INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

23 September 2022 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper B

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

© Institute and Faculty of Actuaries

1 An insurer has issued five identical natural disaster insurance contracts with a single premium of \$1.05 per policy, and a fixed sum assured of \$100. Each policy has a 1% probability of resulting in a claim.

You are given 100 simulated variates $x_i \sim U(0, 1)$ in the 'Q1 Data' tab of the spreadsheet, where a contract results in a claim if $x_i \leq 0.01$.

(i) Calculate the value to the insurer in a single year of each of the five contracts in each simulation. [4]

(ii) Calculate the following:

(a) The expected value of a single contract to the insurer based on the probability above

(b) The expected value of each of the five contracts to the insurer based on the simulations.

[4]

(iii) Explain, using your answers to parts (i) and (ii), why pooling resources is beneficial for an insurer. [6]

The insurer now discovers that all five policyholders live on the same street. The insurer expects that all five contracts will therefore follow the same simulation as the first one.

(iv) Calculate the revised expected value of the contracts based on the simulations. [4]

(v) Explain why the contracts are now loss-making. [3]

(vi) Suggest **two** measures the insurer could have taken to mitigate this issue. [2]

[Total 23]

2 Consider three consultants with utility functions $U_1(w)$, $U_2(w)$ and $U_3(w)$, respectively. Assume that the consultants have zero initial wealth, are non-satiated and risk averse. Their utility functions are:

	Utility function
Consultant 1	$U_1(w) = \ln(w)$
Consultant 2	$U_2(w) = w - 0.00001w^2$
Consultant 3	$U_3(w) = 2(w^{0.5} - 1)$

(i) Plot a chart of the utility for Consultant 1 for a range of wealth from \$0 to \$60,000. [4]

(ii) Comment on your chart in part (i). [4]

Each consultant has been offered a 1-year contract by Company A including a profit-sharing arrangement. The basic fee for each consultant will be \$30,000. In addition, each consultant will also be paid \$Y depending on Company A's success.

$$Y \sim \begin{cases} 5000 \text{ if } X > 100\% \\ 5000 \times \sqrt{X} \text{ if } 0\% \leq X \leq 100\% \\ 0 \text{ if } X < 0\% \end{cases}$$

where X is the 1-year return on the share price of Company A. You have been given the distribution of X in the 'Q2 Data' tab of the spreadsheet.

(iii) Calculate the mean and variance of the fee. [5]

(iv) Calculate the downside semi-variance of the fee. [3]

(v) Calculate the expected utility of the fee for each consultant. [6]

Consultant 1 wishes to negotiate a fixed 1-year fee for their work with no profit-sharing arrangement.

(vi) Calculate the minimum fixed fee, to the nearest \$250, that Consultant 1 would accept instead of the profit-sharing agreement. [3]

(vii) Comment on your answer to part (vi) in the context of Consultant 1's utility function. [3]

[Total 28]

3 An insurer is modelling its probability of ruin over time. The insurer's surplus comprises three items:

- An initial surplus U = \$50 million that is invested in assets
- Premium income of \$1 million per unit time, arriving at the end of each time period
- Claims on the policies issued, paid at the end of each time period.

You have been provided with 100 simulations of claim values in \$m over the next ten time periods in the 'Q3 Data' tab of the spreadsheet.

Assets are assumed to increase in value at a rate of 2% p.a. Once the surplus falls below zero, it is set to zero and remains there indefinitely (i.e. the insurer becomes insolvent).

(i) Calculate the surplus process for the insurer in each simulation. [5]

(ii) Calculate:

(a) the insurer's probability of ruin after 5 years.

(b) the insurer's probability of ruin after 10 years.

(c) the probability that the insurer's surplus after 10 years is larger than its initial value.

[3]

The insurer now decides to model investment returns stochastically. It assumes that returns are uniformly distributed between –2% and 10%. You have been provided with 100 simulations of these returns in the 'Q3 Data' tab of the spreadsheet corresponding to the 100 claim simulations. For example, simulation 1 has claims in period 1 of \$3.85 million and a return of 2.79%.

(iii) Calculate the revised surplus process for the insurer in each simulation. [4]

(iv) Calculate the following revised probabilities:

(a) The insurer's probability of ruin after 5 years

(b) The insurer's probability of ruin after 10 years

(c) The probability that the insurer's surplus after 10 years is larger than its initial value.

[3]

(v) Discuss, with reference to your answers to parts (i) to (iv) inclusive, the implications for the insurer of adopting the stochastic model for investment returns. [8]

[Total 23]

4 An insurer has recently launched a 10-year equity release mortgage product for homeowners. The product includes a guarantee that the loan amount repayable by the borrower will not exceed the value of the homeowner's property when the product matures in 10 years. The risk-free force of interest is 2% p.a.

A homeowner decides to take out a \$62,500 equity release mortgage at time $t = 0$ with a force of interest of 4% p.a. The loan and accumulated interest are paid at maturity. The property is valued at \$100,000 at time $t = 0$.

(i) Calculate the value of the loan at each yearly interval from time $t = 0$ to $t = 10$ inclusive. [2]

You have been given 100 simulations of the annual change in the value of the property, p_t, in the 'Q4 Data' tab of the spreadsheet.

(ii) Calculate, for each simulation, the value of the property at each yearly interval from time $t = 0$ to $t = 10$ inclusive. [3]

(iii) Calculate the expected cost to the insurer at time $t = 0$ of providing the guarantee at time $t = 10$. [4]

(iv) (a) Identify the scenario that corresponds to the 50th percentile for the property value at $t = 10$.

(b) Plot a chart of the loan and property value at yearly intervals from $t = 0$ to $t = 10$ for the 50th percentile scenario.

[5]

(v) Comment on your chart from part (iv)(b). [4]

Now suppose that $1 + p_t$ follows a lognormal distribution with parameters $\mu = 4\%$ and $\sigma = 6\%$. Assume that the property value, loan amount and force of interest remain unchanged.

(vi) Calculate the expected value of the property at time $t = 10$. [3]

(vii) Calculate the cost to the insurer of the guarantee at time $t = 10$. [2]

(viii) Comment on why the values of the guarantee calculated in parts (iii) and (vii) are different. [3]

[Total 26]

END OF PAPER