INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 April 2023 (am)

Subject CM1 – Actuarial Mathematics
Core Principles

Paper B

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 A company wishes to invest in a fleet of 50 vehicles for use by its workforce and will choose between two alternative vehicles for the entire fleet – an Electric Vehicle or a traditional Internal Combustion Engine car.

The Electric Vehicles incur a higher initial cost than the Internal Combustion Engine cars but have lower running and maintenance costs. Worksheet 'Q1 Base' gives details of the costs involved for each type of vehicle.

The company intends to run the fleet for a period of 5 years. It anticipates that the average annual mileage for each vehicle type will be 20,000 miles, spread uniformly over each year.

(i) Calculate, for each vehicle type, the present value of the total cost to the company of buying and running the fleet. You should use a risk discount rate of 4.5% per annum. [25]

(ii) Recommend, with reasons, the most suitable type of vehicle for the fleet, based on your answer to part (i). [2]

The company wishes to consider how its estimate of annual mileage per car would impact its decision on which vehicle type to choose for the fleet.

(iii) Determine the annual average mileage per car, which would result in the same cost, in present value terms, for each fleet. [2]

The company has now proposed to run the fleet for a shorter period of 3 years, and to take account of the sale proceeds of the vehicles at the end of this period, when making its decision. It expects the residual value of the vehicles at the end of 3 years to be 40% of the original purchase price for Internal Combustion Engine cars and 50% for Electric Vehicles.

(iv) Copy your Q1(i) (ii) worksheet to the Q1(iv) worksheet and update your results and recommendation to take account of the company's new proposal, based on an annual mileage of 20,000 miles. [7]

(v) Discuss the suitability of this model as a tool for helping the company select the fleet. [6]

[Total 42]

2 A life insurance company issues a 15-year unitised with-profits contract to a policyholder currently aged 50 exact.

The basic policy information, charging structure and the basis used by the company to carry out profit testing, are all set out in the 'Q2 Base' worksheet.

The unit price increases each year in line with the company's declared bonus interest rate.

You should ignore non-unit reserves.

(i) Calculate the unit cashflows for each year of the policy, per policy in force at the start of the year. [5]

(ii) Calculate the expected non-unit cashflows for each year of the policy, per policy in force at the start of the year. [15]

(iii) Calculate the present value of the profit from the policy at outset. [6]

[Total 26]

3 A company has privately issued three fixed interest bonds with terms of 10 years, 15 years and 25 years respectively. The bonds offer different coupons which are paid half-yearly, the first coupon to be paid in 6 months' time.

The coupon rates, redemption rates and spot rates to use are set out in the worksheet 'Q3 Base'.

(i) Calculate the duration of each bond. [8]

(ii) Explain why a 10-year bond with a coupon rate of 4% would have a lower duration than a 10-year bond with a coupon rate of 2%. [3]

Fifteen years after issue, immediately after the coupon payment then due, the company defaults on the 25-year bond. Bond holders are unable to sell their holdings as the bonds were privately issued so there is no liquid secondary market. The company offers two options to the holders of the 25-year bond:

Option 1: the term of the bond remains at the original term of 25 years, no further coupons will be paid and the bond will be redeemed at 105%.

Option 2: coupons will be immediately reduced to 1.25% per annum, and the term of the bond will be extended by 5 years.

As a result of the default, bondholders expect a 0.5% risk premium over spot rates currently used, which are set out in the worksheet 'Q3 Base'.

(iii) (a) Calculate the present value for the defaulted bond under both options at time 15. [8]

(b) Determine which option bondholders should choose. [2]

(iv) Calculate the duration of the bond at time 15 under the original conditions and under each of options 1 and 2. [5]

Many bondholders decide to select the other option than the one determined in part (iii)(b).

(v) Suggest what other considerations may have driven their decision. [6]

[Total 32]

END OF PAPER