Institute
and Faculty
of Actuaries

Associateship Qualification

Actuarial Statistics (CS1)
Core Principles

April 2023

Actuarial Statistics (CS1)

Syllabus for the 2024 Examinations

This syllabus includes information to support the study of this subject. It will guide you through what you need to learn, application of learning as well as the skills that you need to develop. Information regarding the assessment of this subject is also included.

This syllabus includes:

- Aim of the subject
- How this subject links across the Qualifications
- Subject topics and topic weightings
- Subject objectives
- Assessment information

Aim

Actuarial Statistics provides a grounding in mathematical and statistical techniques that are of particular relevance to actuarial work.

Links across the Qualifications

Associateship Qualification

The principles introduced in CS1 are further developed in CS2 Risk Modelling and Survival Analysis. CM1 Actuarial Mathematics and CM2 Economic Modelling apply the principles and techniques in CS1 to the creation of actuarial and financial models

Topics and Topic Weightings

This subject covers the following topics:

1. Data Analysis [10%]
2. Random variables and distributions [20%]
3. Statistical inference [25%]
4. Regression Theory and applications [30%]
5. Bayesian statistics [15%]

Objectives

1 Data analysis [10%]

Production of simple visualisations and statistics from a data set.

1.1 Describe the purpose and function of data analysis

   1.1.1 Aims of a data analysis (e.g. descriptive, inferential and predictive)

   1.1.2 Stages and suitable tools used to conduct a data analysis to solve real-world problems

   1.1.3 Sources of data and their characteristics, including extremely large data sets

   1.1.4 Meaning and value of reproducible research and the elements required to ensure a data analysis is reproducible

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1.2 Complete exploratory data analysis

    1.2.1 Appropriate tools to calculate suitable summary statistics and undertake exploratory data visualisations

    1.2.2 Interpret and make statistical inferences using Pearson's, Spearman's and Kendall's measures of correlation for bivariate data

    1.2.3 Principal component analysis to reduce the dimensionality of a complex data set

2 Random variables and distributions [20%]

The basic properties and uses of commonly-used probability distributions and the statistical properties of data generated by randomly sampling from a known distribution.

2.1 Understand the characteristics of basic univariate distributions and how to generate samples from them

    2.1.1 Geometric, binomial, negative binomial, hypergeometric, Poisson and uniform discrete distributions on a finite set

    2.1.2 Normal, lognormal, exponential, gamma, chi-square, t, F, beta and uniform continuous distributions on an interval

    2.1.3 Evaluation of probabilities and quantiles associated with these distributions (by calculation or using statistical software as appropriate)

    2.1.4 Poisson process and the connection between the Poisson process and the Poisson distribution

    2.1.5 Generation of basic discrete and continuous random variables using the inverse transform method

    2.1.6 Generation of discrete and continuous random variables using statistical software

2.2 Determine the characteristics of jointly distributed random variables

    2.2.1 Probability function or density function for marginal and conditional distributions of jointly distributed random variables

    2.2.2 The conditions under which random variables are independent

    2.2.3 Covariance, the correlation and the expected value of a function of two jointly distributed random variables

    2.2.4 Mean and variance of linear combinations of random variables

2.3 Evaluate expectations and conditional expectations

    2.3.1 Conditional expectation of one random variable given the value of another random variable

    2.3.2 Mean and variance of a random variable as an expectation of conditional expected values

2.4 Evaluate and apply generating functions

    2.4.1 Moment and cumulant generating functions of a random variable

    2.4.2 Moment calculation via series expansion or differentiation of a generating function

2.5 State and apply the central limit theorem

    2.5.1 Central limit theorem for a sequence of independent, identically distributed random variables

    2.5.2 Comparison of simulated samples from a given distribution with the Normal distribution

2.6 Describe random sampling and the sampling distributions of statistics commonly used in statistical inference

    2.6.1 Random samples from a population

    2.6.2 The sampling distribution of a statistic

2.6.3 The mean and variance of a sample mean and the mean of a sample variance in terms of the population mean, variance and sample size

2.6.4   Basic sampling distributions for the sample mean and variance for random samples from a normal distribution

2.6.5 The distribution of the t-statistic for random samples from a normal distribution

2.6.6 The F distribution for the ratio of two sample variances from independent samples taken from normal distributions

3 Statistical inference [25%]

Use of statistics to make inferences about the process underlying a data set.

3.1 Construct estimators and discuss their properties

   3.1.1 Method of moments for constructing estimators of population parameters

   3.1.2 Method of maximum likelihood for constructing estimators of population parameters

   3.1.3 Efficiency, bias, consistency and mean square error of an estimator

   3.1.4 Comparison of estimators using their mean square error and bias or unbiasedness

   3.1.5 Asymptotic distribution of maximum likelihood estimators

   3.1.6 Bootstrap method for estimating properties of an estimator

3.2 Calculate confidence intervals and prediction intervals

   3.2.1 Confidence interval for an unknown parameter of a distribution based on a random sample

   3.2.2 Prediction interval for a future observation based on a model fitted to a random sample

   3.2.3 Confidence interval for an unknown parameter using a given sampling distribution

   3.2.4 Confidence intervals for the mean and the variance of a normal distribution

   3.2.5 Confidence intervals for a binomial probability and a Poisson mean, including the use of the normal approximation in both cases

   3.2.6 Confidence intervals for two-sample situations involving the normal distribution and the binomial and Poisson distributions using the normal approximation

   3.2.7 Confidence intervals for a difference between two means from paired data

   3.2.8 Bootstrap method to obtain confidence intervals

3.3 Apply the concepts of hypothesis testing and goodness of fit

   3.3.1 Understand the concepts of Null and alternative hypotheses, simple and composite hypotheses, type I and type II errors, sensitivity, specificity, test statistic, likelihood ratio, critical region, level of significance, probability value and power of a test

   3.3.2 Use of basic tests for the one-sample and two-sample situations involving the normal, binomial and Poisson distributions, and apply basic tests for paired data

   3.3.3 The permutation approach to non-parametric hypothesis tests

   3.3.4 Chi-square test to test the hypothesis that a random sample is from a particular distribution, including cases where parameters are unknown

   3.3.5 A contingency (or two-way) table, and use of a chi-square test to test the independence of two classification criteria

4 Regression theory and applications [30%]

Use of statistics to examine and make inferences about the relationships between two or more data sets.

4.1 Understand and use linear regression models

    4.1.1 Response and explanatory variables

    4.1.2 Simple regression model (with a single explanatory variable) and multiple linear regression model (with several explanatory variables)

    4.1.3 Least squares estimates of the slope and intercept parameters in a simple linear regression model

    4.1.4 Use of appropriate software to fit a linear regression model to a data set and interpret the output:

- Perform statistical inference on the slope parameter
- Describe the use of measures of goodness of fit of a linear regression model
- Use a fitted linear relationship to predict a mean response or an individual response with confidence limits
- Use residuals to check the suitability and validity of a linear regression model

    4.1.5 Measures of model fit to select an appropriate set of explanatory variables

4.2 Understand and use generalised linear models

    4.2.1 Binomial, Poisson, exponential, gamma and normal distributions as an exponential family

    4.2.2 Mean, variance, variance function and scale parameter for a generalised linear model binomial, Poisson, exponential, gamma and normal distributions. Evaluate these quantities for the distributions in 4.2.1

    4.2.3 The link function and the canonical link function, referring to the distributions in 4.2.1

    4.2.4 Variables, factors taking categorical values and interaction terms.

    4.2.5 Definition of the linear predictor, including its form for simple models, including polynomial models and models involving factors

    4.2.6 Deviance scaled deviance and estimation of the parameters of a generalised linear model

    4.2.7 Choice of a suitable model using an analysis of deviance and examination of the significance of the parameters

    4.2.8 Pearson and deviance residuals and their use

    4.2.9 Statistical tests to determine the acceptability of a fitted model: Pearson's chi-square test and the likelihood-ratio test

    4.2.10 Fit a generalised linear model to a data set and interpret the output

5 Bayesian statistics [15%]

Use of Bayesian statistics to update prior beliefs about a data-generating process.

5.1 Explain fundamental concepts of Bayesian statistics and use these concepts to calculate Bayesian estimators

    5.1.1 Use of Bayes' theorem to calculate simple conditional probabilities

    5.1.2 Prior distribution, posterior distribution and conjugate prior distribution

    5.1.3 Posterior distribution for a parameter in simple cases

    5.1.4 Use of simple loss functions to derive Bayesian estimates of parameters

    5.1.5 Credible intervals in simple cases

5.1.6 Credibility premium formula and the role played by the credibility factor

5.1.7 Bayesian approach to credibility theory and its use for calculating credibility premiums in simple cases

5.1.8 Empirical Bayes approach to credibility theory and its use for deriving credibility premiums in simple cases

5.1.9 Understanding the differences between the two approaches (Bayes v Empirical Bayes) and the assumptions underlying each of them

Assessment

This subject will be assessed via two timed and online written examination papers.

- CS1 Paper A (CS1A), 3 hours and 20 minutes (including reading time), consisting of a number of questions of varying marks

- CS1 Paper B (CS1B), 1 hour and 50 minutes (including reading time) consisting of a number of questions of varying marks using R (or another pre-specified software package), where relevant, to construct the answers, and Microsoft Word to present the answers.

In order to pass this subject, you must sit both CS1A and CM1B within the same sitting, and achieve a combined mark of a pass.

Topic weighting

The topic weighting percentage noted alongside the topics is indicative of the volume of content of a topic within the subject and therefore broadly aligned to the volume of marks allocated to this topic in the examination. For example if a topic is 20% of the subject then you can expect that approximately 20% of the total marks available in the examination paper will be available on that topic.

Candidates for assessment should ensure that they are well prepared across the entire syllabus. The examination can be composed of questions drawing from any part of the syllabus within any examination sitting and using any command verb. This includes knowledge, techniques, principles, theories, and concepts as specified. Candidates should not rely on past papers alone and should ensure they have covered the entire syllabus as part of their learning and development of this subject. A list of command verbs used in the examinations is included on the IFoA website.

In each examination, candidates will be expected to demonstrate, through their answers, that they have knowledge of, can apply and use higher order skills in this subject:

- Knowledge will be demonstrated through answering questions that assess your understanding of that knowledge as well as through questions that ask you to apply relevant knowledge to scenarios.

- Application will be demonstrated through answering questions which assess that you can identify and apply relevant concepts and skills to solve problems (both numerical and non-numerical).

- Higher order skills will be demonstrated through questions that will assess that you can use relevant knowledge, concepts and skills to solve problems, draw appropriate conclusions, and make meaningful and appropriate comments on those conclusions.

As a guide, in the examination of this subject, you can expect that approximately 20% of the total number of marks for this examination be allocated to the demonstration of knowledge, 65% to application and 15% to higher order.

Qualifications Handbook, Examinations Handbook and Assessment Regulations

Please ensure you read and have understood the Examinations Handbook and Assessment Regulations ahead of your exam as well as the Qualifications Handbook. These are all available on the IFoA website.