# Pricing excess of loss treaty with loss sensitive features: an exposure rating approach

by

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## The problem

- Given:
  - The expected loss cost for the treaty.
  - The characteristics of the portfolio of policies: mixture of lines of business, limits and deductibles.
  - The reinsurance layer: *m* xs *l*
- Estimate an aggregate loss distribution (frequency and severity) that includes all these characteristics.

## The components of reinsurance pricing (I)

- Expected loss cost
  - Experience methods (burning cost, development triangles, etc)
  - Exposure methods (benchmark curve from industry or risk specific)
  - Mixed methods (combination of experience and exposure methods)
- We do not discuss the methods for estimating the loss cost.

## The components of reinsurance pricing (II)

- Premium:
  - Fixed rate
  - Increase or decrease with losses incurred (loss sensitive)
- Other costs: expenses, commissions
  - Fixed % or $ amount
  - Loss dependent
- Profit margin: fixed load or through modelling of cash flows.

## Loss sensitive features

| Feature | Variable component | Description |
|---|---|---|
| Margin plus/ Swing rating | Premium | Premium adjusts with losses incurred times a load, subject to a minimum and a maximum. |
| Profit commission | Commission/ Expenses | Profit is shared with cedant after a load for reinsurer's expenses. |
| Loss Corridor | Losses | Cedant retains part of losses attaching at pre-determined value of LR. |
| Reinstatement | Premium and Loss | Limits the number of total losses. Extra premium is received to reinstate the layer. |
| Annual Aggregate Deductible (AAD) | Losses | Cedant retains the first D losses in aggregate. |

## The need of an aggregate model

- If ***S*** represents the aggregate losses to the layer, then Loss Cost = E[***S***] = E[***X***]E[***N***].
- When premium and expenses vary with losses they become random variables (functions of the aggregate losses ***S***).
- In general Jensen's inequality holds:

$$f(E[S]) \neq E[f(S)]$$

## The need of an aggregate model

- We need to estimate the expected value of premiums and commissions when they are variable.
- Therefore we need an aggregate loss distribution for ***S*** such that

Loss Cost = E[***S***]

## Method 1: Parametric distribution

- Fit a parametric distribution (lognormal, gamma, etc.) using the method of moments.
- E[***S***] given by the loss cost.
- Var(***S***) estimated assuming a Poisson or Negative Binomial distribution for frequency.
- Estimate the parameters.

## Method 1: Parametric distribution

- Very easy to implement and understand.
- It ignores the probability of having zero losses to the layer (not realistic for some lines of business).
- Does not separate frequency and severity distributions.
- Does not account for mixtures of policy limits and deductibles. (E.g. $1m policy limit with no deductible or with $10m deductible).

## Method 2: benchmark severity distribution

- Select an appropriate severity distribution for the line of business. Industry benchmark (ISO) or account specific. Calculate E[***X***].
- Choose a frequency distribution (Poisson or Negative Binomial). Estimate the parameters.
- For Poisson: $\lambda = E[N] = \frac{\text{Loss Cost}}{E[X]}$

## Method 2: benchmark severity distribution

- Compute aggregate losses (Panjer recursion, Fourier Transforms, etc.) See Appendix A.
- Improvement over Method 1: allows for probability of zero and at layer limit.
- When different policy limits are covered the severity might be overestimated since not every claim might reach the full layer limit.

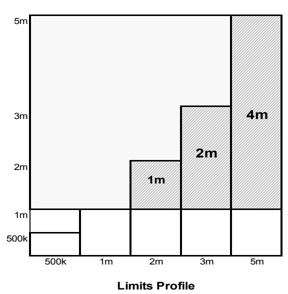
## Method 3: Exposure based severity curve

- Objective: estimate a "blended" severity distribution that:
  - Takes into account all combinations of policy limits and deductibles written by cedant.
  - Allows for multiple lines of business.
- How? : Using the exposure rating method.
- Given this severity, the frequency distribution is estimated as in Method 2.

## Review of the exposure method

- Estimates the proportion of the risk ceded to the reinsurance layer.
- Basic ingredients:
  - Ground-up loss ratio
  - Ground-up severity distribution (benchmark or risk specific)
  - Limits profile: policy limits, deductibles, % of premium for each combination.

## The exposure method for a $4 xs $1m layer




## The formula

- ***X= Ground-up loss severity***
- ***PL***=Policy Limit;
- ***d***=deductible;
- ***Layer: l xs m***

$$\text{Loss Cost} = (\text{S.P.}) * (\text{GU LR}) * \frac{E[X \wedge \min(PL+d, l+m+d)] - E[X \wedge \min(PL+d, m+d)]}{E[X \wedge PL+d] - E[X \wedge d]}$$

Where

$$E[X \wedge a] = E[\min(X, a)]$$

## Estimating frequency with the exposure method

- If we use the exposure method in a layer ***$1 xs m***, it can be shown that the result is the expected frequency in excess of ***m.***
- Given frequency at various attachments the distribution function can be estimated. All math is explained in the paper.
- This is the key result in developing our "blended severity".

## The basic recipe (by line of business)

- Split the layer ***l xs m*** in sub-layers of size ***h*** (small enough to keep resolution but not too small to save computing time).

***h xs m***
***h xs m+h***
***h xs m+2h***
***... ... ...***
***h xs l+m-h***

## The basic recipe (cont'd)

- For each sub-layer estimate the expected frequency using the exposure method.
- Given frequency at each sub-layer, estimate the severity distribution (by line of business)
- With the distribution function estimate the severity density function (by line of business).

## The basic recipe (cont'd)

- Mix all the density functions by LOB weighted by expected frequency to the layer. (Assumes independence between lines)
- All the mathematical details are explained in the paper.
- Result: a "blended" severity curve that takes into account all the policy limit combinations and mixture of lines of business.

## The basic recipe (cont'd)

- With the "blended severity" calculate E[***X***] and then

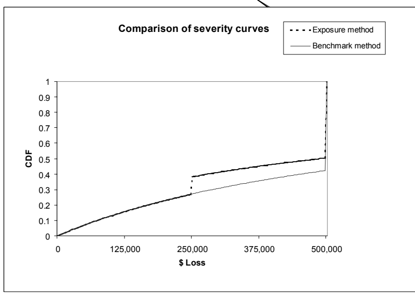
$$E[N] = \frac{\text{Loss Cost}}{E[X]}$$

- Fit a frequency distribution (Poisson or Negative Binomial).
- Compute aggregate losses (Panjer recursion, Fourier Transforms, etc.). Estimate the expected value of all loss sensitive features.

## Worked example: professional liability $500k xs $500k

| | Lawyers | | E&O | |
|---|---|---|---|---|
| Deductible | $10,000 | $25,000 | $50,000 | $50,000 |
| Limit | $750,000 | $1,000,000 | $1,500,000 | $2,000,000 |
| Premium | $1,000,000 | $2,000,000 | $2,000,000 | $3,000,000 |
| FGU LR | 65% | 65% | 75% | 75% |

## Severity distributions



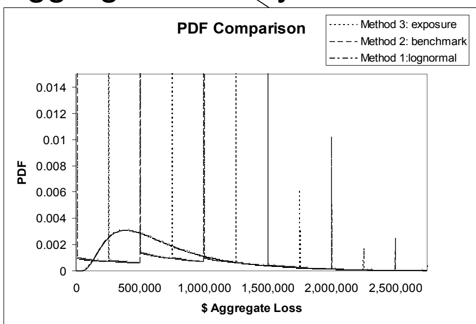

## Assumptions and computation

- Using the expected implied frequency and a variance multiplier of 2 (see Appendix B) we fitted a Negative Binomial distribution.
- Using the severity and frequency distribution we computed the aggregate distribution using Panjer's recursive algorithms.

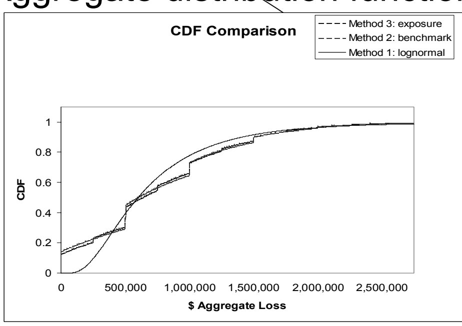
## Loss cost, severity and frequency

| | | |
|---|---|---|
| Expected Loss Cost | $750,000 | |
| Method 2: benchmark severity | Severity | $373,134 |
| | Frequency | 2.01 |
| Method 3: exposure severity | Severity | $351,063 |
| | Frequency | 2.14 |

## Aggregate density function




## Aggregate distribution function




## Calculating the expected value of the treaty features

- For each output of the aggregate losses (0, 1000, 2000,...,100000) defined by the sub-layers calculate the value of the premium, profit commission, etc.
- With the corresponding probability function calculate the expected value of the feature.

If $Y = f(S) \Rightarrow P(Y = y) = P(S = s)$

## Treaty features

- Subject premium: $7.2m
- Margin plus rated: 7% minimum, 12.5% provisional and 18% maximum. Loss load 107.5%.
- Profit commission: 15% after 20% for reinsurer's expenses.
- Brokerage: 10% on provisional.

## Expected results $500k xs $500k

| | Method 1: lognormal | | Method 2: benchmark curve | | Method 3: exposure method | |
|---|---|---|---|---|---|---|
| | Amount | % Prem. | Amount | % Prem. | Amount | % Prem. |
| Prov. Prem. | 900,000 | | 900,000 | | 900,000 | |
| Margin plus | 222,739 | | 167,989 | | 174,238 | |
| Tot. prem. | 1,122,739 | 100% | 1,068,041 | 100% | 1,074,238 | 100% |
| Losses | 750,000 | 66.8% | 750,000 | 70.23% | 750,000 | 69.82% |
| PC | 25,659 | 2.29% | 32,930 | 3.07% | 32,062 | 2.98% |
| Brokerage | 90,000 | 8.02% | 90,000 | 8.43% | 90,000 | 8.38% |
| Marg. CR | 865,659 | 77.1% | 872,930 | 81.73% | 872,062 | 81.18% |

## Comments

- Key difference is the probability of zero losses. The parametric curves do not allow for this.
- If probability of zero losses is high, expected premium is lower and PC is higher.
- Practical relevance for high layers (or CAT layer) that have low frequency.

## Comments (cont'd)

- Communicating the results to underwriters: no need to understand the mathematical details (severity, frequency, Panjer's recursion, etc.) but rather to communicate the relevance of the model in pricing and profitability.

## Practical considerations

- How to choose the size of the sub-layers?
- How to include expenses ALAE?
- When does it fail?: theoretically it always works but:
  - For high frequency layers the resulting aggregate distribution is approximately Normal (CLT).
  - The lognormal might be more reasonable in this case: we need skewness and thicker tail.

## Further aspects to consider

- How to allow for correlations and dependencies between lines of business ceding to the same treaty?
- How to use this technique to assess profitability for multi-layer treaties? (Strong dependence between layers)