

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

8 April 2024 (am)

Subject CM1 – Actuarial Mathematics for Modelling Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 Calculate ${}_{4.25}q_{87.25}$ assuming PMA92C20 mortality and that there is a uniform distribution of deaths between integer ages. You should show your working. [3]

2 An insurance company is required to pay an annuity annually in advance for 4 years. The first payment is \$1.5 million and each subsequent payment is 3% greater than the previous one.

(i) Calculate, showing all working and using an effective rate of interest of 8% p.a., the convexity of the annuity. [3]

The company is considering investing in a single zero-coupon bond of suitable term to cover the present value of the annuity.

(ii) Explain if it is possible to find a zero-coupon bond such that the fund would be immunised against small changes in the rate of interest. [2]

[Total 5]

3 Three bonds, A, B and C, each pay coupons at 4% annually in arrears and are redeemed at 103%. The outstanding terms of the bonds are exactly 1, 2 and 3 years, respectively. The prices of the bonds per £100 nominal are £104, £105 and £106, respectively.

(i) Using the information above and showing all working:

(a) Determine all possible discrete spot rates.

(b) Determine all possible discrete future rates.

[8]

A bank offers a 3-year regular savings plan and adds interest in line with the rates calculated in part (i).

An individual invests £12,000 at the beginning of each of the 3 years.

(ii) Calculate, showing all working, the total accumulated value of the investments at the end of year three. [4]

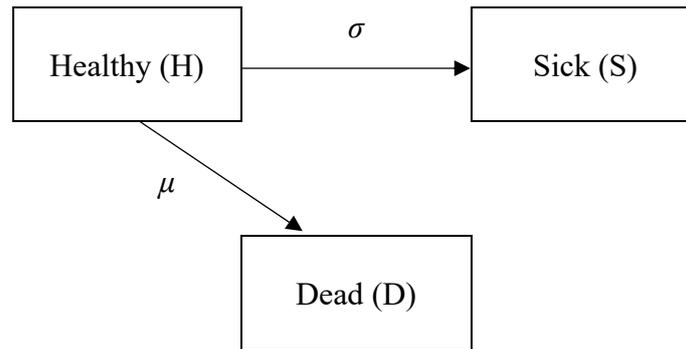
[Total 12]

4 An individual pays £4,000 p.a. into a savings account for 10 years. During the first 4 years, the payments are made quarterly in advance. For the remaining years, the payments are made continuously.

The investor achieves a yield of 6% p.a. convertible quarterly on the investment.

Calculate, showing all working, the accumulated amount in the savings account at the end of 10 years. [7]

- 5 A life insurance company uses the following three-state continuous-time Markov model, with constant forces of transition σ and μ , to price a sickness policy.



The company issues a 25-year policy to a healthy life aged 35 exact.

The benefits provided are:

- £150,000 payable immediately on death from the healthy state.
- £300,000 payable immediately on becoming sick.
- £500 payable at the end of the term of the policy if no claim has been made on the policy.

- (i) Calculate, showing all working, the total expected present value of the benefits. [7]

Basis:

μ : 0.025 for all ages

σ : 0.001 for all ages

Force of interest: 5% p.a.

Premiums are payable continuously for the term of the policy while the policyholder is in the healthy state.

- (ii) Write down an integral formula for the expected present value of the annual premium. You are not required to do any calculations. [2]

The company is investigating introducing a new death benefit that is payable on death from the sick state.

To value this new benefit the transition probability ${}_t p_{35}^{HS}$ would be required.

- (iii) Write down an integral formula for ${}_t p_{35}^{HS}$ in terms of other transition probabilities and forces of transition. You are not required to do any calculations. [2]
- [Total 11]

- 6** A government issues a fixed-interest security paying coupons at a rate of 8% p.a. quarterly in arrears.

The security is to be redeemed at 103% on any coupon payment date from 15 to 18 years after issue, with the exact date of redemption at the discretion of the government.

Investor A, who is liable to pay income tax at 20% and capital gains tax at 25%, purchases the fixed-interest security on the date of issue at a price that gives a net yield to redemption of 6.2% p.a. effective.

- (i) Calculate, showing all working, the price per £100 nominal paid by Investor A. [6]

Investor A sells the fixed-interest security 5 years after purchase to Investor B for £106.50 per £100 nominal.

- (ii) Calculate, using linear interpolation and showing your working, the net effective yield Investor A earns on the total transaction. Give your answer as a percentage rounded to three decimal places. [5]

[Total 11]

- 7 (i) Determine the missing entries, (a) and (b), for the following multiple-decrement table.

<i>Age x</i>	$(al)_x$	$(ad)_x^{\text{death}}$	$(ad)_x^{\text{withdrawal}}$
50	100,000	123	3,100
51	96,777	125	3,050
52	(a)	128	2,950
53	90,524	132	2,800
54	87,592	(b)	2,550
55	84,906	141	2,100
56	82,665	148	1,400
57	81,117	155	950
58	80,012	162	700
59	79,150	173	250
60	78,727	188	0

[2]

- (ii) Calculate the following probabilities, showing all working:

- (a) A life aged 50 exact will not be in the population at the end of the 5th year.
- (b) A life aged 54 exact will remain in the population for at least 2 years.
- (c) A life aged 55 exact will die during the next 3 years.
- (d) A life aged 50 exact will withdraw between the ages of 55 and 59.

[5]

[Total 7]

- 8 A pension scheme provides retirement benefits to a group of 200 pensioners currently aged 65 exact. The scheme pays an annual income of £19,000 annually in arrears if the pensioner is alive. A lump sum of £50,000 is paid at the end of the year of death.

During the first year 3 members died.

- (i) Calculate, showing all working, the mortality profit or loss for the year. [7]
- (ii) Explain why the mortality profit or loss has arisen. [3]

Basis:

Mortality: AM92 Ultimate

Interest: 6% p.a. effective

[Total 10]

- 9 A life insurance company issued a last survivor whole-life assurance policy on 1 January 2020 to a male life aged 60 exact and a female life aged 57 exact. The sum assured of £100,000 is paid at the end of the year of the second death.

Annual premiums of £1,495 are payable annually in advance while at least one of the lives is alive.

Calculate, showing all working, the prospective reserve in respect of this policy at 31 December 2022 if:

- (a) both lives are alive
- (b) only the female life is alive.

Basis:

Mortality: PMA92C20 for the male
PFA92C20 for the female
Interest: 4% p.a. effective

[8]

- 10** The force of interest is a function of time and at any time t (measured in years) is given by the formula:

$$\delta(t) = \begin{cases} a + bt & 0 \leq t < 5 \\ c & 5 \leq t \end{cases}$$

where a , b and c are constants.

You are given $v(t)$, the present value of a unit sum of money due at time t :

$$v(t) = \begin{cases} e^{-[0.04t + 0.01t^2]} & 0 \leq t < 5 \\ e^{-[0.05t + 0.20]} & 5 \leq t \end{cases}$$

- (i) Calculate, showing all working, the values of a , b and c . [5]

£1,000 is invested at $t = 2$ for a period of 7 years.

- (ii) Calculate, showing all working, the accumulated value of this investment at $t = 9$. [3]
- (iii) Calculate, showing all working, the rate of interest earned on the investment in part (ii). Express your answer as a percentage rounded to three decimal places as a nominal rate of interest per annum convertible quarterly. [2]

A continuous payment stream is received at a rate of $5e^{0.03t}$ units per annum between $t = 8$ and $t = 13$.

- (iv) Calculate, showing all working, the present value of the payment stream at $t = 0$. [4]

[Total 14]

11 A life aged 35 exact purchases a 30-year term assurance with the sum assured of £200,000 payable immediately on death. Level monthly premiums are payable in advance throughout the term, ceasing on death.

(i) Calculate, showing all working, the monthly premium. [7]

(ii) Write down an expression for the retrospective reserve after 25 years. You are not required to do any calculations. [3]

The retrospective reserve expressed as a percentage of the sum assured is 3.3%.

(iii) Explain why this percentage is relatively small. [2]

Basis:

Mortality: AM92 Select

Interest: 4% p.a. effective

Initial expenses: £120 at issue plus 30% of the first monthly premium

Renewal expenses: 2% of the second and subsequent monthly premiums

[Total 12]

END OF PAPER