Institute and Faculty of Actuaries

Associateship Qualification

Actuarial Mathematics for Modelling (CM1) Core Principles

Syllabus for the 2024 Examinations

April 2023

Actuarial Mathematics for Modelling (CM1)

Syllabus for 2024 Examinations

This syllabus includes information to support the study of this subject. It will guide you through what you need to learn, application of learning as well as the skills that you need to develop. Information regarding the assessment of this subject is also included.

This syllabus includes:

- Aim of the subject
- How this subject links across the Qualifications
- Subject topics and topic weightings
- Subject objectives
- Assessment information

Aim

Actuarial Mathematics for Modelling (CM1) develops a grounding in the mathematical principles required for actuarial modelling, with a focus on understanding deterministic models and their application to solve financial problems. Alongside this, the related and required understanding of the theory of interest rates and the skills to model cashflows, which may be known or contingent on mortality, morbidity or survival, are also developed.

Links across the Qualifications

Associateship Qualification

Concepts in Actuarial Mathematics for Modelling (CM1) are introduced in Actuarial Statistics (CS1), in particular, a familiarity with probabilities, random variables, expectation and variance, which is required understanding for CM1.

Topics in CM1 are further developed in:

- Economic Modelling (CM2)
  - Extension and application of many concepts and techniques from CM1 to stochastic modelling
- Business Finance (CB1)
  - Application of the knowledge of interest rates to the interpretation of corporate accounts and understanding how corporate operations are financed
- Actuarial Practice (CP1)
  - Applies CM1 techniques to real world problems
- Modelling Practice (CP2)
  - Application and communication of CM1 techniques in a business context
- Communications Practice (CP3)
  - May draw upon concepts and techniques from CM1 in order to answer the CP3 examination questions

Fellowship Qualification

Concepts, principles and techniques from CM1 are further developed in the following Specialist Principles subjects, through solving problems and giving appropriate advice, guidance and recommendations:

- SP1 – Health and Care Principles
- SP2 – Life Insurance Principles
- SP4 – Pensions and other Benefits Principles

Topics and Topic Weightings

1. Theory of interest rates [25%]
2. Equation of value and its applications [20%]
3. Decrement and multiple life models [20%]
4. Pricing and reserving [35%]

Objectives

1 Theory of interest rates [25%]

Understand the principles of time preference theory of interest and the time value of money, including the term structure of interest rates and standard actuarial compound interest rate functions. Apply these principles to real world examples of interest rates, discounting and evaluation of present values of cashflows.

1.1 Show how interest rates may be expressed in different time periods

1.1.1 Relationship between the rates of interest and discount over one effective period, considered arithmetically and by general reasoning

1.1.2 Determine, when given a rate of interest under a specified payment frequency, the equivalent rate under an alternative payment frequency, including:

- Annual effective rate of interest or discount
- Rate of interest of discount payable pthly (p>1)
- Force of interest

1.1.3 Calculate the equivalent annual rate of interest implied by the accumulation of a sum of money over a specified period where the force of interest is a function of time

1.2 Account for the time value of money using the concepts of compound interest and discounting

1.2.1 Accumulate a single investment at a constant rate of interest under the operation of simple and compound interest

1.2.2 Calculate the present value of a future payment by discounting a single investment

1.3 Extend the techniques in 1.1 and 1.2 where appropriate to allow for inflation

1.4 Describe the operation of financial instruments and insurance contracts as a cashflow model (where cashflows may be fixed or uncertain in terms of both amount and timing)

1.5 Calculate the present value and accumulated value for a given stream of cashflows under the following individual or combination of scenarios:

1.5.1 Cashflows are equal at each time period

1.5.2 Cashflows vary with time, which may or may not be a continuous function of time

1.5.3 Some of the cashflows are deferred for a period of time

1.5.4 Rate of interest or discount is constant

1.5.5 Rate of interest or discount varies with time, which may or may not be a continuous function of time

1.6 Evaluate the following annuity and accumulation functions, when given the values for the term, *n*, and the appropriate interest or discount rate function *i*, *v*, *d*, δ, *i*(*p*) or *d*(*p*):

1.6.1 $a_{\overline{n}|}$, $s_{\overline{n}|}$, $a_{\overline{n}|}^{(p)}$, $s_{\overline{n}|}^{(p)}$, $\ddot{a}_{\overline{n}|}$, $\ddot{s}_{\overline{n}|}$, $\ddot{a}_{\overline{n}|}^{(p)}$, $\ddot{s}_{\overline{n}|}^{(p)}$, $\bar{a}_{\overline{n}|}$ *and* $\bar{s}_{\overline{n}|}$.

1.6.2 ${}_{m|}a_{\overline{n}|}$, ${}_{m|}a_{\overline{n}|}^{(p)}$, ${}_{m|}\ddot{a}_{\overline{n}|}$, ${}_{m|}\ddot{a}_{\overline{n}|}^{(p)}$ *and* ${}_{m|}\bar{a}_{\overline{n}|}$.

1.6.3 $(Ia)_{\overline{n}|}$, $(I\ddot{a})_{\overline{n}|}$, $(I\bar{a})_{\overline{n}|}$ *and* $(\overline{Ia})_{\overline{n}|}$ and the respective deferred annuities

1.7 Demonstrate an understanding of the term structure of interest rates

1.7.1 Understand the main factors influencing the term structure of interest rates

1.7.2 Understand and calculate:

- Discrete spot rates and forward rates
- Continuous spot rates and forward rates

1.7.3 Understand and calculate the par yield and yield to maturity

1.8 Understand duration, convexity and immunisation of cashflows:

1.8.1 Demonstrate how the duration and convexity of a cashflow sequence may be used to estimate the sensitivity of the value of the cashflow sequence to a shift in interest rates

1.8.2 Understand, apply and discuss Redington's conditions for immunisation of a portfolio of liabilities

2 Equation of value and its applications [20%]

Understand and apply equation of value principles to evaluate financial problems in particular relating to loan schedules, bond prices, bond yields and project appraisals.

2.1 Understand and apply the concept of an equation of value in terms of:

- Where payment or receipt is certain
- Where payment of receipt is uncertain
- The two conditions required for there to be an exact solution

2.2 Use the concept of equation of value to solve various practical problems

2.2.1 Apply the equation of value to loans repaid by regular instalments of interest and capital. Obtain repayments, interest and capital components, the effective interest rate (APR) and construct a schedule of repayments

2.2.2 Calculate the price of, or yield (nominal or real allowing for inflation) from, a bond (fixed-interest or index-linked) where the investor is subject to deduction of income tax on coupon payments and redemption payments are subject to deduction of capital gains tax

2.2.3 Calculate the running yield and the redemption yield for the financial instrument as described in 2.2.2

2.2.4 Calculate the upper and lower bounds for the present value of the financial instrument as described in 2.2.2, when the redemption date can be a single date within a given range at the option of the borrower

2.2.5 Calculate the present value or yield (nominal or real allowing for inflation) from an ordinary share or property, given constant or variable rate of growth of dividends or rents

2.3 Apply cashflow and equation of value techniques to project appraisals

2.3.1 Calculate the net present value and accumulated profit of the receipts and payments from an investment project at given rates of interest

2.3.2 Calculate the internal rate of return, payback period and discounted payback period and discuss their suitability for assessing the suitability of an investment project

3 Decrement and multiple life models [20%]

Understand how to model uncertain future cashflows, which may depend on the death or survival of an individual, or other uncertain events. Be introduced to the life table, calculation of the mean and variance of the present value of all of the main life insurance and annuity contracts, and their relationship in actuarial

terms. Extend the single decrement model to evaluate health insurance contracts involving two lives as well as the valuation of cashflows in a competing risk environment using multiple state models.

3.1 Demonstrate an understanding of the operation of key assurance and annuity contracts

3.1.1 Understand the following contracts, for example by explaining the timing and nature of the cashflows involved:

- Whole-life assurance
- Term assurance
- Pure endowment
- Endowment assurance
- Whole-life level annuity
- Temporary level annuity
- Guaranteed level annuity
- Deferred benefits

3.1.2 Understand the operation of conventional with-profits contracts, where profits are distributed by the use of regular reversionary bonuses and by terminal bonuses

3.1.3 Understand the operation of conventional unit-linked contracts, where death benefits are expressed as combination of absolute amount and relative to a unit fund

3.1.4 Understand the operation of accumulating with-profits contracts, where benefits take the form of an accumulating fund of premiums, where:

- the fund is defined in monetary terms, has no explicit charges and is increased by the addition of regular guaranteed and bonus interest payments plus a terminal bonus; or
- the fund is defined in terms of the value of a unit fund, is subject to explicit charges and is increased by regular bonus additions (through unit price increases or allocations of additional units) plus a terminal bonus (unitised with-profits)

3.2 Apply formulae for the means and variances of the payments under various assurance and annuity contracts, assuming a constant deterministic interest rate

3.2.1 Life table functions l_x and d_x and their select equivalents $l_{[x]+r}$ and $d_{[x]+r}$.

3.2.2. Describe the meaning of the following probabilities: ${}_np_x$, ${}_nq_x$, ${}_{n|m}q_x$, ${}_{n|}q_x$ and their select equivalents ${}_np_{[x]+r}$, ${}_nq_{[x]+r}$, ${}_{n|m}q_{[x]+r}$, ${}_{n|}q_{[x]+r}$.

3.2.3 Express the probabilities defined in 3.2.2 in terms of life table functions defined in 3.2.1

3.2.4 Use assurance and annuity factors and their select and continuous equivalents, including the extension of the annuity factors to allow for the possibility that payments are more frequent than annual but less frequent than continuous

3.2.5 Use the relationship between annuities payable in advance and in arrear, and between temporary, deferred and whole-life annuities

3.2.6 Use the relationship between assurance and annuity factors using equation of value, and their select and continuous equivalents

3.2.7 Express the mean and variance of the present value of benefit payments as sums / integrals under each contract defined in 3.1.1, in terms of the (curtate) random future lifetime, assuming:

- contingent benefits (constant, increasing or decreasing) are payable at the middle or end of the year of the contingent event or continuously
- annuities are paid in advance, in arrear or continuously, and the amount is constant, increases or decreases by a constant monetary amount or by a fixed or time-dependent variable rate

- premiums are payable in advance, in arrear or continuously and for the full policy term or for a limited period. Where appropriate, simplify the above expressions into a form suitable for evaluation by table look-up or other means

3.2.8 Evaluate the expected accumulations in terms of expected values for the contracts described in 3.1.1 and contract structures described in 3.2.7

3.3 Describe and use assurance and annuity functions involving two lives

3.3.1 Extend the techniques of objectives 3.2 to deal with cashflows dependent upon the death or survival of either or both of two lives

3.3.2 Extend technique in 3.3.1 to deal with functions dependent upon a fixed term as well as age

3.4 Describe and apply methods of valuing cashflows that are contingent upon multiple transition events

3.4.1 Demonstrate an understanding of simple health insurance premium and benefit structures

3.4.2 Describe how a cashflow, contingent upon multiple transition events, may be valued using a multiple-state Markov Model, in terms of the forces and probabilities of transition

3.4.3 Construct formulae for the expected present values of cashflows that are contingent upon multiple transition events, including simple health insurance premiums and benefits, and calculate these in simple cases. This includes regular premiums and sickness benefits are payable continuously and assurance benefits are payable immediately on transition

3.5 Describe and use methods of projecting and valuing expected cashflows that are contingent upon multiple decrement events

3.5.1 Understand the construction and use of multiple decrement tables

3.5.2 Understand the operation of a multiple decrement model as a special case of multiple-state Markov model

3.5.3 Determine dependent probabilities for a multiple decrement model in terms of given forces of transition, assuming forces of transition are constant over single years of age

3.5.4 Determine forces of transition from given dependent probabilities, assuming forces of transition are constant over single years of age

4 Pricing and reserving [35%]

Understand the future loss random variable and its application to the calculation of premiums for conventional life assurance and annuity contracts. Use the prospective and retrospective approaches to calculate reserves, the recursive relationship between reserves, and calculate mortality profit. Project cashflows to profit test life insurance contracts and apply projected cashflow techniques to pricing and reserving.

4.1 Determine the gross random future loss random variable under an insurance contract

4.2 Calculate gross premiums and reserves of assurance and annuity contracts

4.2.1 Calculate gross premiums for the insurance contract benefits listed in 3.1.1 under the following scenarios, or a combinations therefof using the equivalence principle or otherwise:

- Contracts may accept only single premium
- Regular premiums and annuity benefits may be payable annually more frequently than annually or continuously
- Death benefits (which increase or decrease by a constant compound rate or by a constant monetary amount) may be payable at the end of the year of death or immediately on death
- Survival benefits (other than annuities) may be payable at defined intervals other than at maturity

4.2.2 Understand why an insurance company will set up reserves

4.2.3 Calculate gross prospective and retrospective reserves

4.2.4 Understand the equivalence of the prospective reserve and the retrospective reserve under certain conditions, with or without allowance for expenses, for all fixed benefit and increasing/decreasing benefit contracts

4.2.5 Obtain recursive relationships between successive periodic gross premium reserves, and use this relationship to calculate the profit earned from a contract during the period

4.2.6 Understand the concepts of net premiums and net premium valuation and how they relate to gross premiums and gross premium valuation respectively

4.3 Describe and calculate, for a single policy or a portfolio of policies (as appropriate):

- death strain at risk
- expected death strain
- actual death strain
- mortality profit

for policies with death benefits payable immediately on death or at the end of the year of death, policies paying annuity benefits at the start of the year or on survival to the end of the year and policies where single or non-single premiums are payable

4.4 Project expected future cashflows for whole life, endowment and term assurances, annuities, unit-linked contracts and conventional/unitised with-profits contracts, incorporating multiple decrement models as appropriate

4.4.1 Profit test life insurance contracts of the types listed above and determine the profit vector, the profit signature, the net present value and the profit margin

4.4.2 Show how a profit test may be used to price a product, and use a profit test to calculate a premium for life insurance contracts of the types listed above

4.4.3 Show how gross premium reserves can be computed using the above cashflow projection model and included as part of profit testing

4.5 Show how, for unit-linked contracts, non-unit reserves can be established to eliminate ('zeroise') future negative cashflows, using a profit test model

Assessment

This subject will be assessed via two timed and online written examination papers.

- CM1 Paper A (CM1A), 3 hours and 20 minutes (including reading time)

CM1A consists of a number of questions of varying marks where the candidate will need to construct and type answers in Word.

- CM1 Paper B (CM1B), 1 hour and 50 minutes (including reading time)

In CM1B, candidates can expect to answer questions of varying marks, using Microsoft Excel to complete, construct and type their answers.

In order to pass this subject, you must sit both CM2A and CM2B within the same sitting, and achieve a combined mark of a pass.

Topic weighting

The topic weighting percentage noted alongside the topics is indicative of the volume of content of a topic within the subject and therefore broadly aligned to the volume of marks allocated to this topic in the examination. For example if a topic is 20% of the subject then you can expect that approximately 20% of the total marks available in the examination paper will be available on that topic.

Candidates for assessment should ensure that they are well prepared across the entire syllabus. The examination can be composed of questions drawing from any part of the syllabus within any examination sitting and using any command verb. This includes knowledge, techniques, principles, theories, and concepts as specified. Candidates should not rely on past papers alone and should ensure they have covered the entire syllabus as part of their learning and development of this subject. A list of command verbs used in the examinations is included on the IFoA website.

In each examination, candidates will be expected to demonstrate, through their answers, that they have knowledge of, can apply and use higher order skills in this subject:

- Knowledge will be demonstrated through answering questions that assess your understanding of that knowledge as well as through questions that ask you to apply relevant knowledge to scenarios.
- Application will be demonstrated through answering questions which assess that you can identify and apply relevant concepts and skills to solve problems (both numerical and non-numerical).
- Higher order skills will be demonstrated through questions that will assess that you can use relevant knowledge, concepts and skills to solve problems, draw appropriate conclusions, and make meaningful and appropriate comments on those conclusions.

As a guide, in the examination of this subject, you can expect that approximately 5% of the total number of marks for this examination be allocated to the demonstration of knowledge, 80% to application and 15% to higher order.

Qualifications Handbook, Examinations Handbook and Assessment Regulations

Please ensure you read and have understood the Examinations Handbook and Assessment Regulations ahead of your exam as well as the Qualifications Handbook. These are all available on the IFoA website.

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