

# INSTITUTE AND FACULTY OF ACTUARIES

## EXAMINATION

8 April 2024 (am)

### **Subject SP6 – Financial Derivatives Specialist Principles**

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

**1** A callable bond is a bond with an embedded option for the issuer to buy back the bond at a specific time in the future at a predetermined price.

(i) Comment on why an issuer may issue bonds with embedded options. [3]

Consider a 1-year European call option on a 6.75-year bond with a face value of £100 and a current cash price of £96, with a cash strike price of £105 and 3-month, 6-month and 1-year continuously compounded risk-free interest rates all equal to 2%. The bond pays a coupon of 3% per year (with payments made semi-annually). The volatility for the forward bond price,  $F$ , which is assumed to be lognormally distributed, in 1 year is 8% p.a.

(ii) Show that the forward bond price  $F = £94.91$ . [3]

(iii) Calculate the value of this European call bond option using Black's model. [5]

(iv) Comment on the assumptions underlying Black's model. [3]

[Total 14]

**2** An investor previously exempt from central clearing will in due course have to clear its over-the-counter derivatives using a Central Counter Party (CCP) clearing house.

(i) Describe the operation of a CCP. [4]

(ii) Outline the practical considerations arising from the requirement to clear. [5]

[Total 9]

- 3
- (i) Outline why calculating vega using the Black–Scholes model may be inconsistent with this model. [2]
  - (ii) State, in your own words, how the vega of a long vanilla European put option varies with respect to the underlying asset price. [2]
  - (iii) Describe how the vega of a long vanilla European put option varies with respect to the time to maturity. [2]

A company has a portfolio of options on the same underlying stock but with different times to expiry. A summary is provided in the table below:

<i>Option type</i>	<i>Delta of option</i>	<i>Vega of option</i>	<i>Time to expiry of option</i>	<i>Number of long options in the portfolio</i>
Call	0.62	0.45	6 months	2,100
Put	−0.35	0.55	3 months	3,200
Put	−0.41	1.24	1 year	800
Call	0.83	0.97	1 year	3,250

- (iv) Calculate the delta and vega of this portfolio. [2]

The board of the company is concerned about the levels of vega and delta within the portfolio. The board is considering using the following option for hedging:

<i>Option type</i>	<i>Delta of option</i>	<i>Vega of option</i>	<i>Time to expiry of option</i>
Call	0.44	0.41	6 months

- (v) Calculate the investment in the above option and in the underlying asset to make the company’s portfolio both delta and vega neutral. [3]

An investment advisor to the company has suggested that using the standard definition of vega for calculating the overall vega of the portfolio in part (iv) is not appropriate. Instead, they have recommended using a scaled version of an option’s vega by dividing the standard vega of an option by the square root of the option’s time to expiry.

- (vi) Comment on the investment advisor’s suggestion to use this scaled vega for calculating the vega of the company’s portfolio. [3]

[Total 14]

- 4 (i) Explain, in your own words and without using any mathematical symbols, Ito's lemma. [2]

A financial modeller is currently using geometric Brownian motion in a Black–Scholes framework to model a non-dividend paying asset with fixed parameters –  $\mu$  and  $\sigma$ , and price  $S_t$  at time  $t$ , satisfying:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where  $W_t$  is a Brownian motion process under the real probability measure  $P$ .

A banking actuary working with the financial modeller has suggested modifying this stochastic differential equation to allow for discrete jumps in the asset price over the time period  $dt$ .

- (ii) Comment on the banking actuary's suggestion. [3]

The banking actuary's modification is to include an additional term in the stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + \text{Jump}$$

Here,  $\text{Jump} = S_t(J_t - 1)dN_t$ , where  $J_t$  is the value of the jump at time  $t$  and  $N_t$  is the probability of a jump at time  $t$ . Explicitly,  $dN_t = 1$  with probability  $\gamma dt$  and  $dN_t = 0$  with probability  $1 - \gamma dt$ , and  $\gamma$  is a positive non-zero-constant. It is assumed that  $J_t$ ,  $N_t$  and  $W_t$  are independent.

- (iii) Describe what would happen to the asset price ( $dS_t$ ) over a period  $dt$  assuming there is a single jump at time  $t$  ( $dN_t = 1$ ) and  $J_t = 0.8$ . [3]

- (iv) Show that  $E[(J_t - 1)dN_t] = k \gamma dt$ , where  $E$  is the expectation and  $k = E[(J_t - 1)]$ . [2]

- (v) Determine whether the stochastic differential equation for  $dS_t$ , allowing for jumps, has the same drift as the original geometric Brownian motion process for  $dS_t$  ( $dS_t = \mu S_t dt + \sigma S_t dW_t$ ). [2]

In order to apply Ito's lemma to a function  $g(S_t, t)$  (where  $S_t$  satisfies the stochastic differential with jumps), an additional term is added to the standard Ito's lemma:

$$[g(S_t, J_t, t) - g(S_t, t)]dN_t$$

- (vi) Show that  $d \log(S_t) = (\mu - 0.5 \sigma^2)dt + \sigma dW_t + \log(J_t) dN_t$ . [4]

[Total 16]

- 5**
- (i) Define, in your own words, a ‘numeraire’. [1]
  - (ii) Write down an example of a numeraire. [1]
  - (iii) Define, in your own words, a ‘martingale’. [2]
  - (iv) Describe, in your own words, the relationship between tradeable assets and martingales. [2]

$S_t$  is a tradeable asset such that  $S_t = \exp(\sigma W_t + (r - 0.5\sigma^2)t)$ , with  $W_t$  the Brownian motion under the risk-neutral measure,  $Q$ , and  $r$ , the risk-free rate. Further,  $B_t = \exp(rt)$  is the cash bond. Using Ito’s lemma:

$$dZ_t = Z_t(w\sigma dW_t + (wr - r - 0.5w\sigma^2)dt + 0.5(w\sigma)^2 dt)$$

where  $Z_t$  is the discounted value of  $Y_t = S_t^w$ .

- (v) Determine for which values of  $w$  the process  $Y_t$  is tradeable, stating any assumptions made. [3]

Consider an option with underlying asset price  $S_t$ .

- (vi) Comment on the relationship between the market price of risk for the option and the market price of risk for the underlying asset in a Black–Scholes framework. [2]
- [Total 11]

- 6**
- (i) Explain how inflation swaps can be used to hedge a pension scheme’s inflation-linked liabilities. [2]
  - (ii) Outline the risks that a pension scheme would be exposed to if it did use inflation swaps to hedge the scheme’s inflation risk. [3]

A final salary pension scheme offers its members pension payments in euros. The pension payments are increased annually by national average wage inflation, capped at 5%.

One year ago, the trustees of the pension scheme implemented inflation hedging using Eurozone Harmonised Index of Consumer Prices (HICP) inflation swaps.

Recent geopolitical tensions have resulted in significant increases in global inflation rates. The national average wage inflation is currently 8% p.a., and 10-year HICP inflation expectations have also increased to 6% p.a.

- (iii) Discuss the impact of the increased inflationary environment on the pension scheme’s funding position. [5]
- [Total 10]

- 7 (i) Describe a Collateralised Debt Obligation (CDO). [2]
- (ii) Explain how CDOs restructure credit risk. [3]
- (iii) Contrast cash CDOs and synthetic CDOs. [2]

A 5-year synthetic CDO with principal of \$100 million is held within a Special Purpose Vehicle (SPV) and structured as follows:

<i>Tranche (ranked by seniority)</i>	<i>Principal (\$m)</i>	<i>Spread (p.a.)</i>
Senior	40	150 basis points
Mezzanine	50	350 basis points
Equity	10	750 basis points

Payments are made to investors on an annual basis, calculated as the remaining principal at the end of each 12-month period multiplied by the agreed return. The equity and mezzanine tranches only are required to provide collateral equal to the value of their exposure. The collateral is invested by the SPV into high-quality government bonds and attracts interest, paid monthly, at the Secured Overnight Financing Rate (SOFR).

- (iv) Determine the SPV cashflows at inception. [1]

The CDO suffers losses of \$12 million after 2.5 years.

- (v) Determine the SPV cashflows at this point. [1]

No further losses are incurred during the remainder of the life of the SPV. SOFR has been annually compounded at 3% p.a.

- (vi) Calculate the total amount of return paid to each tranche of the CDO over the 5-year period. [4]

- (vii) Comment on the effectiveness of the collateral arrangement. [2]

[Total 15]

- 8 (i) Define, in your own words, ‘systematic risk’ and ‘non-systematic risk’. [2]

A vehicle manufacturer has been producing and selling a range of battery-powered electric vehicles for the past 5 years.

Preliminary modelling of the battery technology indicated that the average battery life should be in excess of 12 years. As part of its marketing strategy, the manufacturer offers a 10-year guarantee on the life of each car’s batteries. The anticipated cost of the guarantee is included in the price of each vehicle. The strategy has been very successful, and the guarantee is now considered an integral part of the manufacturer’s offer.

- (ii) Explain and give two examples of the systematic risks to the manufacturer arising from the guarantee. [3]

Recent projections suggest that this area of the manufacturer’s business will grow significantly.

The finance director has recently become concerned about the risks arising from the issuance of the guarantees and has asked the risk department to investigate different ways of reducing this risk.

To assist with the analysis, the manufacturer has some internal data as well as access to an industry-wide database that can be used to create a reference index for car battery life.

- (iii) Outline how a principal-at-risk bond could be used to transfer the systematic risk. [4]

- (iv) Suggest other ways the manufacturer could address the systematic risk of the guarantee. [2]

[Total 11]

**END OF PAPER**