically, a defined benefit scheme provides pensions linked to final salary, is funded in advance and has an employer paying the balance of cost. Usually, the employer determines the contribution rate based on actuarial advice, although this is subject to the statutory minimum contribution rate set in relation to the MFR. In all such schemes, investment policy is controlled by the trustees (although they are required to consult sponsoring employers).

2.1.2 Over the past 15 years, defined benefit schemes have become significantly more regulated. We now see:

— compulsory triennial MFR valuations;
— production of schedules of contributions (requiring annual re-certification);
— production of statements of investment principles;
— restrictions on the tax-free roll up of scheme surpluses;
— developments in company accounting requirements;
— creation of a debt on the employer when a scheme winds up (although this debt ranks only with other unsecured creditors);
— complicated overriding amendments to the priority order of benefits, where there are insufficient assets on wind up; and
— minimum pension increases (in deferment and in payment).

2.1.3 Changes in the pensions regulatory environment, particularly for defined benefit schemes, are encouraging a trend of reporting clearer financial information. This, in turn, is pushing traditional actuarial methods of funding and investment advice gradually closer to modern financial

methods. Examples of new types of reporting and disclosure include the provision of information to members in the form of the MFR, and to shareholders in the form of FRS 17 (Accounting Standards Board, 2000).

2.1.4 In parallel, the Turnbull report has raised the profile of overall risk management for U.K. companies. This report was produced by the Institute of Chartered Accountants in England & Wales as part of its internal control requirements of the Combined Code on Corporate Governance.

2.1.5 The trend of accounting standards towards transparency through market-based reporting is likely to result in further changes in disclosure requirements for pension schemes. It is less certain whether there will be a corresponding rationalisation of other regulations, for instance the contradictory minimum and maximum funding requirements for U.K. pension schemes.

2.1.6 The overwhelming majority of new schemes set up in the U.K. in recent years have been defined contribution. Employers like cost certainty and cost reduction, but this move has also been driven by:

— the increasing compliance costs of running a defined benefit scheme;
— the increased funding costs of running a defined benefit scheme;
— the risk that new legislation will impose further costs on employers; and
— the introduction of new types of defined contribution arrangements (i.e. group personal pension plans and stakeholder pensions), which offer a reduced burden on employers.

2.2 *Current Actuarial Advice*

2.2.1 These are the key features of a typical actuarial valuation of a defined benefits scheme:

— a contribution rate will be recommended (and, in practice, a strong steer has usually been agreed beforehand with whichever party sets the contribution rate);
— the valuation method (usually the projected unit method) will target pensions earned to the date of the valuation, but with projected salary increases;
— the valuation assumptions are often 'off-market' (usually taking advance credit for equity out-performance, and sometimes taking assets into account at an off-market value);
— the actuary has scope to adjust the off-market assumptions in arriving at a contribution recommendation;
— a comment is included about the suitability of investments; and
— the actuary can choose whether to disclose the discontinuance position on an insurance cost basis, a cash equivalent basis, or a 'closed fund' basis. The first of these is market driven, the second is, in effect, prescribed, given that many schemes have adopted the MFR as their cash equivalent basis, and the third is left as a matter of 'actuarial judgement'.

2.2.2 Valuation reports are not required to disclose the (economic) cost of the benefits, which is a key item of information for the sponsoring employer. Similarly, valuation reports are not required to disclose how much of the benefits could have been secured if the scheme had wound up at the valuation date, which ought to be one of the more interesting items for trustees and members. The omission of these important items reduces the level of transparency of actuarial valuation reports.

2.2.3 Valuation reports are also not required to provide:

— indications of how the funding position will change in the future, given the recommended contribution rate and the scheme's investment strategy; or
— comments on other risks which may affect the value of the employer's and the members' stakes in the scheme.

2.2.4 Finally, it is often the case that the same techniques (and sometimes the same actuarial basis) as used for the formal actuarial valuation are used for:

— strategic advice to the employer, e.g. on benefit design; and
— benefit calculations, e.g. where members trade one benefit for another of the same 'value', as determined by the actuary.

2.3 *Current Investment Strategies*

2.3.1 Current investment strategies are typically characterised by:

— large holdings in equities;
— active investment management, with the objective of out-performing a peer group or benchmark;
— holdings in gilts not constrained to the duration of the liabilities;
— limited exposure to bonds other than gilts; and
— control of risk being indirect only (by limits on particular asset classes and by target levels of out-performance).

2.3.2 Trustees are required to set out their investment strategy in a statement of investment principles. These often state that the trustees' objectives are broadly:

(1) to ensure that funds are available to pay the benefits as and when they fall due; and
(2) to maximise the return on the investments, subject to an acceptable level of risk.

2.3.3 The statement does not usually set out:

— how risk is quantified;
— whose risk it is; or
— what level of risk is acceptable.

2.3.4 There is sometimes, however, a statement that the two risk objectives are, to a degree, contradictory; the first point directs the trustees to a stance of minimum risk, while the second indicates that risks will be inherent in the investment allocation.

2.3.5 References are sometimes made in the statement to the statutory minimum funding requirement. However, the MFR is not investible, i.e. there is no asset that moves in line with MFR liabilities. Despite this, many schemes have framed their investment strategies around the assumption that the best way of ensuring that the scheme meets the MFR is to track the asset allocation implied by MFR regulations. The Pensions Board has partly addressed this investibility issue by proposing that the MFR measure of liabilities is modified, to be based wholly on bond yields.

2.3.6 In practice, investment strategies are often set on the basis of either:

— experience and general rules of thumb (which tend to differ, depending on the investment consultancy), e.g. "assets covering salary-related liabilities should be invested in 75% equities and 25% bonds"; or
— an 'asset-liability model', where random scenarios are generated from a plausible economic model, and the results are examined to steer the trustees towards an appropriate investment strategy.

2.3.7 The rule of thumb approach has no quantitative risk justification. It is our understanding that trustees and companies who ask for a more robust risk assessment are often steered towards asset/liability modelling by their actuarial or investment advisers.

### 2.4 *Asset/Liability Models — the Promise*

2.4.1 An asset/liability model (ALM) uses an economic model to produce stochastic simulations of returns on asset classes and other relevant economic data, e.g. inflation. The output from this model is used in calculating (approximate) liability values at different time horizons. Given an investment strategy and a contribution rate, the model produces a range of funding levels at future times.

2.4.2 The introduction of ALMs was heralded as a step forward in the financial management of pension schemes. We could quote many practitioners. Clark (1992) is typical:

> "I firmly believe that, for those involved in the funding of long-term liabilities, cash-flow modelling techniques and asset & liability models provide the way ahead."

2.4.3 Unfortunately, the typical application of an ALM in the pensions context seems to amount to little more than producing 'funnels of doubt', which serve only to demonstrate the (unsurprising) conclusion that the

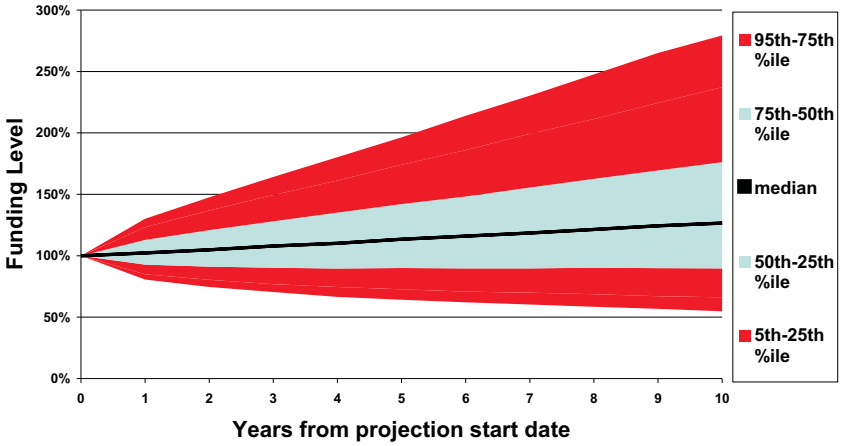



Figure 2.1. Funding level — equity investment strategy and 15% contributions

further into the future projections are made, the more uncertainty there is about key outputs.

2.4.4 Figure 2.1 shows a typical 'funnel of doubt' for one possible ALM output, in this case funding level, for a scheme with the following characteristics:

— 100% equity investment;
— liabilities valued on a market-consistent 'projected unit' method (PUM) basis (without any additional investment mismatch reserve);
— fixed contribution of 15% of payroll; and
— no discretionary benefit increases.

2.4.5 An initial ALM chart is often contrasted with a second chart that shows the results for a different investment strategy. For example, Figure 2.2 shows an equivalent chart assuming that the trustees follow a bond investment strategy (50% gilts and 50% index-linked gilts). Again, the conclusions which can be drawn from the chart, i.e. that bond investment results in a lower dispersion of funding levels, but has a lower expected return, provides little in the way of new information.

## 2.5 *ALMs — the Reality*

2.5.1 The stark contrast between the charts in Figure 2.1 and Figure 2.2 is beneficial when explaining investment risks to trustees. However, this use as an educational aid is undermined by the common practices of:

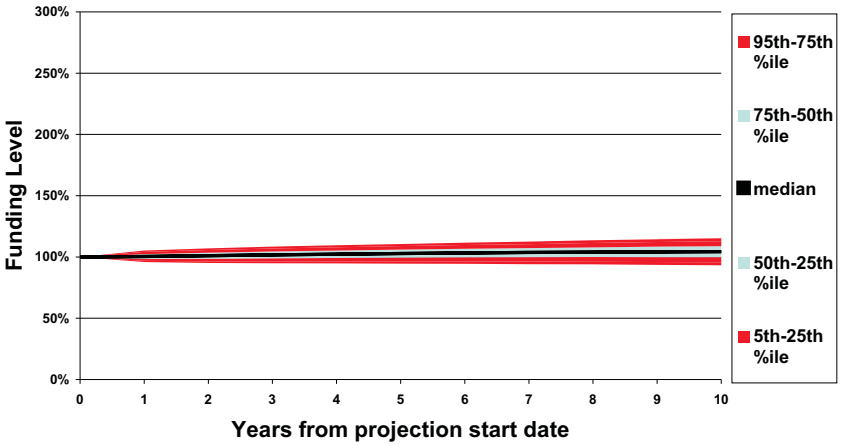



Figure 2.2. Funding level — bond investment strategy and 15% contributions

— discarding the worst scenarios (e.g. cutting the chart off at the 5th percentile); and
— labelling outcomes simply as 'unfavourable' or 'very unfavourable', without identifying the percentiles that these outcomes refer to.

2.5.2 We conclude that, instead of being a valuable tool for risk management, ALMs are actually used more as a qualitative method for explaining risks. Although the existence of many of these risks is often appreciated, the move to market-consistent approaches has served to highlight how the *magnitudes* of some risks were disguised, both in traditional actuarial valuations and in the standard application of ALMs.

2.5.3 The inability to apply ALMs quantitatively stems from having ill-defined objectives. For example, Yakoubov, Teeger & Duval's (1999), commentary on their own funnel of doubt diagrams includes the phrases "unacceptable risk of a below 100% funding level" and "unacceptable decline in the median funding level". In quantitative terms, one would want to know to whom the risk is unacceptable and what the critical level of risk is.

2.5.4 We are interested, in this paper, in the ability of ALMs to provide *quantitative* assessment of risks. We work on the premise that the prime purpose of such a study should be to answer the question: "How does a given strategy affect the value of the interests of different stakeholders, including allowance for risk?"

2.6 *Other Features of ALMs*

2.6.1 What other information do ALMs provide? The models can provide an assessment of the likelihood of a shortfall (i.e. a funding level below 100%) occurring at a particular time. For example, if the scheme considered in Section 2.4 adopts an equity strategy, this leads to a shortfall at the end of 10 years in 30% of the scenarios, while, with a bond strategy, a shortfall occurs in 26% of the scenarios.

2.6.2 What is lacking is a method of quantifying the value of the different strategies. We could assess both the frequency and the magnitude of the shortfalls. For the example scheme, the equity strategy led to a shortfall in 30% of cases, and the average shortfall (given that a shortfall occurs) was £87 million. When a bond strategy was adopted, a shortfall occurred in 26% of the scenarios, and the average shortfall was £19 million. However, these statistics still fail to measure the value to the stakeholders of these different strategies.

2.6.3 It is tempting for the unwary to combine the probabilities and the magnitude of the shortfalls to arrive at a value, but this approach suffers the same defects as pricing options using direct (or real world) probabilities. Some insurers have recently found the flaw in this simplistic approach in relation to their policy guarantees. For further information, see the 'Parable of the Bookmaker' of Baxter & Rennie (1996), or any introductory text on option pricing.

2.6.4 One way of defining a measure of the risk inherent in an investment strategy is the cost of buying an out-performance option that pays out the amount of the deficit at the end of the projection period. This is nothing other than shortfall insurance.

2.6.5 To price these shortfalls correctly, we need either to derive 'risk neutral probabilities' or to use deflators (discussed in Section 4.3). In either case, we find that the cost of this protection for the equity strategy is £111 million, while for the bond strategy the cost is just £10 million.

2.6.6 The cost of providing downside protection is a useful measure of the risks inherent in different investment strategies. It could be argued that this approach is too pessimistic, as it costs the downside protection while taking no account of the upside that the equity investment offers.

2.6.7 This could be taken into account by selling the upside. For example, a call option that pays the value of the assets in excess of a given funding level, say 120%, could be sold. For the equity strategy outlined, the value of this call is £48 million. Although this provides a non-trivial realisation price for giving up the upside, it still falls well short of the £111 million cost of protection against under-funding. The results are summarised in Table 2.1.

2.6.8 The implied discount rates (the rate required to discount the average shortfall cost to arrive at the market value of the option) are negative for the put options, but in excess of the expected return on equities

Table 2.1. Pricing investment under and out performance

| | Shortfall protection for equities | Shortfall protection for bonds | Value of upside for equity investment |
|---|---|---|---|
| Option type | Put | Put | Call |
| Strike level | 100% funding | 100% funding | 120% funding |
| Investment | Equities | Bonds | Equities |
| Probability of option paying out | 30% | 26% | 54% |
| Average payout (given payout > 0) | £87m | £19m | £239m |
| Average payout | £26.5m | £4.8m | £130m |
| Deflated cost | £111m | £10m | £48 m |
| Implied discount rate | −13.3% | −7.3% | +10.5% |

for the call option. This is a familiar feature in option pricing, where put options have implied discount rates below the risk free rate, and call options have rates in excess of the expected return on the underlying asset. This reflects the gearing implicit in holding an option.

2.6.9 These pricing numbers are useful in quantifying the risks when choosing an investment strategy. However, they are still not sufficient to give the full picture, which involves the quantification of transfers of value between the different stakeholders.

2.6.10 Another common attempt to answer the risk/reward question, in the context of the pension scheme, has been to produce graphs that show 'risk' versus expected return. At first sight, such graphs are similar to those that arise in modern portfolio theory. Different portfolios are constructed and a boundary known as the 'efficient frontier' is found, where, for a given expected return, the 'risk' (usually taken to be the standard deviation) of the portfolio is minimised. A typical plot is shown in Figure 2.3; the dots represent different possible portfolios and the black line traces portfolios on the efficient frontier.

2.6.11 Applying these risk and return plots to a corporate entity is, however, missing the point. They may apply when considering the trade off between risk and return for an individual investor (assuming one had the appropriate measure of risk), but, in a corporate context, they could only make sense if we plotted them separately for each stakeholder and if we included in the analysis all the stakeholders' other financial assets and liabilities. In other words, seeking to optimise portfolios in this manner, for a corporate entity, is the equivalent of seeking an optimal portfolio for only a *portion* of an individual's wealth. Not only does this make little sense, but it leads to a blanket solution for individual stakeholders, irrespective of their actual financial state and risk preferences.

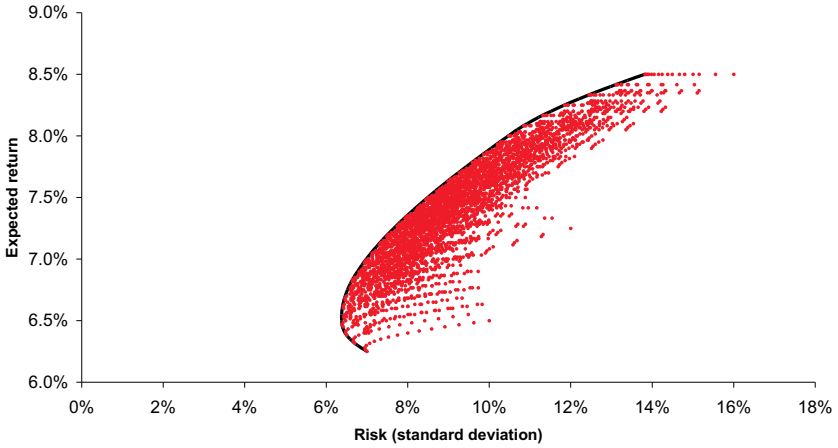



Figure 2.3. Risk and return plot

2.6.12 ALMs do provide some useful, if 'scheme-centric', insights into pension developments; but the various possible approaches described above do not tell us enough to be able to place *values* for each set of stakeholders on the different possible strategies.

2.7 *Agency Costs*

2.7.1 Stakeholders cannot make optimal financial decisions unless they have accurate information. This applies as much to pension funding as, for example, to new capital projects. Accounting standards and other regulations are examples of attempts to inform all interested parties about a wide area of corporate issues, including pensions.

2.7.2 Shareholders often appoint managers to run a business on their behalf. However, it is unlikely that the managers will be motivated purely to maximise shareholder value. Instead, managers may seek to maximise some combination of their pay, job security and future job prospects.

2.7.3 Shareholders can monitor their appointed managers or provide incentives to align the managers' interests with the shareholders' interests. These arrangements collectively constitute what has come to be known as corporate governance. The additional costs of corporate governance, together with any reduction in shareholder value resulting from their managers not acting to optimise shareholder value, are known as agency costs. The seminal reference on agency costs is Jensen & Meckling (1976), although, since then, whole consultancy businesses have been built on the back of advising how to reduce agency cost — further, and more up-to-date,

details are set out in Chew (1999). Shareholders have to trade the cost of corporate governance against the cost of the managers not acting solely in their interests.

2.7.4 In general, the more opaque an economic contract, the harder it is to assess its value, and the more likely it is to give rise to agency costs. Pension provision, particularly defined benefit provision, is very opaque, and is, therefore, exposed to considerable risk of agency costs.

2.7.5 The corollary is that the better members understand their scheme, the more easily they can place a value on the benefits being offered. This suggests that shareholder value is created when pension scheme design is simple and communications are effective.

2.7.6 In summary, financial transparency acts as a driver to recognising economic values, and so to reducing agency costs. Accordingly, we welcome the main proposals contained in FRS 17.

## 3. Proposed Approach

### 3.1 *A Value-Based Approach*

3.1.1 The trend for more disclosure and information leads us to seek a method whereby actuarial advice could become cleaner and clearer. This would leave us to focus on the 'added-value' elements that actually matter to the stakeholders. Achieving this aim requires the actuarial profession to take on board some key concepts from finance theory. Actuarial knowledge of liability structures will remain an integral skill in providing robust advice.

3.1.2 We suggest that the above objective could be met if actuaries:

(1) disclosed clearly the effect of different courses of action on the various stakeholders; and
(2) used the same market-consistent basis for their advice, regardless of the recipient of the advice.

### 3.2 *Improvements to the Current Approach*

3.2.1 Advice in relation to benefit design, pension scheme funding and investment is often:

— opaque in derivation, and at odds with basic economic constraints, leaving actuarial advice open to external criticism and vulnerable to manipulation; and
— given without assessing transfers of wealth between different stakeholders, leaving actuaries and their clients unclear about who really benefits.

3.2.2 Switching to a narrower range of acceptable actuarial bases would, in itself, be a significant step towards clarity and simplicity. One very relevant example is that it is fruitless to generate a debate about the size of the 'equity risk premium' to use in funding bases, because risk premiums do not appear in

the assessment of economic cost (see e.g. Copeland & Weston, 1988; or Brealey & Myers, 1996). In any case, the equity risk premium is an untestable figure, with estimates of the annual rate ranging from 2% to 10%.

3.2.3 There are also advantages for the profession if we adopt a benchmark approach that is internally consistent and coherent. We could then:

— defend our advice in public more robustly;
— minimise the number of U-turns, such as those on the MFR and on company pensions accounting; and
— use the benchmark to cross-reference off-market advice.

3.3 *Comments on 'Long-Term' Advice*

3.3.1 Actuarial advice has often been characterised, and justified, by appeal to the 'long term'. For example, the 'long-term view' has been given as the reason for the heavy equity investment by U.K. pension schemes.

3.3.2 Can we unpick the 'long-term view', and reconcile it with the economic value approach? The answer is that the 'long-term view' omits explicit consideration of the risks attaching to investment. So, for example, advance credit is taken for some or all of the expected returns on equities, but no allowance is made for risks.

3.3.3 It is not so easy to ignore risks in the short-term view, because this is the end of the spectrum at which disclosure and the regulators bite; and even if disclosure and regulation were relaxed, the risks themselves would not go away.

3.3.4 The explanation of the 'long-term view', under which risks are selectively ignored, is that it assumes that the employer has infinite resources to absorb any series of reverses that might cause the company to go into liquidation or to wind-up its pension scheme.

3.3.5 In this paper, we suggest that, if we are to deal clearly and fairly with the stakes of all the stakeholders, we must adopt techniques that incorporate risks in a market-consistent way, rather than rely on techniques based on a riskless 'long-term view'.

3.4 *Market-Consistent Assumptions*

3.4.1 The value of any cash flow stream is equivalent to the value of a matching asset. Accrued pension liabilities (excepting mortality risk) can be priced by a bond portfolio. Hence, discounting liabilities at rates implied by bonds produces an economically-consistent price for the liabilities. We are aware that a shortage in duration of available bonds may give rise to reinvestment risk for very long dated liabilities, but these are typically the least financially significant part of the total pension liability being considered (i.e. younger members, who tend to have low accrued pensions). We use the terms 'price' and 'value' interchangeably, as price is the monetary expression of value.

3.4.2 There is a debate about whether gilt or corporate bond yields should be used when valuing the liabilities. The difference in yield between, for example, investment grade corporate bonds and gilts is the sum of the liquidity premium and the credit risk premium.

3.4.3 Pension promises are typically illiquid, and hence the matching portfolio should also contain illiquid assets. There are technical issues with incorporating allowances for illiquidity in a simple model, because, for instance, it implies that no clear market price exists for the asset, and hence it is difficult to value liquidity risk. In this paper, we have used gilt rates for our modelling.

3.4.4 The issue of credit risk of a pension scheme is, we think, more clear-cut. Credit risk can, and should, be modelled directly, because the credit risk will favour one party at the expense of another, and therefore involves an exchange of wealth. A subjective adjustment to a discount rate is an inaccurate and potentially misleading way of dealing with this risk.

### 3.5 *Projected Unit Funding Method*

3.5.1 The general overall method for funding U.K. pension schemes is the 'projected unit' method (PUM), as set out in the actuarial guidance note 'GN26: Pension Fund Terminology'. This method has typically been allied with slowly changing, off-market actuarial assumptions and 'smoothed' (i.e. also off-market) values of assets. These practices still have a significant following — see, for instance, the summary of the range of actuarial assumptions used by different actuaries in PwC (2000).

3.5.2 As related by Greenwood & Keogh (1997), the preference for PUM dates from a period when discontinuance benefits were easily covered. However, times have changed:

— statutory revaluation in deferment (and the fall in inflation expectations) means that discontinuance benefits are, in current economic conditions, primarily index-linked;
— the limited price indexation of pensions in payment means that pension increases are now generally prescribed rather than discretionary;
— the fall in long-dated interest rates, following the 1998 Russian debt crisis, means that the cost of securing accrued benefits, especially those with fixed increases (e.g. in deferment GMPs), has increased;
— the statutory MFR (in combination with the debt on the employer regulations) means that much of the deficit in a pension scheme sponsored by a still solvent employer can be recovered; and
— PUM 'past service reserves' are not always greater than the value of discontinuance benefits.

3.5.3 Against this background, it seems odd to us that actuaries should continue to focus on an ongoing funding method that has a subjective measure as its a funding target.

3.5.4 In the current subjective circumstances, the interactions between ongoing and discontinuance methods are difficult to unravel. We suggest that the solution is to start from something objectively measurable, i.e. the discontinuance position, and to use this as a benchmark from which to determine contributions.

### 3.6 *Defined Accrued Benefit Method*

3.6.1 McLeish & Stewart (1987) put the case for using an alternative method to PUM very eloquently:

> "It seems to us ... that the prime purpose of funding an occupational pension scheme must be to secure the accrued benefits ... in the event of the employer being unable or unwilling to continue to pay at some time in the future. To that end, the contributions would have to be sufficient both to pay the benefits as they fell due for as long as the scheme continued, and also to establish and maintain a fund which would be sufficient to secure the accrued benefits in the event of contributions ceasing and the scheme being discontinued, whenever that might occur.''

3.6.2 The method that McLeish & Stewart recommended, known as the 'defined accrued benefit method' (DABM), is consistent with valuing accrued liabilities in a manner that ensures that the discontinuance position is covered. The DABM takes no account of future salary increases in assessing a scheme's funding position, and so does not place a liability on the balance sheet for them. With these characteristics, and substituting fully market-consistent assumptions, the DABM becomes a coherent method for funding defined benefit schemes in the current regulatory environment.

3.6.3 It is worth noting that, in 1987, the funding levels implied by DABM were well below the norm at that time. Regulatory requirements have since then closed the gap between discontinuance reserves and ongoing reserves, especially when ongoing valuations frequently resort to off-market assumptions.

3.6.4 If the party responsible for determining the company contribution rate in a defined benefit scheme wishes to fund a scheme at a lower level than 100% of accrued benefits, then this might be a reasonable course of action. However, we believe that actuarial advice should be clear about this, and that conflicts will arise if we use opaque methods and off-market assumptions that have the effect of presenting such schemes as fully funded. The use of the DABM, coupled with market-consistent assumptions and a target fund that reflects the required level of security (measured against the level of benefits which can be secured on discontinuance), would make this clear.

3.6.5 We are not suggesting that it is necessary for schemes to be 100% funded on a discontinuance basis (although many members may quite reasonably have assumed that this is the objective, given the communications material that they have received to date). We are suggesting that we should

disclose the basis on which a scheme is being funded in such a way that all parties can understand their position or stake. This point is relevant to the way in which the Government's proposals on security for occupational pensions may develop.

## 4. Overall Framework

### 4.1 *Economic System and Stakeholders*

4.1.1 We first consider the scope of the economic system by identifying the stakeholders. For a typical U.K. pension scheme, the stakeholders will include:

— current pension scheme members;
— current employees who are potential future members;
— company shareholders (or, more generally, the owners of the enterprise);
— company creditors, especially holders of loan stock issued by the company;
— the Government, represented by the Inland Revenue;
— professional advisers, including wind-up administrators and investment managers, a group whom we have called collectively 'consultants'; and
— 'externals', a combined stakeholder group of suppliers (including the benchmark cost of labour for the industry), less customers.

4.1.2 Some of these categories can be further subdivided. For example, we could split pensioner and active pension scheme members, these two groups effectively being subject to different economic contracts.

4.1.3 If we were to consider only a subset of the stakeholders, then an impression could be given that a particular funding method or investment strategy would benefit all parties. Of course, this would usually be happening at the expense of the stakeholder excluded from the analysis.

### 4.2 *Pensions — an Economic Contract*

4.2.1 An occupational pension is an economic contract between employer and employee. The totality of these contracts is significant in the context of the U.K. economy; in the U.K. private sector there were 151,000 funded pension schemes recorded in the Government Actuary's 1995 assessment (Government Actuary's Department, 1998) and assets held by U.K. pension schemes, as a whole, are about £800bn, according to the Faculty and Institute of Actuaries (2000).

4.2.2 We can put these contracts into a framework, by considering the objectives of the different stakeholders and the economic system in which they all operate. Within our framework, which could be applied to any economic contract, we place values on the interests of different stakeholders, and examine how they are affected by different strategies.

4.2.3 The characteristic of the stakeholder approach is that it tracks and values the cash flows generated within the economic system under various strategies. Although actuaries do not generally use this approach, it should be more within their 'comfort zone' than for many other finance professionals.

4.3 *Deflators*

4.3.1 The method of assessing stakeholder values involves stochastic projections of asset and liability cash flows. Each projection represents a simulation of the various cash flows within the defined system, and we use deflators to value these cash flows. Deflators work as stochastic discount factors, and each simulation produces a different realised value for the deflator. The amount of particular cash flows differs between simulations; equities, for example, have more volatile cash flows than gilts. The cash flows are multiplied by the deflator, and the average is taken over all simulations. The effect is that the cash flow in a particular projection is weighted according to the economic characteristics emerging in that projection. The consequence of using deflators is that cash flows for separate asset classes discount back to their market value, i.e. they retain economic coherence. The values of liability cash flows reflect the economic characteristics of each projection, and their present values are derived in the same way.

4.3.2 Deflators were first introduced by Arrow (1953), although their first appearance in the actuarial literature was at the 1991 AFIR International Colloquium (Ami *et al*., 1991). Smith (1996) introduced the concepts to the U.K. profession, and, more recently, an introduction to deflators for actuaries has been published by Jarvis, Southall & Varnell (2001). A rigorous approach to deflators in a conventional Brownian motion setting can be found in Duffie (1996).

4.3.3 The advantages of using a model based on deflators are as follows:

— values can be placed on the stakes of all stakeholders;
— the values will be consistent with market conditions if the deflator model is calibrated against relevant market indicators;
— the alternative risk neutral approach, which involves changing the measure of the underlying distribution so that the risk premium is zero (because the result is independent of the value of the risk premium), appears to have confused actuaries unused to the mathematical formalism; and
— many market models, such as those frequently used to price options, rely on the existence of a perfect dynamic hedge (which equates to the unrealistic assumption of a complete market), whereas deflators can be generalised to incomplete markets.

4.3.4 The deflator technique allows us to replace the ALM 'funnels of

doubt' in Section 2.4 by an analysis of how the overall value of the enterprise, including pension assets, is shared between the various stakeholders. This gives us a framework to compare who wins and who loses as a result of particular decisions and strategies, which we will explore in detail in Section 6.

4.3.5 When setting up a deflator-based model, there is an issue in deciding what tax treatment should be applied to the cash flows modelled. This reflects the different tax treatments that cash flows receive in the hands of different investors. We have modelled the cash flows for a tax-exempt investor, as this is consistent with the financial assets being held in the pension scheme. Where tax flows directly impact the system, we have modelled these explicitly (i.e. income tax and corporation tax).

4.3.6 If we include in our analysis other stakeholders who face differential tax rates, then, for a model calibrated to non-taxed cash flows, we would have the possibility of tax arbitrages. These arbitrages would not be a flaw of the model, as tax arbitrages do exist. We encourage others to take up the challenge implicit in Modigliani & Miller's famous irrelevance proposition 1 (Modigliani & Miller, 1958), and explore how corporations can take full advantage of differential taxation.

### 4.4 *The Economic Value of Pension Promises*

4.4.1 When an employer operates a defined contribution scheme, the economic value of the pension promise is equal to the market value of the assets in the scheme. When an employer operates a defined benefit scheme, the market value of the assets in the scheme is clearly not a measure of the economic value of the promise. For example, if we consider the extreme case of a completely unfunded defined benefit promise, the pension promise is still a liability, and, as such, has an economic value.

4.4.2 In these terms, a defined benefit promise is effectively a debt, i.e. a liability of the employer, upon which a value may be placed. Recognising this, we can identify debt instruments traded on financial markets that are closest to replicating the pension promise in cash flow terms. The market value of these instruments can be used to establish an objective economic value for pension promises.

4.4.3 In comparison, traditional actuarial methods have not been directed towards putting an economic, or market, price on defined benefit pension promises. The exception is at discontinuance, where actuarial valuations normally coincide with economic valuations.

### 4.5 *Transfers of Value*

4.5.1 A first order analysis suggests that switching assets within a pension scheme cannot, following Black (1980) and Tepper (1981), create value; but switching pension investments can, and usually does, result in a transfer of economic value between stakeholders.

4.5.2 For example, if surpluses are shared between the sponsor and members, switching assets into equities may have the effect of increasing the value of the stakes of members, particularly pensioners. This is because the extra volatility in funding level resulting from equity investment means that pensions are more likely to receive discretionary increases.

4.5.3 The question is: "Who has paid for this increase in the value of the members' stake?" In a closed system, an increase in value for one set of stakeholders results in a reduction for another set. Many strategic actions result in wealth transfers between stakeholders, and, in some cases, these transfers are unintentional.

4.5.4 For sound and transparent financial management, we need to help our clients to identify the winners and losers from any action, and to assess the magnitude of the wealth transfer for them.

4.6 *Modelling the Employees' Stake*

4.6.1 The remuneration received by the employee, excluding pension provision, is modelled as the benefits received in excess of the market rate for labour in the relevant industry. This is the net gain to the employee for exchanging his or her labour for cash benefits.

4.6.2 The cash benefits modelled take account of benefits in addition to basic pay, including an element of profit-related pay, and a differential rate of pay is included for non-pension scheme members.

4.6.3 There is a trade-off between the provision of cash benefits and pension provision (i.e. deferred pay). In the event of pension provision being reduced, it would be logical to expect the employees to seek compensation by negotiating higher benefits elsewhere in their remuneration package, see Exley, Mehta & Smith (1997). It would then follow that shareholders would not gain at the employees' expense by curtailing pension benefits.

4.6.4 However, strategic decisions do affect the value of the pension promise. For example, in Section 6 we show how a change of investment policy or actuarial assumptions can alter the stakes of different stakeholders. In practice, we do not see evidence of employees negotiating higher salaries as a result of a change in the pension scheme's funding basis or investment strategy. This may be due, in part, to the lack of clarity in pension reporting that makes it difficult for employees to value pension promises accurately. We also note that pensioners are not in a position to negotiate higher benefits to compensate for reduced pensions. Therefore, we have not modelled compensating effects on the non-pension benefits that may result from strategic decisions affecting the pension scheme.

4.6.5 The pension promise has many similarities to corporate debt, both as a financial instrument and the credit worthiness associated with it. If corporate debt is issued for a specific project, and then diverted to fund other higher risk projects, the capital markets will view any subsequent fund raising by the corporation with suspicion, and charge accordingly. (This is

discussed in Copeland & Weston, 1988). Similarly, if a corporation promises a pension and then undermines the credit-worthiness of that promise (by under-funding the scheme, say), employees will place a lower value on the pension entitlement, and try to negotiate other compensation, either with the same company or elsewhere. We view the assessment of the value placed on pensions by employees as an area worthy of further research.

## 5. Results — Introduction

### 5.1 *Details of Statistical Model*

5.1.1 Details of the model developed to value the financial interests of the stakeholders in a pension arrangement are given in the appendices.

5.1.2 All the results presented are based on projections of the cash flows over a period of 10 years. This time horizon was used, as testing indicated that it was sufficient for the stakes calculated to be accurate to within 0.1% of the total value of the combined entity.

### 5.2 *A Base Case for Comparison*

5.2.1 In this section we specify the benchmark case that we use for comparative purposes in presenting the main results.

5.2.2 The benchmark pension scheme has the following features:

— pension discontinuance liabilities are £230 million at the outset;
— the market value of the ongoing pension liabilities at the start of the projection is £267 million;
— the ongoing funding method is the PUM;
— valuation assumptions are derived from current market conditions;
— salary inflation is assumed to be 2% in excess of price inflation;
— it is 100% funded on an ongoing basis at the start of the projection period;
— assets are invested 100% in equities;
— the value of pensioner liabilities is initially 45% of total scheme liabilities on the PUM basis; and
— the surplus is measured on an ongoing basis, and is split, 20% to augment pensions in payment and 80% to reduce company contributions over a period of five years.

5.2.3 The sponsoring company has the following features:

— there is a total value of realisable assets of £385 million;
— there is a total combined value of the company and pension scheme of £652m, of which PUM liabilities are 41%;
— it has a dividend policy of distributing 50% of its reserves, subject to the dividend not changing by more than 10% in any year;

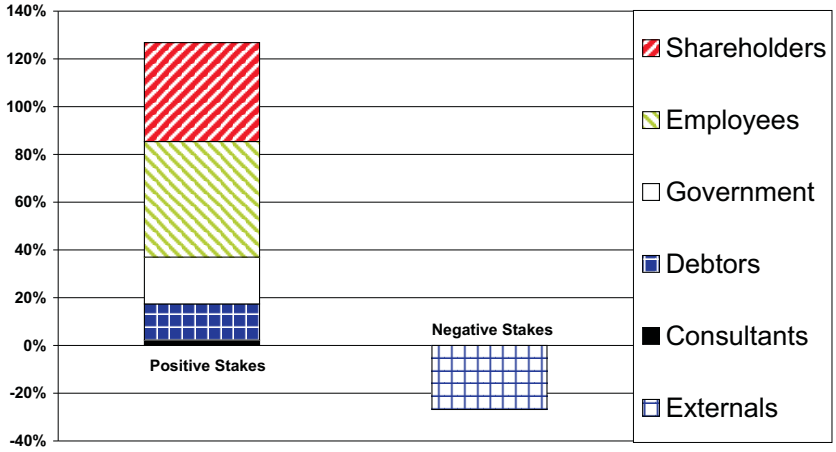
— it enters liquidation when shareholder funds become negative; and
— it is financed by loan stock, share capital and shareholder funds, such that the initial borrowing ratio, defined as total loans divided by shareholder capital and shareholders funds, is 35%.

5.2.4 The capital of the benchmark company was constructed to reflect the median gearing of companies in the FTSE All Share Index (source Datastream).

5.3 *Results for the Base Case*

5.3.1 For the base case, the stakes of the different stakeholders are shown in Figure 5.1. The stake of the shareholders represents the market value of the equity of the company. In the base case, the value of this equity, i.e. the company's equity market capitalisation, is £270 million. Where we show the effects of changes in the shareholders' stakes, these represent the change in share value consistent with the market's current valuation of the company. We note that empirical tests would be unable to validate the size of these changes, as it is not possible to isolate the impact of a single new piece of information on a share price.

5.3.2 The sum of all the stakes is equal to 100% of the value of the combined entity of the company and pension scheme. Employees (which group includes ex-employees) have the largest stake, representing 48% of the




Source: Datastream

Figure 5.1. Stakes for a PUM equity strategy

total value of the entity; this represents their total benefits package, including pension provision. The other major stakeholders are shareholders (41%), Government (20%) and debtors (15%). In contrast, the share of externals is negative, because it acts as a measure of the economic value added by the firm's operations. The externals' stake measures the payments by the company to suppliers for materials and services (including the benchmark cost of labour in the industry), less the revenues it receives from its customers. For a company that normally operates profitably, the stake of externals will, therefore, be negative.

5.3.3 In the base case, the company enters liquidation and the pension scheme is wound up in 4.0% of the scenarios. This figure is consistent with data on company loan stock defaults reported in Moody's Investment Service report (2000). We have not modelled the option for the company to wind up the pension scheme whilst its trading activities continue.

5.3.4 The lender of last resort (See Appendix A for the details), although included in the modelling, has been omitted from the charts, because this stake (and change in stake) was insignificant compared with the stakes of the other stakeholders.

5.4 *Tests to be Considered*

5.4.1 In the next section we consider a range of different strategies that we compare to the base case. The strategies that we consider explicitly are:

— funding strategy;
— investment strategy;
— scheme surplus distribution and deficit amortisation periods; and
— dividend policy.

5.4.2 In addition, we shall consider the relation between scheme size, scheme maturity and investment strategy.

5.5 *Risks Highlighted by the Results*

The tested strategies result in different divisions of the overall wealth of the enterprise between the different stakeholders. The changes in stakes are the result of different risks faced by each of the stakeholders. Below we highlight some of the risks that stakeholders may face. Note that, by considering a closed economic system, for every loser there is a winner, hence a risk that has a negative effect for one stakeholder will create a positive effect for other stakeholder(s):

— *Benefit leakage*. If surpluses are shared with the employees, then this will make a positive contribution to the employees' stake. The size and frequency of these discretionary increases will be influenced by the investment strategy.
— *Strong company balance sheet*. This could be the result of a slow pace of pension funding or a low rate of distribution of company funds to

shareholders. Whatever the cause, extra reserves on the balance sheet will decrease the risk of the company entering liquidation. Hence, salaries continue to be paid and pension rights accrue in more scenarios.

— *Low funding level*. This will stem from a slow pace of funding or from a volatile funding level. Employees will often lose out, due to the reduced security of benefits.
— *Volatile funding levels*. On the downside, these could result in default on the pension promise; equally, on the upside, where surplus is shared, extra discretionary benefits will arise.
— *Option to default on pension promise*. The shareholders may avoid paying the pension promised if the company enters liquidation and insufficient funds are available to meet discontinuance rights. In some circumstances, a high-risk strategy may be beneficial to the shareholders, as they have the option to default if the strategy fails, but may reap rewards if it is successful. High borrowing ratios and high-risk investment strategies would enable the shareholders to maximise the value of this option.
— *Failing to minimise tax*. If funds are not allocated to minimise tax liabilities, the Government will take a higher share of the economic value of the entity.
— *Leakages to advisers*. If two strategies are the same in other respects, then the strategy that minimises the payment to external advisors should be preferred.
— *Wind-up risk*. If the company enters liquidation, the pension scheme will be wound up. Employees are at risk from there being insufficient funds to meet the pension promises made to date. They also lose the benefit of future salary and pension accrual.

5.6 *Cash Flows Excluded from the Analysis*

5.6.1 The model considers all cash flows between the company (including the pension scheme) and other parties who have a financial interest in the scheme. As such, it is a micro-economic model focusing on a single company (or industry).

5.6.2 As with any model, the object is not to produce an exhaustive description of all aspects of the real world, but to focus on the key aspects that enable insights to be gained. We have, therefore, prescribed the model to include those stakeholders identified in Section 4.1. In our view, for the analysis of a corporate pension scheme, redefining the closed system to include additional stakeholders would result in second order effects that would not alter the conclusions regarding the company and pension scheme.

5.6.3 The model does not consider cash flows between third parties. For example, tax payments made by employees to the Government as a result of other income are not modelled. Similarly, the Government's stake does not include the tax flows arising from dividends being paid net of tax, as opposed

to debt payments that are made pre-tax. If these tax flows were included, they would show an increase in the Government's stake from pension schemes holding equities.

5.6.4 When analysing any financial problem, the stakeholders and cash flows to be modelled will be determined by the focus of the investigation. If we were to consider the Government's overall interest in pension provision, other effects would need to be included. For example, by making pension provision, employees are removing their entitlement to means-tested social security benefits in retirement. A complete model of all Government involvement would have to include all social security benefits, as well as the tax incentives offered for pension provision.

5.6.5 There are also cases where a class of stakeholders may not behave as a homogeneous group. For example, consultants who are competing for fees may choose a course of action that does not maximise the total consultants' stake. This could still be a rational course of action, if a consultancy can maximise its fee income at the expense of other consultancies with which it competes.

## 6. Results — Detail

### 6.1 *The Impact of Different Funding Strategies*

6.1.1 In this section we consider how different strategies affect the stakes of the different stakeholders. We look at the pace of funding, at investment policy, and at the size of the scheme in relation to the company.

6.1.2 First, we consider the pace of funding and compare new strategies with the base case described in the previous section (market-consistent PUM). The different funding strategies considered are:

— market-consistent DABM;
— PUM basis incorporating 1% p.a. loading on the discount rate;
— PUM basis incorporating 6% p.a. loading on the discount rate; and
— market-consistent PUM, with contributions fixed as a percentage of payroll.

6.1.3 These strategies are all examples of altering the pace of funding. In each case, the pension scheme is set up to be 100% funded on the ongoing basis. Choosing different bases alters the absolute amount of assets in the pension scheme. The value of the combined entity of the company and the pension scheme is maintained by a compensating adjustment to the company's reserves. This ensures that an economic entity with the same total value is considered.

### 6.2 *Moving from PUM to DABM*

6.2.1 The effect of moving to a market-consistent DABM funding

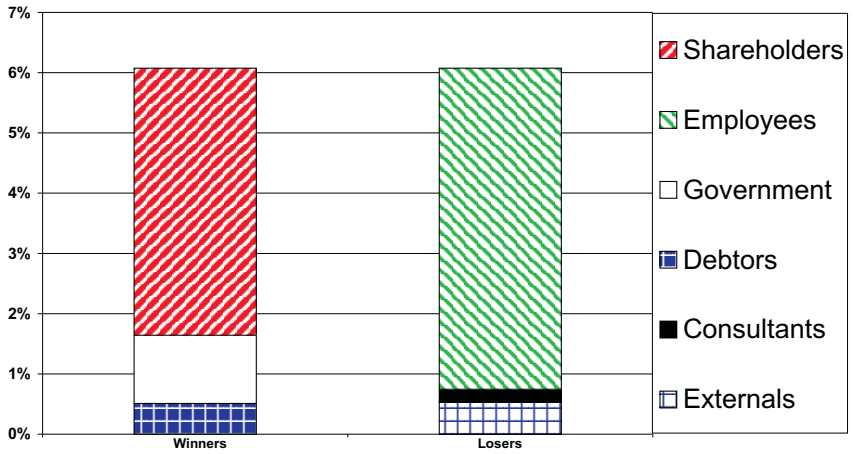



Figure 6.1. Changes in stakes — moving to a DABM equity strategy

strategy is shown in Figure 6.1. The changes in stakes are measured as a percentage of the total value of the economic entity, i.e. £652 million. For example, in Figure 6.1 the 5.3% reduction in the stake of employees translates to a monetary decrease of £35 million. The company entered liquidation in 4.0% of scenarios with the PUM funding strategy; under a DABM strategy this decreased to 2.0%.

6.2.2 The clear winners from the change in funding method are shareholders, primarily at the expense of employees. This is the result of the slower pace of funding implied by the DABM.

6.2.3 The gain in the stake of shareholders arises from the reduced number of liquidations and because the slower pace of funding increases the chance of the pension promise not being met in full in the event of a company liquidation. This increase in the shareholders' stake, of 4.4% of the total value of the combined entity, is equivalent to an appreciation of 11% in the share price.

6.2.4 The Government also gains from this strategy, as money held within the company is subject to a higher average rate of tax than money inside the pension scheme. More tax is payable to the Government, and tax also continues to be paid in more scenarios. Debt holders also gain from the reduced numbers of liquidations.

6.2.5 Consultants (which group includes investment managers) receive less in fees, because there are fewer wind-ups, and there is less money in the pension scheme on which investment management fees are charged. Externals have a decrease in their stake (i.e. a larger negative stake), as the lower number of wind-ups results in them supplying the company in more

scenarios. However, the largest losers from the reduced pace of funding are employees. Employees lose out in the event of the pension scheme winding up, because they are more likely not to receive their pension entitlement. This outweighs the impact of employees receiving salaries and pension benefits in more scenarios.

6.2.6 There are other ways in which the pace of funding can be altered. The next section looks at different ways of adjusting the pace of funding, assuming that, as in the base case, the pension scheme continues to invest in equities.

### 6.3 *Adding a Loading onto the Discount Rate*

6.3.1 In practice, many actuarial valuations start from market-derived assumptions, and then increase the discount rate used. If an attempt is made to justify the increase, it is usually based on claiming advance credit for expected excess equity returns over bonds (the equity risk premium). This premium, however, is compensation for the additional risk associated with equity investment, and is already reflected in the price. Hence, loading the discount rate double counts the risk premium (see Gordon, 1999).

6.3.2 Figure 6.2 demonstrates the magnitude of the change in stakes from adding a 1% p.a. loading (i.e. addition) onto the discount rate implied by the market when valuing non-pensioner liabilities. This change reduces the percentage of scenarios where the company enters liquidation to 2.7% from the base case of 4.0%.

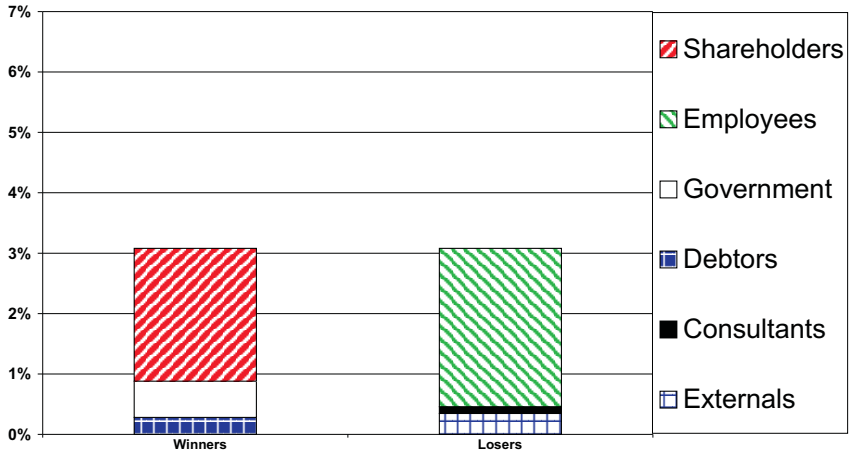



Figure 6.2. Changes in stakes — adding a 1% p.a. loading onto the discount rate for a PUM funding strategy

6.3.3 The winners and losers are the same as for the change to the DABM funding method.

6.3.4 Many have argued that 1% p.a. is a significant under-estimate of the expected excess return from investing in equities, and that a larger loading should be used. Indeed, commentators point to historic out-performance of equities over bonds in excess of 6% p.a. (over certain periods in the U.K. and the United States of America) as demonstrating that actuarial methodology is 'chronically conservative'.

6.3.5 We have noted that the higher expected return on equities is merely compensation for the higher risk. However, it is still informative to investigate the winners and losers from using a higher loading on the discount rate. Figure 6.3 demonstrates the winners, from using a 6% p.a. loading on the discount rate up to retirement in the PUM funding basis (note the change of scale on the *y* axis). This change reduces the percentage of scenarios where the company enters liquidation to 1.0% from the base case of 4.0%.

6.3.6 The winners and losers have not altered, but the shift in wealth from employees to shareholders is significant. In monetary terms, we have a combined entity of company and pension scheme with a value of £652 million, where the pension scheme has £267 million of liabilities (ongoing market consistent PUM basis). Using the 6% p.a. loading on the discount rate increases the shareholders stake by £59 million (9% of the total entity, or a 22% share price appreciation) and decreases the stake of employees by £70 million (11% of the total entity).

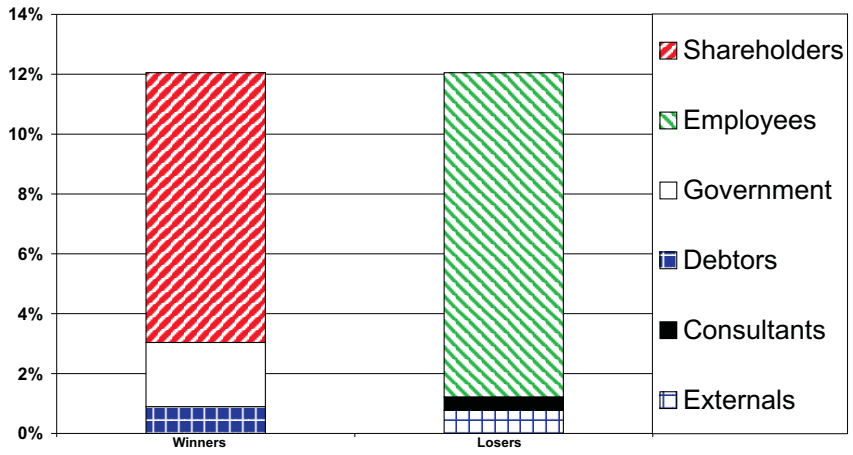



Figure 6.3. Changes in stakes from adding a 6% p.a. loading onto the discount rate for a PUM funding strategy

6.4 *Pace of Funding — Summary*

Table 6.1 summarises the impact on the different stakeholders from a variety of possible methods of slowing the pace of funding.

Table 6.1. Impact of slowing the pace of funding

| | | |
|---|---|---|
| Shareholders | *Gain* | Money retained in the company is available for distribution, also benefit from fewer liquidations. |
| Employees | *Lose* | Less security in the pension scheme resulting in reduced benefits in the event of the pension scheme winding-up. |
| Government | *Gain* | Higher tax rate on money outside of the pension scheme. |
| Debtors | *Gain* | Stronger balance sheet, fewer defaults. |
| Consultants | *Lose* | Less money in the pension scheme on which fees can be earned, and receivership income diminishes. |
| Externals | *Lose* | Reduced number of wind-ups results in externals contributing more to the operations of the continuing entity. |

6.5 *Stability of Contributions*

6.5.1 Stability of the contribution rate is often listed as a priority when considering a funding method. Rather than actuaries adjusting assumptions to provide the rate required, the sponsor could agree to pay a fixed contribution rate, irrespective of the funding position of the scheme. This would provide a defined benefit scheme with the stability of contributions associated with defined contribution schemes.

6.5.2 Figure 6.4 shows the effect of using a fixed contribution rate of

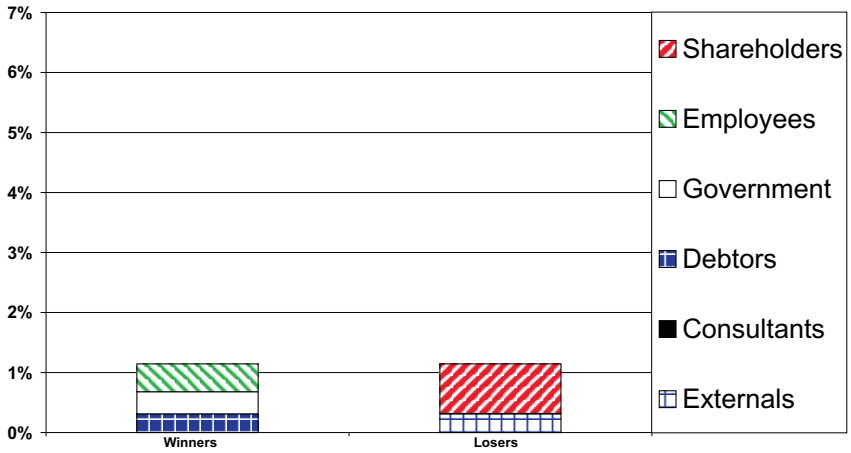



Figure 6.4. Effect of paying a fixed contribution of 20% of payroll

20% of the payroll, irrespective of whether the scheme is in surplus or in deficit. With this strategy, the percentage of scenarios where the company entered liquidation decreased to 2.7% from the base case of 4.0%.

6.5.3 When the scheme is in surplus, the sponsor continues to pay contributions of 20% of payroll without any reduction to amortise surplus. Similarly, contributions are not increased when the scheme is in deficit. Figure 6.4 shows that employees benefit from this strategy, mainly at the expense of shareholders.

6.5.4 This transfer of wealth to employees could be interpreted as too high a contribution being paid into the scheme. Turning this around, we could search for a contribution rate that did not alter the stake of employees.

6.5.5 Figure 6.5 shows the effect of paying a fixed contribution of 15% of payroll into the scheme, irrespective of whether the scheme is in surplus or in deficit. Paying 15% of payroll reduces the percentage of scenarios where the company enters liquidation to 2.4% from the base case of 4.0%.

6.5.6 Paying 15% of payroll results in the stake of employees not altering, compared with paying the contribution derived from a market-consistent PUM valuation. The PUM basis increases contributions when the scheme is in deficit, so reducing the risk to employees of default on the pension promise. The fixed contribution of 15% of payroll results in contributions being maintained, even if the scheme is in surplus, and so employees benefit from the discretionary increases.

6.5.7 Figure 6.5 also shows that shareholders are losers. Hence, paying a fixed contribution is an inefficient strategy, as shareholders lose, but

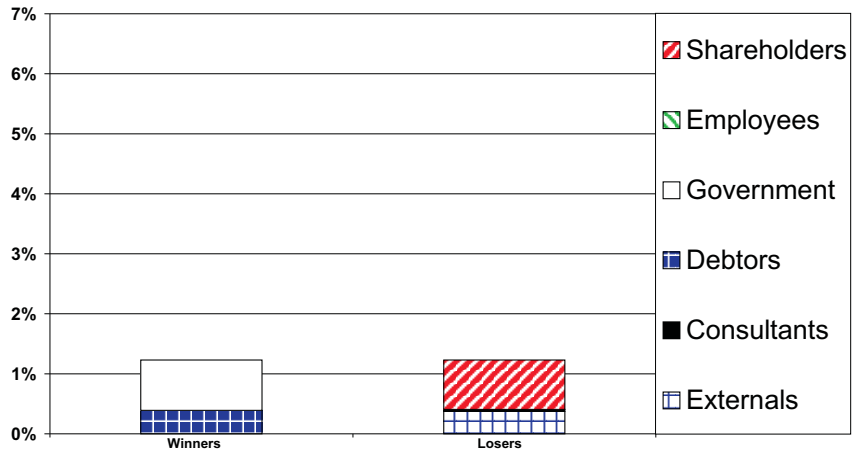



Figure 6.5. Effect of paying a fixed contribution of 15% of payroll

employees do not gain; the main beneficiary is the Government. Under a fixed contribution strategy, the combined stakes of employees and shareholders decrease, compared with paying a higher average and more volatile contribution implied by a market-consistent valuation.

### 6.6 *Funding Strategy — Key Stakeholder Summary*

The combined stakes of the key stakeholders, i.e. shareholders and employees, decrease as the pace of funding decreases (because less money is held in the pension scheme tax shelter). This effect increases as the pace of funding decreases, as is summarised in Table 6.2.

Table 6.2. Impact of different strategies on shareholders and employees

| New strategy | Shareholders | Employees |
|---|---|---|
| DABM funding | +4.4% | −5.3% |
| PUM 1% p.a. risk premium | +2.2% | −2.6% |
| PUM 6% p.a. risk premium | +9.0% | −10.9% |
| Fixed 20% contributions | −0.8% | +0.5% |
| Fixed 15% contributions | −0.8% | 0.0% |

### 6.7 *The Impact of Investment Strategy*

6.7.1 Another key area where actuaries influence the financial management of pension schemes is the setting of investment strategy. We start by considering two situations:

— moving to a bond investment strategy (using market-consistent PUM funding methodology); and
— the interaction between changing to a bond investment strategy and payment of discretionary benefits.

6.7.2 In each case we define a bond strategy to be 50% investment in fixed gilts and 50% in index-linked gilts.

6.7.3 The effect of moving to a bond investment strategy for a market-consistent PUM funding method is shown in Figure 6.6. This change in investment strategy reduces the percentage of scenarios where the company enters liquidation to 2.1% from the base case of 4.0%.

6.7.4 The winners from the strategy are shareholders, who gain as a result of smaller surpluses having to be shared with employees. The size of this change is equivalent to a 6% appreciation in share price. The Government also gains, as the lower funding level volatility means that surplus funds are not generated in the tax shelter of the pension scheme (although not all cash flows relevant for a comprehensive analysis have been modelled, as noted in ¶5.6.3). The lower volatility of the funding level also

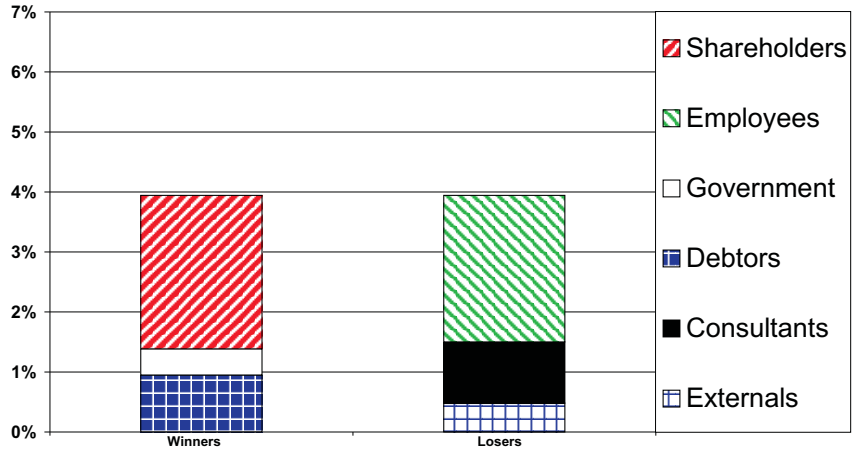



Figure 6.6. Moving to a bond based strategy with PUM funding strategy

feeds through into a lower volatility of pension contributions, which benefits the debt holders. All the winners benefit from the reduced number of liquidations.

6.7.5 The stakeholders who lose from the move to the bond investment strategy are employees, consultants and externals.

6.7.6 The employees lose out, as the bond investment strategy generates less surplus from which discretionary increases can be paid. The consultants lose from the lower fund management charges and fewer wind-ups. The share of externals decreases, as, in more scenarios, they continue to provide the raw materials and services that the company uses to generate profits.

6.7.7 The 'employees' stakeholder group comprises a number of sub-categories, pensioners, actives, non-members and new entrants. The change in investment strategy affects these sub-categories in different ways, as illustrated by Figure 6.7.

6.7.8 Pensioners are the only group significantly affected. It is unclear, from Figure 6.7 alone, the extent to which the change in stakes from the new investment strategy is caused by the sharing of surplus. To investigate this, we repeat the comparison, but with no surplus leakage to employees.

6.7.9 The switch to a bond investment strategy without surplus sharing is the same strategy change that we considered in Section 2.4, where we investigated the answers provided by ALMs. The first difference is that the stakeholder model gives us additional information about winding up. The bond strategy reduces the percentage of scenarios where the company enters

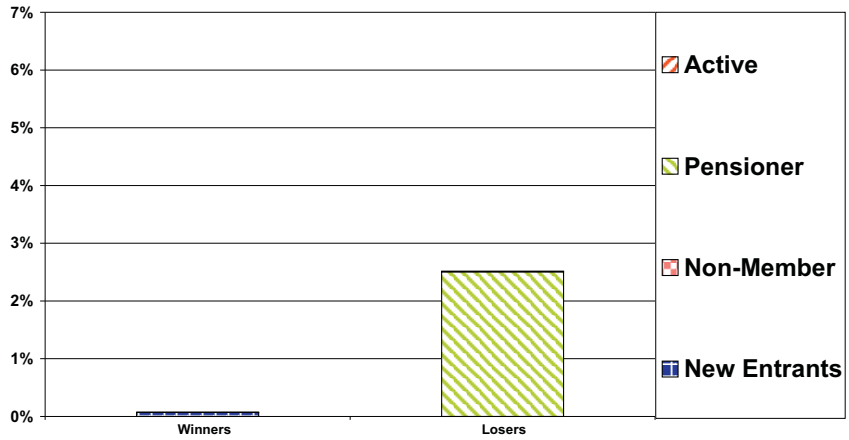



Figure 6.7. Impact on the different employee stakes of moving to a bond based strategy with PUM funding strategy

liquidation from 4.0% to 1.7%. ALMs implicitly assume that a sponsor backing a pension scheme has infinite resources. Figure 6.8 shows the change in stakes when a bond investment strategy is adopted and no surplus is

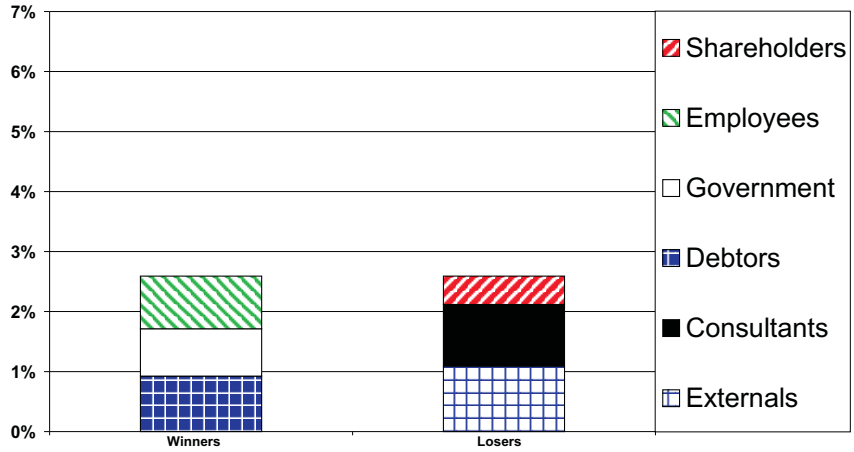



Figure 6.8. Moving to a bond based strategy with PUM funding strategy and no surplus distribution to employees

distributed. We can see that, not only do employees gain from this change in strategy, but the extent of their gain is quantified. This quantitative information cannot emerge as output from a traditional ALM.

6.7.10 If surplus in the pension scheme is used solely to reduce future contributions from the sponsor, and not to enhance pensions in payment, then the employees' stake is larger for a bond-based strategy. This effect is demonstrated by Figure 6.8. The employees' stake in the combined economic entity of the pension scheme and the company is a function of the investment strategies adopted, and whether the surplus is shared. Table 6.3 shows the size of the employees' stake for all the combinations considered.

Table 6.3. Impact of investment strategy on the employees' stake

| Investment strategy | Employees' stake as a proportion of the total |
|---|---|
| Equities (with surplus distribution) | 48.4% |
| Bonds (with surplus distribution) | 46.0% |
| Equities (no surplus distribution) | 43.4% |
| Bonds (no surplus distribution) | 44.3% |

6.7.11 Similar results were obtained when the investment strategy was changed under the DABM funding method.

## 6.8 *Summary — Move to Bond Strategy*

The impact on all the stakeholders is set out in Table 6.4.

Table 6.4. Impact of a change to bond investment

| | | |
|---|---|---|
| Shareholders (with surplus distribution) | *Gain* | Lower volatility in funding level leads to less benefit leakage. Also fewer wind-ups benefit shareholders. |
| Employees (with surplus distribution) | *Lose* | As discretionary increases are less valuable. |
| Shareholders (no surplus distribution) | *Lose* | Option to default on pension promises in the event of the company entering liquidation has decreased in value. |
| Employees (no surplus distribution) | *Gain* | Lower level of volatility reduces wind-ups, benefit from receiving pensions and pay. |
| Government | *Gain* | Receive tax in more scenarios. |
| Debtors | *Gain* | Fewer defaults, lower volatility. |
| Consultants | *Lose* | Lower investment fee income and fewer company liquidations. |
| Externals | *Lose* | Reduced number of wind-ups results in externals contributing more to the entity. |

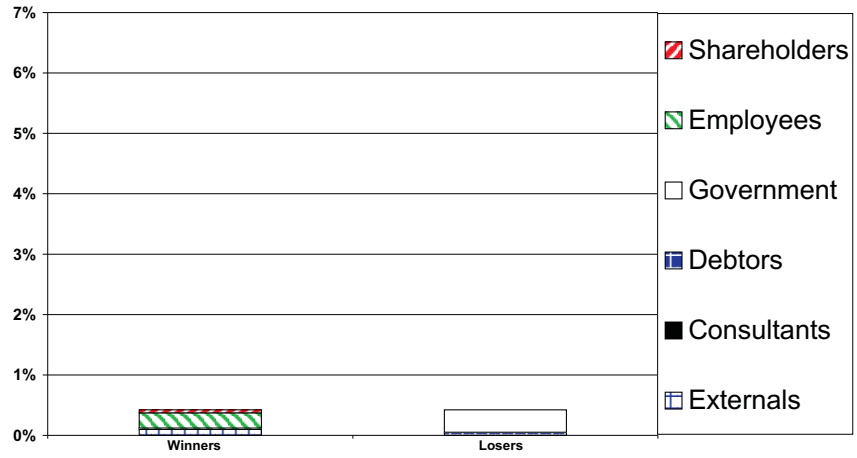



Figure 6.9. Changing the deficit correction period to 5 years for a PUM strategy with equity investment

6.9 *Impact of Deficit Amortisation Periods and Surplus Distribution*

6.9.1 The analysis so far has assumed that deficits are amortised over a 10 year period, and surpluses repaid as contribution reductions are spread over a 5 year period. We now investigate the impact of varying the deficit amortisation period.

6.9.2 Figure 6.9 shows the impact on the different stakeholders of moving to a deficit correction period of 5 years (from 10 years) for a scheme using a PUM market-consistent valuation and investing in equities. Shortening the deficit correction period to 5 years increases the percentage of scenarios when the company goes into liquidation to 4.7% from the base case of 4.0%.

6.9.3 Note that both shareholders and employees are winners as a result of this change. Although the percentage changes are small compared with some of the other strategies, this result is significant, because it illustrates that the key stakeholders can benefit in aggregate.

6.9.4 Employees benefit because the shorter deficit correction period is equivalent to an increased pace of funding, which, as seen earlier, increases their stake. This increase is sufficient to offset the loss that they experience due to the increased number of liquidations. More surprising is the gain experienced by shareholders. Although they pay for the increased rate of funding implied by the shorter deficit correction period, this is offset by the tax efficiency of this strategy. Consultants gain from higher funding in the pension scheme and more company liquidations. Externals also gain (i.e their stake is less negative), because of the increased number of liquidations.

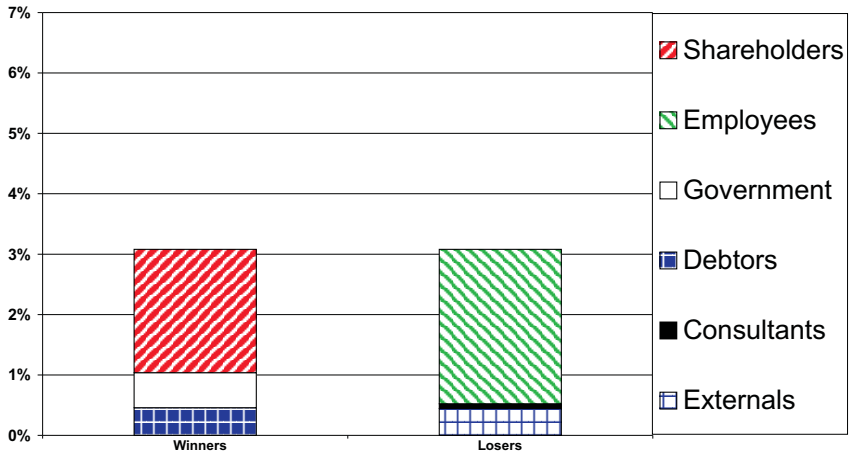



Figure 6.10. Extending deficit correction period from 5 to 10 years for PUM funding strategy with 1% p.a. loading on the discount rate

6.9.5 The losers are the Government, which receives less tax, as more money is sheltered in the pension scheme, and debtors, who suffer a higher rate of default.

6.9.6 We can also analyse the impact of changing the deficit correction period in the proposal for amending the MFR submitted by the Faculty and Institute of Actuaries (2000). Under this proposal, the period for achieving 100% funding on the new MFR basis is extended from 5 to 10 years. Although this is just one of a number of proposed changes in the MFR basis, the impact on the stakeholders is interesting. This is shown in Figure 6.10 where a 1% p.a. loading on the discount rate for non-pensioners has been assumed. With a 10 year deficit correction period, the percentage of scenarios where the company enters liquidation is 2.7% (as noted in ¶6.3.2). With a 5 year period, liquidations are at the 4.5% level.

6.9.7 Figure 6.10 is very different from Figure 6.9, with the shareholders now being winners from a longer deficit correction period. This is a result of the interaction of the deficit correction period and the 1% p.a. loading on the discount rate.

6.9.8 The 1% p.a. loading on the discount rate means that a lower absolute funding level is targeted for the pension scheme. This brings the assets in the scheme down towards (or even, in some cases, below) the assets needed to meet the discontinuance funding level. The option that the shareholders have to default on the pension promise comes closer to being 'in the money', and so its value increases. A longer deficit correction

period means that less money is paid into the scheme, so this option to default remains 'nearer the money' for longer, and so has greater value. Hence, for a 1% p.a. loading on the discount rate, the stake of shareholders increases significantly with the deficit correction period. However, when no loading on the discount rate is used, the option is 'out of the money', has little value, and the tax shelter effect of the pension scheme is more important.

6.9.9 The similarity between Figure 6.10, where the deficit correction period is extended to 10 years, and Figure 6.2, where an additional 1% p.a. loading on the discount rate was introduced, is striking. This suggests that the proposed increase in the deficit correction period, for the amendment to the MFR, produces a similar transfer of wealth from employees to shareholders as increasing the loading on the discount rate used by 1% p.a. However, the interaction between deficit correction periods and additions to the discount rate are non-linear. In Table 6.5 we summarise the effects of changing the deficit correction period and making additions to the discount rate for non-pensioners. Comparing the different strategies demonstrates that lengthening the deficit correction period from 5 to 10 years has a larger adverse impact on the employees' stake than adding a further 1% p.a. to the discount rate (i.e. using a 2% p.a. addition). This shows that members' interests would be better served by maintaining the current deficit correction period, even at the expense of a higher addition to the discount rate. Interestingly, the sponsor's stake does not change significantly.

Table 6.5. Summary of different strategies and their impact on shareholders and employees

| Strategy | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Funding method | PUM | PUM | PUM | PUM | PUM |
| Risk premium | 0% p.a. | 0% p.a. | 1% p.a. | 1% p.a. | 2% p.a. |
| Deficit correction period | 10 years | 5 years | 10 years | 5 years | 5 years |
| Stakes: | | | | | |
| — shareholders | 41.1% | 41.4% | 43.6% | 41.6% | 41.7% |
| — employees | 48.4% | 48.7% | 45.8% | 48.4% | 48.1% |

## 6.10 *Surplus Sharing — Benefit Leakage*

6.10.1 In this section we quantify the effect of surplus sharing with employees, i.e. the cost of benefit leakage. This is an area where actuaries agree that providing extra benefits represents a cost to the employer, but, to date, there has been little objective quantification of this particular cost.

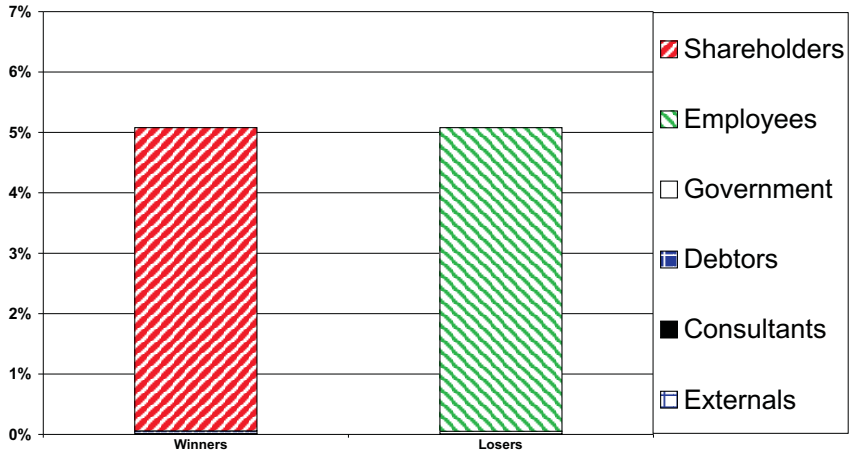



Figure 6.11. Impact of removing discretionary increases for pensioners for a scheme investing in equities

6.10.2 Figure 6.11 shows the change in the stakes from removing the rights of pensioners to share in surplus when a market consistent PUM valuation basis is used. For this approach, the percentage of scenarios resulting in the company entering liquidation reduces slightly to 3.8% from the base case of 4.0%.

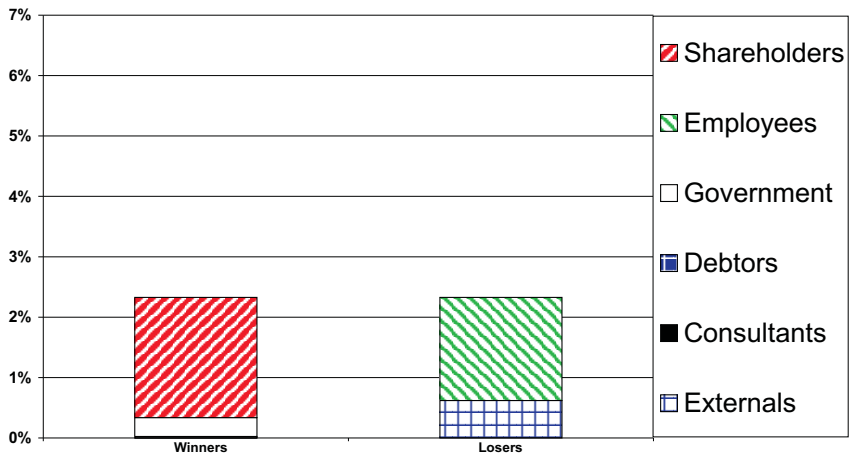



Figure 6.12. Impact of removing discretionary increases for pensioners for scheme investing in bonds

6.10.3 Unsurprisingly, removing discretionary benefits transfers wealth from employees to shareholders. However, the advantage of the approach adopted is that it allows this transfer to be quantified. The transfer of value from employees to shareholders is £32 million.

6.10.4 We can also quantify the amount of the transfer resulting from the investment in equities. Figure 6.12 shows the equivalent figure for removing the rights to discretionary increases where the scheme invests in bonds. In this case, the percentage of scenarios resulting in the company entering liquidation falls from 2.1% to 1.7% (as in ¶6.7.9).

6.10.5 With a bond investment strategy, Figure 6.12 shows us that the discretionary increases have more than halved in value from the viewpoint of employees. This is reflected in the change in the stake for shareholders.

6.10.6 The model's 'rule' for surplus sharing can be tailored for particular cases.

6.11 *Dividend Policy*

6.11.1 In this section we investigate briefly the impact of dividend policy. The base case assumes a dividend policy of distributing 50% of reserves, subject to the constraint that dividends can only be changed by a maximum of 10% in any one year.

6.11.2 We now consider the PUM market-consistent basis with equity investment, but where the company elects not to distribute dividends at all. The results are shown in Figure 6.13. The impact of this change in strategy is to reduce the percentage of scenarios resulting in the company entering liquidation to 1.7% from the base case of 4.0%.

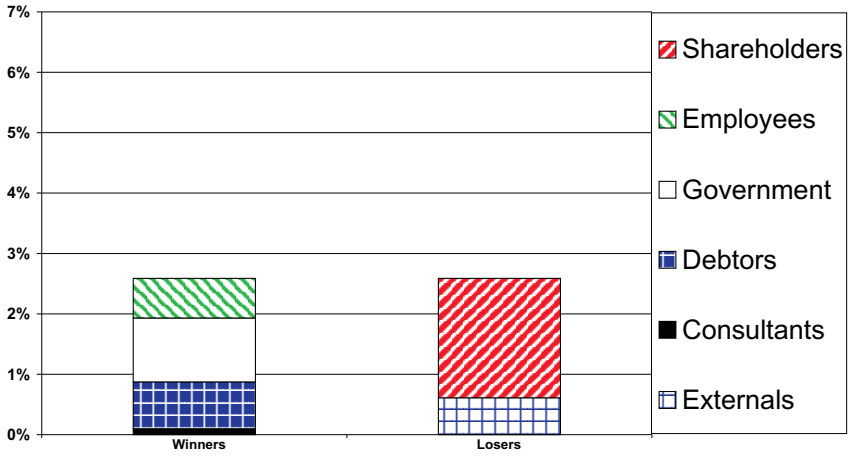



Figure 6.13. Impact of paying zero dividend

6.11.3 Clearly, retaining more funds reduces the likelihood of the company going into liquidation. Employees and debtors both benefit from this. Consultants also gain, because investment fees earned outweigh the loss of income from liquidating the company. The Government also receives higher taxes, as the company remains as an ongoing, tax-paying entity in more scenarios.

6.11.4 The losers are shareholders, who do not receive dividends, and externals, who provide their services to the company in more scenarios, due to the lower number of liquidations.

6.11.5 This illustrates how areas traditionally outside the actuary's consideration have a significant impact on the stakes of the different parties with financial interests in the pension scheme.

6.11.6 Another example is the level at which a company is forced to enter liquidation. Testing shows that the trigger level for liquidation also has a significant impact of the stakes.

6.12 *Impact of Scheme Maturity*

6.12.1 Whilst investment strategy or the funding method may be altered, the size and maturity of a scheme are not parameters over which the same control can be exercised. They are, however, important characteristics of a scheme that will have an influence on the value of the different stakes.

6.12.2 In this section we look at a more mature scheme. We alter the membership profile of our base case, by halving the number of non-pensioner members at all ages, and by increasing the numbers of pensioners, to maintain the value of the ongoing liabilities and the pensioner profile. This results in the pensioner liabilities increasing from 45% to 72% of the total liabilities.

6.12.3 As the structure of the firm is now changed (including the workforce, and hence the profitability per employee), we cannot directly compare this to our base case. Instead, we consider changes in strategy for the mature scheme, and compare this with the effects noted for the original base case analysed to date.

6.12.4 Figure 6.14 shows the impact on the wealth stakes of the different shareholders from changing the investment strategy for a mature scheme from 100% equities to 100% bonds. For our mature scheme, with an equity strategy, the base level of liquidations is 6.8%. The new base level is higher than the original base of 4.0%, because the scheme has a higher proportion of pensioners who share in the surplus. A bond strategy reduces this percentage to 5.2%.

6.12.5 Comparison with the change in investment strategy for a less mature scheme (see Figure 6.6) shows considerable similarity. The winners and losers are the same — only the magnitude of the changes has altered.

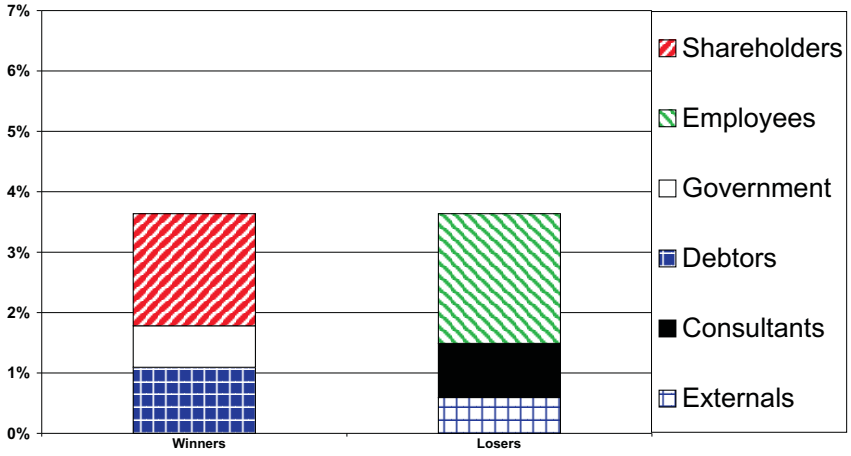



Figure 6.14. Effect of changing investment strategy of mature scheme from 100% equities to 100% bonds

6.12.6 This interesting result indicates how the winners and losers from a change in investment strategy are not primarily dependent on the maturity of the scheme. This is in distinct contrast to traditional actuarial practice, which has advocated bond investment strategies for mature schemes and equity strategies for schemes composed mainly of active members.

6.12.7 The key reasons for the gains and loses are the same as those set out in Table 6.4.

6.13 *Impact of Scheme Size*

6.13.1 In this section we consider a scheme that is larger in proportion to the combined economic entity. The assets and liabilities of the scheme are increased by 50%. To maintain the overall value of the total economic entity, the share capital, loan capital and reserves are reduced. The ratio between the share capital, loan capital and reserves is maintained in the same proportions as before. This change results in the value of the past service liabilities, as measured on the ongoing PUM market basis, increasing from the original base case of 69% to 159% of the value of the company.

6.13.2 As with the mature scheme, the structure of the economic entity has altered, and it is not meaningful to compare the results directly with our original base case. Therefore, we consider the impact of changing the investment strategy for this large scheme, and compare the impact of this change with the impact of the equivalent change on the smaller scheme.

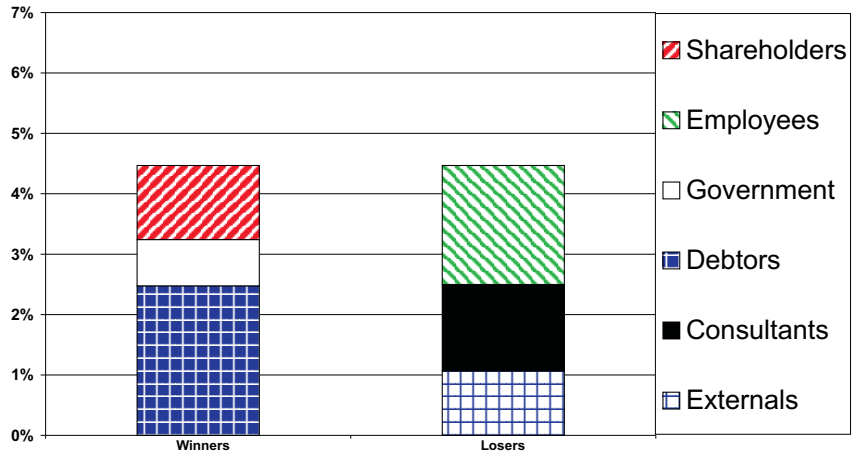



Figure 6.15. Change in stakes moving from 100% equities to 100% bonds in a large scheme

6.13.3 The change in stakes from moving to a bond investment strategy is shown in Figure 6.15. For this change, the percentage of scenarios resulting in the company entering liquidation reduces from 13.8% (equity-based) to 5.8% (bond-based). This reduction in liquidations demonstrates the well-known risk faced by a sponsor with an equity investment policy for a scheme that is sizeable in relation to the company.

6.13.4 The actual winners and losers are the same as for the original base case (Figure 6.6) and for the mature scheme (Figure 6.14). This reinforces the conclusion that demographics ought to be less of a driver of investment policy than is currently the case.

## 7. Conclusions

### 7.1 *Direct Conclusions from the Modelling*

7.1.1 In this paper we have outlined a new framework for viewing pensions advice in a wider corporate context. Our three main conclusions are:

— from the viewpoint of each stakeholder separately, the framework allows us to identify strategies that benefit one stakeholder at the expense of others;

— neither scheme funding policy nor investment policy significantly affects the value of the *combined* stakes of the key stakeholders, i.e. the owners of the company (the equity holders) and the beneficiaries (the employees); and

— if the aim is to increase the value for key stakeholders, that is the enterprise as a whole, actions which reduce costs, including agency costs, may be more beneficial than searching for optimal funding or investment methods.

7.1.2 In terms of other effects, we note that the Government and employees do not generally both gain from the same strategy, because having assets in a pension scheme acts as a tax shelter. As a corollary, we observe that shareholders, debt holders and the Government often gain simultaneously. Although this suggests that the pension playing field can be tilted against employees, care is needed here, because this effect is partly due to the way in which we have defined the economic system. For example, we do not include in our model the state benefits that the Government may provide for pensioners if their occupational scheme fails.

7.1.3 In relation to defined benefit schemes, our main theme is that the stakeholder framework should be used to advise the party responsible for setting the contribution rate. In particular, the effect on other stakeholders of different contribution rates and investment strategies should be explored.

7.1.4 Pension costs are becoming more transparent, and we recommend that pensions actuaries meet the challenge that this poses by:

— extending the scope of their advice to encompass all parties to the pension arrangements; and
— using deflators as an additional valuation tool for the objective assessment of economic interests for the extended range of parties.

7.1.5 We note that actuarial valuations using market assumptions do represent an objective measure, but that their scope is limited by the 'scheme-centric' approach, and does not lead to the identification of winners and losers.

7.2 *Suggestions about Disclosure*

7.2.1 Our suggestions for greater clarity are:

— Member benefit statements should disclose the percentage of the accrued benefits that could be secured in the event of the scheme being wound up. The statements would define as 'funded' the accrued benefits that could be secured, and define as 'unfunded' the remaining benefits that could not be secured.
— Trustee reports to members should disclose both the current investment policy and the trustees' view of the minimum risk portfolio. This should be supplemented by a statement from the trustees explaining, in stakeholder terms, why the risk inherent in the difference between the two portfolios was appropriate to the scheme and to the enterprise as a whole.
— Actuarial valuation reports should disclose:

- as objective and meaningful a solvency level as possible, i.e. the estimated cost of securing the benefits if the sponsoring company ceases to exist; this would be either through an existing insurance company or, if the liabilities meant that they were unacceptable to current insurers, the solvency level, if the scheme was converted into an insurance company;
- the target level of scheme solvency, which we stress is not necessarily 100%, and the corresponding contribution requirement; and
- a financial measure (e.g. the cost of insurance) of the risk of the recommended contributions failing to meet the target funding level (including accruing benefits) until the contribution rate is next reviewed, taking into account the scheme's investment policy.

— Asset liability studies should show the economic interests of all stakeholders, and be presented using (stochastic) discounted values which are consistent with current market conditions.

7.2.2 These disclosures would represent a major step forward in the clarity of pension scheme reporting, and enable stakeholders to make significantly better-informed decisions.

7.3 *Priority of Pension Scheme Deficits*

7.3.1 It seems odd to us that deficits in pension promises, which are, in effect, deferred pay, should rank only with other unsecured creditors. Accordingly, there is a strong case for advancing the priority of pension scheme deficits ahead of all unsecured creditors, and possibly further, although the potential disruptive impact of this on corporate financing would need to be investigated.

7.3.2 Advancing the priority of pension promises would:

— accord more closely with member expectations; and
— allow more reliance to be placed on the company covenant by scheme trustees.

7.3.3 There is no free lunch — in effect, wealth would be transferred from other unsecured creditors (and possibly secured creditors, depending on the implementation) to pension scheme members. However, this approach might provide the Government with a politically acceptable solution to the issue of how to handle the review of occupational pensions.

7.4 *Further Research*

There are a number of areas for further research and developments, including some effects that we have chosen not to model. These areas include:

— modelling companies close to insolvency;
— remuneration effects, such as the need for employers to maintain and communicate the value of employee benefits to avoid staff losses, and frictional costs for employees;
— the impact of liquidity effects (which could be material, given that pension promises are very illiquid);
— allowing for the systematic element of the mortality risk inherent in pension promises — pricing this would be easier if an insurance company were to securitise the mortality element of its annuitant population;
— modelling a wider group of stakeholders, for example including means-tested state benefits and tax on investments;
— behavioural effects, for example the propensity of individual stakeholders within a particular stakeholder group to take actions that seek to maximise their stake within that group; and
— the magnitude of agency costs and the scope for adding value to the enterprise, as a whole, by better management of such costs.

## Acknowledgments

We would like to thank Andrew Smith for his original work that stimulated many of the concepts in this paper. In particular, Appendix B is a virtual transcript of his unpublished work (Smith, 2000).

We would also like to acknowledge the work of those who have spearheaded the debate in the actuarial profession in recent years. In particular, Exley, Mehta & Smith, whose (1997) paper we consider to be a seminal paper for the profession. Many of the ideas developed in this paper have their origin in that paper. We are grateful to the authors for that paper and the numerous other papers that they have contributed in recent years.

We would also like to thank:
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The views and opinions expressed in this paper are ours, and should not be interpreted as those of our firm, the Institute or the Faculty of Actuaries, or any other organisation with which we are associated. Responsibility for errors and omissions is ours alone.

## REFERENCES


AMI, D., KAST, R. & LAPIED, A. (1991). Generalised arrow pricing to understand financial markets. *2nd AFIR International Colloquium*, **4**, 1-26.

ARROW, K. (1953). The role of securities in the optimal allocation of risk bearing. (English translation, 1964). *Review of Economic Studies*, **31**, 91-96.

ACCOUNTING STANDARDS BOARD (2000). *Financial reporting standard 17.* ASB Publications.

BAXTER, M.W. & RENNIE, A.J.O. (1996). *Financial calculus; an introduction to derivative pricing.* Cambridge University Press.

BLACK, F. (1980). The tax consequences of long-run pension policy. *Financial Analysts Journal*, **36**, 21-28.

BREALEY, R.A. & MYERS, S.C. (1996). *Principles of corporate finance (fifth edition). International edition.* McGraw-Hill.

CLARK, G.J. (1992). Asset & liability modelling — the way ahead? Paper presented to the Staple Inn Actuarial Society.

CHEW, D.H. (1999). *The new corporate finance: where theory meets practice (second edition). International edition.* Irwin McGraw-Hill.

COPELAND, T.E. & WESTON, J.F. (1988). *Financial theory and corporate policy (third edition).* Addison-Wesley.

DUFFIE, D. (1996). *Dynamic asset pricing theory (second edition).* Princeton University Press.

EXLEY, C.J., MEHTA, S.J.B. & SMITH, A.D. (1997). The financial theory of defined benefit pension schemes. *B.A.J.* **3**, 835-966.

FACULTY AND INSTITUTE OF ACTUARIES (2000). *Occupational review of the Minimum Funding Requirement.* Faculty and Institute of Actuaries.

GORDON, T.J. (1999). The price of actuarial values. Paper presented to the Staple Inn Actuarial Society.

GOVERNMENT ACTUARY'S DEPARTMENT (1998). *Occupational pension schemes 1995 — tenth survey by the Government Actuary).* HMSO.

GREENWOOD, P.M. & KEOGH, T.W. (1997). Pension funding and expensing in the minimum funding requirement environment. *B.A.J.* **3**, 497-549.

JARVIS, J.J., SOUTHALL, F. & VARNELL, E. (2001). Modern valuation techniques. Paper presented to the Staple Inn Actuarial Society.

JENSEN, M.C. & MECKLING, W.H. (1976). Theory of the firm: managerial behavior, agency costs and ownership structure. *Journal of Financial Economics,* **3**, **No 4**, 305-360.

MCLEISH, D.J.D. & STEWART, C.M. (1987). Objectives and methods of funding defined benefit pension schemes. *J.I.A.* **114**, 338-424.

MODIGLIANI, F. & MILLER, M.H. (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review*, **48**, 261-297.

MOODY'S INVESTMENT SERVICE (2000). *Historical default rates of corporate bond issuers, 1920-1999.* Downloadable file from www.moodys.com/defaultstudy

PWC (2000). *PricewaterhouseCoopers Pensions*, August 2000. PDF file downloadable from www.pwcglobal.com.

SMITH, A.D. (1996). How actuaries can use financial economics. *B.A.J.* **2**, 1057-1175.

SMITH, A.D. (2000). Internal Bacon & Woodrow paper.

TEPPER, I. (1981). Taxation and corporate pension policy. *Journal of Finance*, **36**, 1-13.

YAKOUBOV, Y.H., TEEGER, M.H. & DUVAL, D.B. (1999). The TY model. Paper presented to the Staple Inn Actuarial Society.

## APPENDIX A

## APPLICATION OF A STAKEHOLDER MODEL

A.1 *Inputs and Calibration*

A.1.1 The stakeholder model values all the cash flows between stakeholders. We calculate the value of each stake in a manner that is consistent with observed market prices. The model requires the following inputs:

— the pension scheme details, including benefit structure, investment strategy and solvency level;
— the company's financial structure, last year's accounts, dividend policy and the trigger for a company entering liquidation;
— the market value of the company and the pension scheme;
— income tax, corporation tax and National Insurance rates; and
— economic and statistical inputs for the stochastic economic model.

A.1.2 Details of the company's capital structure are also required, as this contains information about the risks faced by different stakeholders. For example, the level of gearing, reserves available and the dividend cover all affect the risk in holding shares in a company.

A.1.3 The basic input to the model is the total value of the company and the pension scheme, i.e. the total wealth in the system at the outset. The model then describes how strategic decisions affect the division of this wealth.

A.1.4 The value of the company at any time is assumed to be the total realisable value of all the assets available in the event that the company enters liquidation, allowing for any discontinuance pension scheme deficit at market value. The value of the firm measured in this way does not recognise the value of future profits to the shareholders.

A.1.5 Although the pension scheme's discontinuance deficit is included in the accounting valuation of the company, the assets held in the pension scheme are not. Hence, the combined value of the company and pension scheme is the value of the company, as described in ¶A.1.4, plus the market value of the assets in the pension scheme.

A.1.6 The stakes represent the economic value to the different stakeholders in the combined economic entity. If the equity (and debt) of a company is quoted, we would have a market assessment of the stakes of the shareholders and loan stockholders. In that case, the model would be calibrated to these market values.

A.1.7 The shareholders' and loan stockholders' stakes (i.e. the equity and debt market value) differ, from time to time, from the accounting value of the company, as described in ¶A.1.4, because the market assesses the value of future cash flows.

A.1.8 Although the stakes of different stakeholders alter from time to time, the total value of the combined entity remains unchanged, as we are dealing with a closed economic system. To the extent that the shareholders' stake exceeds the realisable asset value of the company after meeting all prior claims, there must be an offsetting change in other stakes. In our model, the main example of an offsetting change is the externals stakeholder (i.e. the difference between suppliers and customers). For a profitable company, the externals will have a negative stake that represents the value of a cash stream, which we label as future oligopoly profits. If the company ceases to trade, the externals' stake reverts to zero, as they make no further contribution to the economic system.

A.2 *Stakeholder Model*

A.2.1 The model tracks the cash flows between the following stakeholders:

— employees, both members of the pension scheme and non-members;
— shareholders;
— loan stock holders;
— the Government, as a raiser of taxes;
— externals (the cash flows made to suppliers less payments received from customers);
— consultants and advisers; and
— a lender of last resort (who will ensure that receivers' fees are always paid).

A.2.2 The deflated value of all the future cash flows receivable by a stakeholder provides a market-consistent value of their stake. To achieve full convergence of deflated value to current value would require an infinite summation over time and an infinite number of simulations. In Appendix B we describe a methodology under which the deflated values of the stakes converge rapidly.

A.2.3 Our starting assumption is that all stakeholders, apart from the Government, wish to maximise the value of their stake in an enterprise, on the basis of information available to them. We assume that the Government adopts a passive role and maintains the current level and structure of taxation and benefits.

A.2.4 In addition, we introduce a lender of last resort, who meets the costs of winding up in those circumstances where the residual company assets are not sufficient. The introduction of this lender is a device that simplifies the modelling, by providing for cash flows that would otherwise move outside our defined economic system. The modelling work carried out indicates that the size of this stake is negligible in comparison to those of the other stakeholders. Therefore, we have omitted the lender of last resort from the charts in the results section.

A.2.5 All cash flows between the different stakeholders, including the lender of last resort, are modelled. For each cash flow attributed to a stakeholder, there will be a compensating opposite cash flow for other stakeholders. Many of these cash flows are defined by specific rules; examples are tax on earnings paid by individuals and pensions received at retirement. Other cash flows need decision rules to be specified, including, for example, what discretionary benefit increases will be granted (and in what circumstances) and what the dividend policy of the company will be. We have made assumptions about these decision rules in our model, but this is an area where practitioners can gain insights by varying the assumptions.

A.2.6 The operating structure of the company is an inherent part of the model. We introduce the simplifying assumptions that the company outsources all functions, other than core production, and has no fixed operational assets; all reserves are then assumed to be invested in realisable assets. The following cash flows relating to the company are then considered:

— sales revenue less supplier costs;
— investment income on the company's assets;
— staff costs relative to a benchmark;
— payments to service debt;
— contributions to the pension scheme; and
— fund management fees and other consultancy costs.

A.2.7 From these cash flows, we construct a profit and loss account from which we deduce a first order estimate of corporation tax payable. Given an assumed dividend policy, the change in reserves is calculated, enabling a skeleton balance sheet to be constructed.

A.2.8 Staff costs are modelled relative to an assumed benchmark level for the industry, the rationale being that the value of the benefits received by the employee is the excess over the market rate for that job. The benefit that an employee derives from employment is the excess remuneration over the market rate for his or her labour. So, in the event of the company winding-up, his or her financial loss is limited to the loss of any benefit in excess of the benchmark. The corollary is that the rates of benefit above the benchmark will affect the profitability of the company. For this purpose, we consider employee benefits to be salary, uplifted by a percentage to reflect other benefits receivable, including any profit-related pay. We keep pension benefits separate from the assessment of the employees' total remuneration package to reflect their deferred nature.

A.2.9 The shareholders receive any dividend paid, plus, in the event of the company entering liquidation, any residual worth after all prior claims on the assets have been met.

A.2.10 The process of identifying all the cash flows received or paid is repeated for all the other stakeholders in the system.

A.2.11 While actuaries will be familiar with modelling some of the items described, e.g. staff costs and pension scheme contributions, they will not be so familiar with others. However, extensive data are available for the quantities required, and actuaries ought to view this modelling exercise as an application of core cash flow modelling skills.

A.2.12 In the event of the company ceasing to trade, a distribution of remaining assets must be made. The cash flows need to be specified in terms of the priorities of the different stakeholders (and, as necessary, within sub-divisions of the stakeholders).

A.2.13 Initial conditions need to be set, which include the capital structure of the company and details of the pension scheme. The pension scheme details include:

— benefit structure;
— initial solvency level; and
— investment strategy.

A.2.14 Once the cash flows and priorities are fully specified, a stochastic model is used to simulate all the future cash flows to the stakeholders. These cash flows are then weighted, using deflators, and averaged to calculate the present value of each stake. Different strategies can then be compared; for example, we can vary the investment strategy of the pension scheme to identify how different stakeholders are affected.

A.3 *Economic and Investment Outputs*

A.3.1 One of the necessary components for the projection is the stochastic investment model. The minimum requirement for this model is that it produces:

— returns on the investment asset classes;
— oligopoly profit for the company;
— inflation, salary growth, and any other economic variables needed for the liability cash flows; and
— deflators to provide economic consistency.

A.3.2 Stochastic models are now commonly used in actuarial work, but in order to provide meaningful results for value-based work, the model must be able to produce deflators.

A.3.3 To describe how the company's profits fluctuate with economic conditions, we model a stochastic variable that we call 'oligopoly profit'. These profits are the income from sales after deducting supplier costs and the benchmark staff costs for the industry. We assume that a company achieves oligopoly profit by operating in a market without perfect competition; if a market had perfect competition, there would be no oligopoly profit.

A.3.4 By specifying the market portfolio (i.e. the universe of investible assets), the deflator approach allows us to price the systematic risk in the

market. The systematic risk in oligopoly profit is defined by the correlation between oligopoly profit and market return. The non-systematic element of risk in oligopoly profit is uncorrelated with the market return, and therefore does not affect market prices.

A.3.5 This has parallels with the capital asset pricing model (CAPM) for calculating the expected return on a stock. However, whereas the CAPM operates in a one-period framework, the deflator-based approach generalises the model to multi-period pricing.

## APPENDIX B

## EVALUATING THE STAKES IN AN ENTERPRISE

B.1 *Valuing Cash Flows*

B.1.1 Our aim is to calculate the value of each stakeholder's stake. The ideal would be to value the cash flows received by each stakeholder at every future time in every simulation. By calculating the expectation of these deflated cash flows, we would derive the value of each stake.

B.1.2 This approach is impractical, because it requires an infinite summation over time and an infinite number of projections to ensure that the deflated values converge to current market values.

B.1.3 In this appendix we outline a robust alternative, that deals with time horizons that are sufficiently long to include business planning horizons, but over which, given sufficient simulations, the deflated values of the stakes converge. This appendix draws heavily on the ideas developed in Smith (2000), to whom we are indebted.

B.1.4 We start by considering proxies for the value of the stakeholders' stakes in the economic system. We term these proxies 'accounting values', as they report the value of different stakes that may not necessarily concur with the economic value of the stakes. Under certain restrictions, we show that the following equivalence holds:

$$\frac{1}{D(s)}\sum_{u=s+1}^{t}\mathsf{E}_s[D(u)\times Acc\_valadd(u)] + Acc\_val(s) =$$

$$\frac{1}{D(s)}\sum_{u=s+1}^{t}\mathsf{E}_s[D(u)\times CF(u)] + \frac{1}{D(s)}\mathsf{E}_s[D(t)\times Acc\_val(t)]$$

where:

— $D(t)$ is the deflator at time $t$;
— $Acc\_valadd(t)$ is the accounting value added between $t-1$ and $t$;
— $Acc\_val(t)$ is the accounting value at time $t$;
— $CF(t)$ is the cash flow received immediately before time $t$; and
— $\mathsf{E}_s$ is the expectation operator given information up to time $s$.

B.1.5 This equation has an intuitive explanation. As $t \to \infty$ the right hand side becomes the deflated value of all future cash flows, the value of the stake.

B.1.6 Therefore, we see that there are two ways of valuing a stake. We can either sum all deflated cash flows and add in the deflated terminal accounting value, or we can take the initial accounting value and sum the deflated accounting value accruing in all future time periods.

B.1.7 This identity provides a route forward that avoids the difficulties of summing infinite series over infinite numbers of simulations. By choosing an appropriate accounting convention, we can arrange for the sum of the deflated accounting value added figures to converge. The economic value of any stake can then be evaluated through the accounting value added figures.

B.2 *Accounting Values*

B.2.1 Stakeholders usually have their stakes in an enterprise reported periodically as accounting numbers. These numbers will differ from the market value and true economic worth of the stake, because, for example, accounting values only take account of existing contracts, while future cash flows will depend on future contracts as well. Furthermore, accounting numbers may not be on a market value basis, and include arbitrary rules for depreciation and amortisation.

B.2.2 Despite the lack of a clear relationship between accounting values and the underlying economic value of a stake, accounting numbers have the advantage of providing an alternative, off-market, measure that can be calculated.

B.2.3 For a given accounting convention, we define the accounting profit as the change in accounting value over the period plus any cash flows received. Hence:

$$Acc\_profit(t) = CF(t) + Acc\_val(t) - Acc\_val(t-1)$$

where $Acc\_profit(t)$ is the accounting profit between $t-1$ and $t$.

B.3 *Measures of Added Value*

B.3.1 To derive a measure of added value, we need to compare the change in the accounting value with the opportunity cost, i.e. the cost incurred by investing in the business, as opposed to making some alternative investment. We refer to the return on the alternative investment as the cost of capital.

B.3.2 The accounting value added can then be written as:

$$\begin{aligned} Acc\_valadd(t) &= Acc\_profit(t) - Cost\_cap(t) \times Acc\_val(t-1) \\ &= CF(t) + Acc\_val(t) - [1 + Cost\_cap(t)] \times Acc\_val(t-1) \end{aligned}$$

where $Cost\_cap(t)$ is the cost of capital between $t-1$ and $t$.

B.3.3 By using a stochastic model, the calculation of the cost of capital is greatly simplified, because we avoid the need to identify an investment with a similar risk. We can use the market return on any alternative investment, because the stochastic model captures the risk (and return) in the

simulations. This gives us the freedom to choose a convenient investment when calculating the cost of capital.

B.4 *Accounting Measures and Deflated Cash Flows*

B.4.1 Using the deflator property (see ¶C.2) we can write:

$$\mathsf{E}_{u-1}\big(D(u) \times [1 + Cost\_cap(u)]\big) = D(u-1).$$

B.4.2 Hence, multiplying through by $Acc\_val(u-1)$ gives:

$$\mathsf{E}_{u-1}\big(D(u) \times [1 + Cost\_cap(u)] \times Acc\_val(u-1)\big) = D(u-1) \times Acc\_val(u-1).$$

B.4.3 Using the tower law of conditional expectations, we can evaluate this expression at an earlier time $s$. Hence, for $s \leq u-1$, we have:

$$\mathsf{E}_s\big(D(u) \times [1 + Cost\_cap(u)] \times Acc\_val(u-1)\big) = \mathsf{E}_s\big(D(u-1) \times Acc\_val(u-1)\big).$$

B.4.4 Similarly, we can write a generalised expression for the expected accounting value added, when evaluated at an earlier time $s$. Using the equation from ¶B.3.2:

$$\begin{aligned}\mathsf{E}_s\big(D(u) \times Acc\_valadd(u)\big) = {} & \mathsf{E}_s\big(D(u) \times CF(u) + D(u) \times Acc\_val(u)\big) \\ & - \mathsf{E}_s\big(D(u) \times [1 + Cost\_cap(u)] \times Acc\_val(u-1)\big).\end{aligned}$$

B.4.5 Substituting for the final term, using ¶B.4.1, gives:

$$\begin{aligned}\mathsf{E}_s\big(D(u) \times Acc\_valadd(u)\big) = {} & \mathsf{E}_s\big(D(u) \times CF(u) + D(u) \times Acc\_val(u)\big) \\ & - \mathsf{E}_s\big(D(u-1) \times Acc\_val(u-1)\big).\end{aligned}$$

B.4.6 Summing over values of $u$, from $u = s+1$ to $t$, we find that intermediate $Acc\_val$ terms cancel to give:

$$\begin{aligned}\sum_{u=s+1}^{t} \mathsf{E}_s\big(D(u) \times Acc\_valadd(u)\big) = {} & \sum_{u=s+1}^{t} \mathsf{E}_s\big(D(u) \times CF(u)\big) + \mathsf{E}_s\big(D(t) \times Acc\_val(t)\big) \\ & - \mathsf{E}_s\big(D(s) \times Acc\_val(s)\big).\end{aligned}$$

B.4.7 Rearranging, we obtain the expression in the equation in ¶B.1.4. The final expression is independent of the cost of capital, and the inclusion of the cost of capital was purely for algebraic simplicity.

B.5 *Application to Companies and Pension Schemes*

B.5.1 To simplify the use of accounting measures for pension schemes and their sponsors, we will require that:

— all stakeholders who pay or receive cash flows from the scheme or sponsor are included in the analysis;
— the total accounting value for all stakeholders is equal to the market value of the economic system (the market value of shares and loan stock in the company plus the market value of assets in the pension scheme); and
— the cost of capital is defined to be equal to the return on the aggregate funds invested.

B.5.2 We now can consider the different stakeholders, denoted $j = 1$ to $n$, such that:

$$Total\ market\ value(t-1) = \sum_{j=1}^{n} Acc\_val(t-1, j).$$

B.5.3 Immediately before time $t$, this market value will have grown by the return on the assets, which has been defined to be the cost of capital. At time $t$ cash flows are made, hence the market value at time $t$ is:

$$Total\ market\ value(t) = \sum_{j=1}^{n} Acc\_val(t-1, j) \times \big(1 + Cost\_cap(t)\big) - \sum_{j=1}^{n} CF(t, j).$$

B.5.4 This is also equal to the sum of the accounting values of all the stakes of the different stakeholders.

B.5.5 We can now calculate the total accounting value added for all parties:

$$\sum_{j=1}^{n} Acc\_valadd(t, j) = \sum_{j=1}^{n} CF(t, j) + \sum_{j=1}^{n} Acc\_val(t, j) - \sum_{j=1}^{n} Acc\_val(t-1, j) \times \big(1 + Cost\_cap(t)\big).$$

B.5.6 However, from ¶B.5.1, we know that this is equal to zero, as we require that the total accounting value must equal the aggregate market value. The above formula represents an important insight into the coherent

analysis of financial stakes. It says that value cannot be created, not even accounting value, for all parties at once. If one stakeholder gains, another stakeholder must lose out.

B.6 *Defining an Accountancy Standard*

B.6.1 To this stage, our analysis has been independent of the actual accounting standard. We are now ready to select a standard from which we can evaluate the accounting value added, and hence calculate the stakes of different stakeholders. We require an accounting standard that is consistent with the conditions in ¶B.5.1, and, in particular, we require the aggregate accounting value for all stakeholders to be equal to the market value of the economic entity.

B.6.2 Let us consider two extreme choices of accounting convention. This will inform our choice of an appropriate convention.

B.6.3 One possibility would be to set the accounting value of a particular stake equal to zero for all time periods. This results in no capital gains to the accounting profit, the only element in the accounting profit being the cash flows. It is clear that this is no more than cash flow accounting. This accounting standard is easy to implement, but does not assist us in calculating the values of different stakes.

B.6.4 A more radical accounting standard would be to use the deflated value of future cash flows. Our result in ¶B.1.4 now does not depend on the time horizon $t$. Also, by choosing an appropriate definition of the cost of capital, we can reduce the variability, and hence reduce the number of simulations required to calculate the stakes.

B.6.5 The problem with this second approach is that we are essentially required to know the answer in advance (if we know the deflated values of the future cash flows initially, we would not have to use projections and deflators to recover the answer that we already knew).

B.6.6 The practical implications are that we need to choose an accounting method that provides rapid convergence, but is also easy to calculate. The accounting method selected should provide a proxy for the stake. The more approximate this proxy, the larger the number of simulations that will be required to recover the true economic values.

B.6.7 For some stakeholders there are clear choices for the accounting stakes, for example, holders of loan stock may use the capital value of the loan, the shareholders may use share capital plus reserves. Other stakeholders' stakes are less easily defined, for example the Government's stake or that of receivers.

B.6.8 The work in this appendix has demonstrated that the choice of accounting standard does not affect the results obtained, but it will affect how quickly the results produced converge to the stakes of the different stakeholders. Therefore, the choice of accounting standard is a matter of reducing the computing power needed.

## APPENDIX C

## ECONOMIC MODEL

C.1 *Specification*

C.1.1 We model the returns on cash and inflation using a bivariate auto-regressive process. We first model the continuously compounded rates of return (or 'forces of return').

C.1.2 We let $I(t)$ denote the force of inflation in the period $(t-1, t)$ and $R(t)$ for the nominal force of interest in the interval $(t, t+1)$. Hence, at time $t$, both the values $I(t)$ and $R(t)$ are observable.

C.1.3 The values of returns on cash and inflation over the subsequent period are calculated as:

$$\begin{bmatrix} I(t+1) \\ R(t+1) \end{bmatrix} = A \begin{bmatrix} I(t) \\ R(t) \end{bmatrix} + (1-A) \begin{bmatrix} IMU \\ RMU \end{bmatrix} + \begin{bmatrix} IE(t+1) \\ RE(t+1) \end{bmatrix}$$

where:
— $IMU$ is the long term mean force of inflation;
— $RMU$ is the long term mean force of interest (return on cash); and
— $A$ is an auto-regressive parameter.

C.1.4 The randomness to the inflation and interest rate processes are provided by:

$$\begin{bmatrix} IE(t) \\ RE(t) \end{bmatrix}$$

so-called random innovations, a series of independent identically distributed (iid) random vectors, that have zero mean and a variance covariance structure:

$$\begin{bmatrix} IVAR & IRCOV \\ IRCOV & RVAR \end{bmatrix}.$$

C.1.5 In the case where normally distributed innovations are used, this can be seen to be a bivariate version of the Vasicek interest rate model, providing full-term structures for both nominal and real interest rates.

C.1.6 The term structure model is calibrated to the observed market yields on nominal and index-linked bonds. The remaining parameters are derived from the volatility and covariances of the bonds chosen for calibration. The model is structured so that all nominal bonds are constrained to mature at their par value (and index-linked bonds mature at their par, adjusted for experienced inflation).

C.1.7 The force of average earnings growth in the interval $(t-1, t)$ is denoted by $N(t)$. This is modelled relative to the force of inflation by:

$$N(t) = I(t) + NMU + NSD.NE(t)$$

where:
— $NMU$ is the long-term earnings growth in excess of inflation;
— $NSD$ is the standard deviation of earnings growth relative to inflation; and
— $NE(t)$ is an independent random innovation with zero mean and unit variance.

C.1.8 Returns on other asset classes are modelled relative to the return on cash. The force of return $F_i(t)$, on the $i$th asset in the interval $(t-1, t)$, is given by:

$$F_i(t) = R(t) + MU_i + \sum_{j=1}^{n} L_{ij}E_j(t)$$

where:
— $R(t)$ is the force of return on cash in the period $(t, t+1)$;
— $MU_i$ is the risk premium on the $i$th asset in excess of cash;
— $E_j(t)$ is the $j$th random independent innovation;
— $L_{ij}$ denotes the sensitivity of the return on the $i$th asset class to the $j$th random innovation; and
— $i$ and $j$ take values 1 to $n$, where $n$ is the number of asset classes.

C.2 *Deflators*

C.2.1 A deflator is a stochastic process such that the product of the total return index for a tradeable asset and the deflator is a martingale. In other words, the deflator $D(t)$ satisfies the following identity:

$$D(t) \times TR_i(t) = \mathsf{E}_t[D(t+j) \times TR_i(t+j)] \quad \forall\, j > 0$$

where:
— $TR_i(t)$ is the total return index for asset $i$; and
— $\mathsf{E}_t$ represents the expectation operator conditional on the information available at time $t$.

C.2.2 Given the simple structure of the economic model described above, it is possible to derive explicit expressions for the deflators. Using subscripts to denote the $i$th asset, we see from the martingale property that:

$$1 = \mathsf{E}_t\left[\frac{D(t+1)}{D(t)}\frac{TR_i(t+1)}{TR_i(t)}\right]$$

$$1 = \mathsf{E}_t\left[\frac{D(t+1)}{D(t)}\exp\left(R(t) + MU_i + \sum_{j=1}^{n} L_{ij}.E_j(t)\right)\right].$$

C.2.3 This expression indicates a functional form for the deflator. The characteristic exponent $\psi(p)$, for a random variable $X$, is defined as $\mathsf{E}[\exp(pX)] = \exp(\psi(p))$. We can then show that the deflator ratio will have the following format for some set of $\xi_j$:

$$\frac{D(t+1)}{D(t)} = \exp\left(-R(t) + \sum_{j=1}^{n} \xi_j E_j(t) - \sum_{j=1}^{n} \psi(\xi_j)\right).$$

C.2.4 Combining this expression with the total return index, we see that:

$$MU_i = \sum_{j=1}^{n} \psi(\xi_j) - \sum_{j=1}^{n} \psi(\xi_j + L_{ij}).$$

C.2.5 The choice of distribution will determine the functional form of $\psi(p)$, and hence, using ¶C.2.4, the set of $\xi_j$ can be recovered. Using this set of $\xi_j$ and the sampled observations from the chosen distributions enables deflators to be evaluated.

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