INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2023 (am)

Subject CP2 – Modelling Practice
Core Practices

Paper Two

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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Exam requirements

Read the background document, which describes the scenarios that have been modelled and documented for this project and the work that remains outstanding.

Read the audit trail that has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

You should assume that your colleague's calculations have been checked and are correct.

1 Spreadsheet model

Expand the spreadsheet model to:

(i) repeat the calculations for the expected revenue, expenses, profit and profit margin (as a percentage of revenue) assuming the average temperature over the 12-month period is less than 20°C. [5]

(ii) calculate the revised level of monthly costs per cohort such that the profit margin under the adverse weather scenario is the same as the profit margin under the normal weather scenario. [4]

You should ensure that the additional work you undertake on the spreadsheet contains appropriate self-checks. [Sub-total 9]

2 Charts

Construct suitable charts to illustrate:

(i) the frequency distribution of the natural logarithm of birth weights for suitable LN(weights) buckets across the ten cohorts. [2]

(ii) a comparison of the amounts of the different types of cost and profits:

(a) under the normal weather scenario.

(b) under the adverse weather scenario based on the original monthly cost per cohort.

(c) under the adverse weather scenario based on the revised monthly cost per cohort.

[**Note:** The comparison is to cover totals across all ten cohorts.] [3]

(iii) a comparison of the cost components and profit, all as proportions of revenue, under normal weather against the values under the adverse weather scenarios before and after monthly cost reduction. The comparison is to cover totals across all ten cohorts. [3]

[Sub-total 8]

3 Summary document

Prepare a summary document of around five to seven pages, capturing the main features and results of the work done by you and your colleague. You can assume that the summary is being prepared for your manager, a senior actuary, who will present the work to the farmer.

Your summary should include the following:

- The purpose of the project, data, method and assumptions used by you and your colleague
- Results, including relevant tables and charts
- Commentary on the results
- Key conclusions
- Suggested next steps.

Commentary on the results should cover, but not be limited to:

- analytical comments on each stage of the results, including explaining patterns in the results and any unusual features.
- an explanation of the differences between the results under the various scenarios modelled.

Next steps need to be specific to the project, with some mention of why each is valid. The summary should cover the full scope of the project, including the current approach that was modelled in the spreadsheet provided.

You are not required to add to or amend the audit trail.

Marks available for the summary:

(i)	Methodology (including purpose, data, method and assumptions).	[26]
(ii)	Results, including charts.	[5]
(iii)	Commentary on results and conclusions.	[22]
(iv)	Next steps.	[20]
(v)	Drafting.	[10]
		[Sub-total 83]
		[Total 100]

Background

A farmer has been breeding chinchillas for sale as pets to a local market. After the chinchillas are born, the farmer will look after the chinchillas for a period of 12 months, at which point, they will have become fully grown and reached their maturity weight. Once the chinchillas are fully grown, the farmer will sell the chinchillas that meet the weight criteria in the market. The market will only accept chinchillas that weigh 5 kilograms (kgs) or more. The farmer will keep chinchillas weighing more than 6.5 kgs for breeding. The farmer will not sell animals weighing less than 5 kgs.

The farmer is worried about their expected profits from the sale of the chinchillas at the market and how these profits may be affected by local weather conditions. The farmer has engaged your firm to project expected future profits for the latest ten cohorts of chinchillas that have been born.

The farmer has provided data on these cohorts of chinchillas. For each cohort, 30 chinchillas are born. The data provided by the farmer shows the birth weights of each of the 30 chinchillas for each cohort. The chinchillas' birth weights are expected to follow a lognormal distribution with parameters mu = 1.15 and sigma = 0.1. A reputable agricultural journal has specified that the breed owned by the farmer has a maximum birth weight of 4.0 kgs and a minimum birth weight of 2.4 kgs.

The farmer has also provided sample monthly growth rates in the chinchilla's weights over a 12-month period for the 30 sample chinchillas.

The farmer has provided the costs of breeding and looking after the chinchillas over the 12-month period, which are:

- the birthing cost of $2.00 per chinchilla.
- a variable feed cost of $0.50 per kg of the maturity weight for the whole 12-month period.
- monthly cost, per cohort, of $30.00 for labour and utilities (part of this monthly labour cost is a discretionary bonus that the farmer can adjust depending on the profitability of the ten cohorts).

The current market price for selling chinchillas is $5.00 per kg.

Our firm's analytics team has provided the survival probabilities for chinchillas over the first 12 months of their lives. The expected survival probability is related to the natural logarithm of the chinchilla's birth weight. The analytics team has provided a table of survival probabilities based on this factor.

The analytics team has also advised that both the survival probabilities and the growth rates of the chinchillas are dependent on weather conditions. If the average temperature over a 12-month period is less than 20°C (considered to be adverse weather conditions), then the monthly growth rates will be 10% lower than the farmer advised rates for that 12-month period. The survival probabilities will also be 7.5% lower for all birth weights.

Your colleague has started the modelling and has completed:

- the calculation of the maturity weights for base scenario.
- identification of which animal stock satisfies the market requirements.
- calculation of expected revenue.
- the expected expenses per cohort.
- the overall expected profit and profit margin (% of revenue) for all ten cohorts.

Your manager has asked to complete the following additional calculations:

(i) Calculate the expected revenue, expenses, profit and profit margin as a percentage of revenue for the ten cohorts assuming the average temperature over the 12-month period is less than 20°C (an adverse weather scenario).

(ii) Calculate the revised level of monthly costs per cohort such that the profit margin under the adverse weather scenario is the same as the profit margin under the normal weather scenario.

Your manager also asked you to construct a number of charts, details of which are set out in the exam requirements.

Your manager has asked you to prepare a summary covering all elements of the work (both the original work your colleague completed and the additional modelling you are undertaking).

Your summary should include the following:

- Purpose of the project, data, method and assumptions used by you and your colleague
- Results, including charts
- Commentary on the results and key conclusions
- Suggested next steps.

You are not expected to include the additional modelling you undertake in the audit trail, but your results should be included in the summary.

Audit trail

Objective

A farmer is worried about their expected profits from the sale of chinchillas at a local market and how it can be affected by weather. The Excel model calculates the profit margin under normal weather conditions.

Data

The farmer has provided birth weight data as well as the monthly growth rates in the weights of the chinchillas for ten cohorts of 30 chinchillas from the farm. The associated costs of the project were also provided by the farmer.

The analytics team has provided the probability of survival over the first 12 months under different weather conditions.

Data checks

The birth weights have been confirmed to be within the expected range for the breed owned by the farmer. This is done using Excel functions MIN and MAX for each column in cells B36:K37 of the 'Data' tab. The expected minimum and maximum weights are included in cells B39:K40 and checks are included in cells B43:K44 by checking that the actual minimum/maximum weights are higher/lower than the expected minimum/maximum weights, with the minimum and maximum weights being sourced from a reputable agricultural journal.

The growth rates have been checked for reasonability to be within the range 0% to 100% in cells M36:X37 of the 'Data' tab.

Probabilities of survival have been checked and confirmed to be within zero and one in cells B166:C170 of the 'Parameters' sheet.

Birth weights were checked for completeness, which was satisfactory as the data had 300 entries (30 chinchillas per cohort × ten cohorts). No outliers were picked up in the data.

Assumptions

1. Interest rate earned on the cashflows can be ignored given the cashflows span over a relatively short period.
2. Expense inflation can be ignored.
3. Data provided and decrements will be appropriate for the future periods.
4. Deaths occur halfway through the 12-month period.

Methodology

Parameters

This sheet includes:

- the survival probability tables in cells B3:C164.
- the survival probability at fair weather in column C.
- the parameters for lognormal distribution in cells F2:G3.
- weight restrictions applied to mature stock ready for market in cells J2:K3.
- expenses in cells K9:K11, split into fixed birthing expense per animal, variable component based on weight of animal (for feed cost) and a level monthly expense.
- the market price (provided in cell K7).
- the time to mature (provided in K13).

Growth factors and survival probabilities

This worksheet calculates the monthly growth factors observed for each animal over a period of 12 months as well as the survival probability based on the natural logarithm of birth weight for each animal.

Using the data provided, the natural logarithm of weight is calculated in cells B5:K34 using the Excel function LN(birth weight).

Cells A36:D39 check the expected lognormal distribution parameters against that from the data. The average and standard deviation for each cohort is calculated in cells B41:K42 using the AVERAGE and STDEV.S functions.

Growth factors (1 + growth rate), where growth rate represents the percentage gain in weight per month, have been calculated in cells M5:X34. The growth rates were picked from the 'Data' tab.

Using a VLOOKUP function, the probability of survival for each chinchilla is calculated in cells AA5:AJ34 based on natural logarithm of birth weight. The survival probabilities are looked up from the table given in the 'Parameters' tab. Checks are included in cells AA36:AJ37 to ensure all probabilities are between zero and one.

Expected profits

This worksheet calculates the maturity weight and the revenue from the chinchillas surviving the 12-month period.

Maturity weight has been calculated as birth weight × PRODUCT (growth factors across all 12 months) for all 30 chinchillas in each cohort. The results of these calculations are found in cells B5:K34. Birth weights have been pulled from the data provided and the growth factor is linked in from the sheet 'Growth Factors & Survival Pr'. The total maturity weight is calculated in cells B36:K36 by summing the weights of the 30 chinchillas in each column (i.e. each cohort). The number of chinchillas below the minimum weight and above the maximum weight is calculated in cells B37:K38 for each cohort using the COUNTIF functions and the individual weight data for each cohort and the relevant limit.

Cells Y5:AH34 provides the expected maturity weight eligible for marketing. The table therefore excludes maturity weights lower than minimum allowed at the market and those which qualify for breeding by using IF and OR excel functions.

Expected revenue is calculated in row 38 as the sum of (qualifying weights × probability of survival) × market price per unit weight of $5. This is calculated for each cohort. The total revenue across all ten cohorts is calculated.

Cells M5:W34 provides the weight of animals fed (total fed weight), taking into account that the ones that died would have been fed for about half of the period, assuming death occurs halfway through the 12-month period. The table therefore takes the maturity weight × (1 – (1 – survival probability) ÷ 2) for each chinchilla. The total fed weight is used to estimate the cost of feed for the cohort described below.

Expense breakdown for each cohort is provided in cells X45:AH47.

- Row 45 calculates the birthing cost for each cohort as the birth cost per animal from the 'Parameter' sheet cell K9 × the number of animals born.
- Row 46 calculates the feeding cost, which is based on the fed weight of all animals (cells N36:W36) × the feeding cost from the 'Parameter' sheet cell K10.
- Row 47 is the level monthly cost calculated as number of months (12) × monthly expense from 'Parameter' sheet cell K11.

The total expenses for each cohort are calculated in cells Y39:AH39 by summing the three expenses above. The total expense across all ten cohorts is calculated in cell AJ39.

In cells Y40:AH40, profit for each cohort is calculated as the difference between revenue in row 38 and total expense in row 39. The profit margin is calculated in cell AJ41 as aggregate profits across all ten cohorts divided by the aggregate revenue.

END OF PAPER