INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

5 May 2020 (am)

Subject SP6 – Financial Derivatives
Specialist Principles

Time allowed: Three hours and fifteen minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

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1 A large company carries out business in its home country, X, and in another country, Y. Each country has a different currency. The company makes extensive use of a commodity available to buy only in Country Y, which it purchases using the currency of Country Y.

(i) Define a quanto option and give a suitable example based on the company and the commodity described above. [4]

(ii) Assess the effectiveness for the company of using quanto options to attempt to fully hedge the risks associated with the future buying of the commodity in the commodity market. [6]

[Total 10]

2 (i) Define an exotic option. [1]

(ii) State the main differences between:

- a cash-or-nothing binary put option; and
- an asset-or-nothing binary call option.

[2]

An exotic option has an underlying asset with price $S(t)$ at time t.

The underlying asset price process under the risk neutral measure P is given by $dS(t) = S(t)(r - q)dt + S(t)\sigma dW(t)$, where r is the constant interest rate, q is the constant dividend yield of the underlying asset, $W(t)$ is P Brownian motion and σ is the constant volatility of the underlying asset price.

(iii) Write down an expression for $S(T)|F_t$ under the measure P, where F_t is the filtration. [2]

Suppose Q is a measure equivalent to P with Q Brownian motion $W^Q(t)$ and $\gamma = -2\sigma$ is an F_t previsible process satisfying $W^Q(t) = W(t) + \int_0^t \gamma ds$.

(iv) Derive the stochastic differential equation for $dS(t)$ and the expression for $S(T)|F_t$, both under the measure Q. [3]

(v) Determine the Radon–Nikodym derivatives $\frac{dQ}{dP}$ (which is P measurable) and $\frac{dP}{dQ}$ (which is Q measurable and noting $\frac{dP}{dQ} = \left(\frac{dQ}{dP}\right)^{-1}$). [3]

[Total 11]

3 E-Coin is a recently launched digital currency that does not involve a central bank. The number of E-Coins is currently limited, but new E-Coins are occasionally issued. There is a cap on the total number of E-Coins that will likely be reached in a few years. E-Coin is gaining in popularity and an established exchange is about to launch an E-Coin future.

(i) Determine whether the E-Coin would be considered an investment asset or a consumption asset. [2]

(ii) List the items that the exchange would need to specify in the E-Coin future. [3]

The exchange is concerned that many of the E-Coin futures may be traded by less experienced investors with limited financial resources for speculative purposes.

(iii) Describe the risks to the exchange of these less experienced investors trading in the new E-Coin futures. [3]

(iv) Explain how the exchange can mitigate the risks described in part (iii). [3]

[Total 11]

4 A life insurance company offers a wide range of savings products where it collects an annual fee from thousands of policyholders each year. At any time, policyholders can transfer their savings to another life insurer. The key risk to the life insurer is that more policyholders than expected lapse their policies, meaning the life insurer will not benefit in future from subsequent annual fees.

A bank wishes to offer the life insurance company structured securities and over-the-counter (OTC) contracts to help hedge lapse risk. The bank already offers a wide range of structured securities and OTC contracts to help life insurance companies hedge mortality risk.

(i) Describe how the bank could use the concepts underlying the following contracts to offer the life insurer products to hedge lapse risk:

- mortality swaps
- principal-at-risk mortality bond
- survivor cap.

[6]

The bank will offer securities on a lapse index calculated by the life insurer.

(ii) Define moral hazard. [1]

(iii) Set out the moral hazard associated with using a lapse index derived by the life insurer based solely on the life insurer's lapse experience. [2]

(iv) Suggest ways in which the bank could alter the contracts in part (i) to mitigate the moral hazard, assuming a lapse index on the life insurer's lapse experience must be used. [2]

(v) Outline other ways, other than the contracts in part (i), that the life insurer could mitigate lapse risk. [3]

[Total 14]

5 An investment bank has a payment to make in T years of value $1{,}000{,}000X(S(T))$ where $X(S(T)) = \max(K - |0.5S(T) - K| - 2|S(T) - K|, 0)$.

Here $S(T)$ is the value of the notional share value of a manufacturing company at time T and K is a fixed positive value.

(i) Draw a graph of $X(S(T))$ for variable values of $S(T)$. [5]

(ii) Prove, stating any assumptions made, that $X(S(T))$ can be constructed from:

- buying 2.5 European call options with expiry in T years and with strike $0.8K$;
- selling 4 European call options with expiry in T years and with strike K; and
- buying 1.5 European call options with expiry in T years with strike $\frac{4K}{3}$.

All of these options have the underlying as the stock price of the company. [6]

The investment bank is considering purchasing the derivatives above in sufficient quantity to hedge the $1{,}000{,}000X(S(T))$ payment. The stock of the manufacturing company is not traded on an exchange (as it is held privately) and instead a derived stock price is determined by a calculation. The investment bank is considering using the traded shares of a similar company as an underlying asset for the vanilla derivatives above.

(iii) Assess the basis risk to the investment bank in using vanilla derivatives with an underlying of the traded stock of another similar manufacturing company. [3]

[Total 14]

6 Insurance companies in a developed country have investment portfolios of long dated corporate bonds to match their liabilities. The regulatory regime requires that these insurers must invest only in bonds with credit ratings of BBB– or higher ('Investment Grade' bonds). When the credit rating of a bond is downgraded to BB+ or lower ('Sub-Investment Grade'), or defaults, the insurer is required to immediately sell the downgraded bond and buy an Investment Grade bond of their choice.

(i) Explain why the value of a bond may fall when it is downgraded to Sub-Investment Grade. [2]

(ii) Explain how the regulatory regime described above will help reduce credit risk. [2]

(iii) Outline the limitations of using credit ratings in the above regulatory regime to reduce credit risk. [4]

An insurer has a large portfolio of corporate bonds, including a $100m exposure to a 15-year bond that currently has a credit rating of BBB ('Big Bond'). The insurer is considering hedging the costs associated with Big Bond being downgraded to a rating of BB. The insurer has been offered the following standard Credit Default Swap (CDS) from a bank with the following terms:

- underlying = Big Bond
- notional of CDS = $100m
- term of CDS = 3 years
- CDS spread = 2%, which is paid at the end of each year, contingent on Big Bond not defaulting during that period (i.e. there is no accrual payment on default).

The insurer assumes:

- a recovery rate of 40% .
- the probability of Big Bond defaulting during a year conditional on no earlier default is 4% .
- defaults occur at the end of the year.
- risk-free interest rates are assumed to be 0% at all terms.

(iv) Demonstrate that the value of the CDS to the insurer is $1.4m. [4]

The insurer has calculated that if Big Bond immediately downgrades to a credit rating of BB:

- the cost of selling the downgraded Big Bond and buying an appropriate Investment Grade bond would be $30m.
- the value of the CDS would increase from $1.4m to $8.2m based on an assumed probability of Big Bond defaulting during a year conditional on no earlier default of 8% when it is rated BB.

(v) Calculate the required notional of the CDS for the insurer to hedge its $30m costs associated with an immediate Big Bond downgrade. [2]

(vi) Determine, without performing any further calculations, whether the required notional calculated in part (v) to hedge the fixed $30m cost would increase, decrease or remain approximately unchanged in each of the following separate scenarios:

(a) The term of the CDS is increased from three years to 10 years.

(b) The insurer assumes a recovery rate of 50% (rather than 40%).

[3]

(vii) Suggest practical challenges associated with using the increase in value of the CDS to fund the expected $30m costs associated with the downgrade of Big Bond to a credit rating of BB. [3]

[Total 20]

7 A defined benefits pension fund is currently underfunded and has a significant proportion of its investment portfolio invested in equities with a current value of 20 million in the local currency.

(i) Describe the typical uses of derivatives in its investment portfolio. [4]

The pension fund is concerned about the risk of extreme falls in the equity market. The trustees of the pension fund are considering several options to manage this risk.

(ii) Explain why reducing the equity exposure may not meet the wider objectives of the trustees. [2]

To better manage the risk, the trustees are considering using derivatives.

(iii) Describe how put options could be used to manage the risk. [3]

The pension fund's equity investment has a similar weighted composition as the main equity index used in the country. Its hedging strategy is to spend 0.5% of its equity portfolio value at the start of each calendar month on buying two-month European put options on the equity index that are 30% below the current underlying price (of the equity index).

On the first day of the calendar month the options are purchased using the following information:

The equity index value is 2,000.

Strike price	*Option price at the start of the current trading day*	*Number of options traded on the previous day (millions)*
1,350	0.02	32
1,400	0.11	2
1,450	0.01	47

(iv) Demonstrate that the number of options required to implement the hedge as detailed above is 909,091. [2]

(v) Comment on how the hedge could be implemented given the current market conditions. [3]

The number of options as calculated in part (iv) was eventually purchased in the market at the price shown in the table above at the start of the calendar month. At the end of the calendar month, before the options are sold, the equity index has fallen 20% and the implied volatility has risen to 50%.

Risk-free interest rate: 2% (continuously compounded)
Equity index dividend yield: 4% (continuously compounded)

(vi) Demonstrate that the Black–Scholes value of the put options held is 19,158,000. [4]

(vii) Assess the effectiveness of the hedging strategy at the end of the calendar month. [2]

[Total 20]

END OF PAPER