



Institute
and Faculty
of Actuaries

EXAMINERS' REPORT

CM1 - Actuarial Mathematics

Core Principles

Paper A

September 2023

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula, or other textbook works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
November 2023

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work, focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations; candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used, or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions. Failure to do so can lead to fewer marks being awarded. In particular, where the instruction, “showing all working” is included and the candidate shows little or no working, then the candidate will be awarded less marks even if the final answer is correct.

Where a question specifies a method to use (e.g. determine the present value of income using annuity functions) if a candidate uses a different method, the candidate will not be awarded full marks, indeed, the candidate might even be awarded no marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

There appeared to be many candidates who had underestimated the quantity of study required for this subject. The nature of the online exam format meant that there was little on the paper that could be answered via knowledge based answers alone.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, only well prepared candidates show enough of their working to fully benefit from this.

The Examiners felt that the “open book” nature of the online exam led some candidates to rely on their notes much more than if the exam had been “closed book”. The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they

would do if the exam were of the traditional “closed book” format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 54
1557 presented themselves and 492 passed.

Solutions for Subject CM1A September 2023

Q1

(a)

$$\bar{A}_{50} \approx (1.05)^{0.5} \times 0.3 = 0.307408523 \quad [1/2]$$

Other valid approximations $\bar{A}_{50} \approx \frac{i}{\delta} \times 0.3$ or $\bar{A}_{50} \approx (1 + \frac{1}{2} \times i) \times 0.3$

(b)

$$A_{50} = 1 - d\ddot{a}_{50} \Rightarrow 0.3 = 1 - \left(\frac{0.05}{1.05}\right)\ddot{a}_{50} \Rightarrow \ddot{a}_{50} = 14.7 \quad [1]$$

$$a_{50} = \ddot{a}_{50} - 1 = 14.7 - 1 = 13.7 \quad [1/2]$$

(c)

$$\bar{a}_{50} = \ddot{a}_{50} - \frac{1}{2} = 14.7 - \frac{1}{2} = 14.2 \quad [1]$$

Or $\bar{a}_{50} = a_{50} + \frac{1}{2}$ or $\bar{A}_{50} = 1 - \delta\bar{a}_{50} \Rightarrow \bar{a}_{50} = \frac{1 - \bar{A}_{50}}{\delta}$

(d)

$$\ddot{a}_{50}^{(4)} = 14.7 - \frac{3}{8} = 14.325 \quad [1]$$

[Total 4]

Question 1 was well answered by the majority of the candidates.

Q2

A - Models can look impressive. [1/2]

The borrowing department needs to ensure the model has been tested and checked against real world results in the area for which it will be used. [1/2]

Any errors in the model will be transferred from the lending to the borrowing department. [1/2]

There is a danger that the borrowing department do not understand the model and use it as a 'black-box' without adequate scrutiny of the results. [1/2]

[Marks available 2, maximum 1 1/2]

B - Models rely heavily on data input. [1/2]

It is not clear where the borrowing department obtained the updated data. [1/2]

Are these data in the correct format and consistent with the data used by the original department? [1/2]

[Marks available 1 1/2, maximum 1 1/2]

C -Need to understand the uses to which it can safely be put. [1/2]

There may be assumptions underlying the model (and/or proxy model) that may not be appropriate for the borrowing department. [½]
 The borrowing department may be interested in a part of the model which was heavily approximated when the proxy model was created. [½]
 The projection period may be too short. [½]
 There may be limitations to the range of, for example, bank base rates for which the model behaves sensibly (if it doesn't look at possible shock events). [½]
 Proxy models are, by definition, models of more complex models. They are simplifications which produce results more quickly than the original models. [½]
 However, they will also be less accurate. Unless the borrowing department understands this limitation, they may not be able to interpret the results appropriately. [½]
 [Marks available 3½, maximum 1½]

D - It is not possible to include all future events. [½]
 Unexpected changes in, for example, legislation in the future may render the results of the model invalid. [½]
 New trade agreements may hugely alter the volumes of import/export. [½]
 A change in government may alter the government's tax take. [½]
 [Marks available 2, maximum 1½]

E - It may be difficult to interpret some of the outputs of the model. [½]
 The borrowing department may not have the expertise to interpret the results. [½]
 They may instead just seize on the best- or worst-case results. [½]
 [Marks available 1, maximum 1]
[Total max 4]

Candidates who simply reproduced core reading without making their comments specific to the question gained little credit.

Q3

(i)

${}_5q_{xy}^1$ represents the probability that the first death of (x) and (y) occurs between time 5 and time 6 AND that the first death is (x). [2]

(ii)

$${}_5q_{xy}^1 = \int_5^6 {}_t p_x \times {}_t p_y \times \mu_{x+t} dt \quad [2]$$

$${}_t p_x = \exp\left(-\int_0^t \mu_{x+s} ds\right) = e^{-\int_0^t 0.025 ds} = e^{-0.025t}$$

Now, we have

$$\text{Similarly, we have } {}_t p_y = e^{-0.01t} \quad [1]$$

Thus, we have:

$$\begin{aligned}
 {}_5|q_{xy}^1 &= \int_5^6 e^{-0.025t} \times e^{-0.01t} \times 0.025 dt \\
 &= \int_5^6 e^{-0.035t} \times 0.025 dt \\
 &= 0.025 \left[\frac{e^{-0.035t}}{-0.035} \right]_5^6 \\
 &= \frac{0.025}{0.035} \times (e^{-0.035 \times 5} - e^{-0.035 \times 6}) \\
 &= 0.0206234
 \end{aligned}$$

[2]

[Total 7]

Part (i) Many candidates misunderstood what had to happen to life y in the 6th year. Whilst many candidates understood that x had to die first, many mistakenly thought y also had to die in the same year.

Part (ii) A common error was to state incorrect integral boundaries.

Q4

We need to derive dependent probabilities of surrender.

Force of surrender given by:-

$$\mu_{t=1}^{surr} = -\ln(1 - 0.15) = 0.1625189$$

$$\mu_{t=2}^{surr} = -\ln(1 - 0.10) = 0.1053605$$

[1]

Force of mortality given by: -

$$\mu_{x=67}^{mort} = -\ln(1 - 0.017824) = 0.01798476$$

$$\mu_{x=68}^{mort} = -\ln(1 - 0.019913) = 0.02011394$$

[1]

Dependent probabilities given by: -

$$(ap)_{x=67} = \exp(-0.1625189 - 0.01798476) = 0.834850$$

$$(ap)_{x=68} = \exp(-0.1053605 - 0.02011394) = 0.882078$$

[1]

Dependent surrender probabilities; -

$$(aq)_{x=67}^{surr} = \left(\frac{0.1625189}{0.1625189 + 0.01798476} \right) \times (1 - 0.834850) = 0.148695$$

$$(aq)_{x=68}^{surr} = \left(\frac{0.1053605}{0.1053605 + 0.02011394} \right) \times (1 - 0.882078) = 0.099018$$

[2]

[Total 5]

Many candidates did not attempt this question.

It appears many candidates did not read the question properly or did not seem to recognise that this was a straightforward question on multiple decrement tables.

Q5

Let R denote the initial annual rental rate.

Then, we have:

$$\begin{aligned}
 0 &= -24,000 + R\bar{a}_{\overline{1}|12\%} + (R+1,500)\bar{a}_{\overline{1}|12\%}v_{12\%} + (R+3,000)\bar{a}_{\overline{1}|12\%}v_{12\%}^2 + (R+4,500)\bar{a}_{\overline{1}|12\%}v_{12\%}^3 + 9,000v_{12\%}^4 \\
 &= -24,000 + R\bar{a}_{\overline{1}|12\%} \times (1 + v_{12\%} + v_{12\%}^2 + v_{12\%}^3) + 1,500\bar{a}_{\overline{1}|12\%} \times (v_{12\%} + 2v_{12\%}^2 + 3v_{12\%}^3) + 9,000v_{12\%}^4 \\
 &= -24,000 + R\bar{a}_{\overline{1}|12\%} \times \ddot{a}_{\overline{4}|12\%} + 1,500\bar{a}_{\overline{1}|12\%} \times (Ia)_{\overline{3}|12\%} + 9,000v_{12\%}^4
 \end{aligned}$$

[4]

And, we find R such that:

$$\begin{aligned}
 R \times (1.058867 \times 0.8929) \times (1 + 2.4018) &= 24,000 - 1,500 \times (1.058867 \times 0.8929) \times 4.6226 - 9,000 \times 0.63552 \\
 \Rightarrow R &= \text{£}3,645.39 \text{ per annum}
 \end{aligned}$$

[2]

[Total 6]

A common error was to set the first payment to $R-1,500$, but not give R as the answer.

As this question covered only a 4-year period, the answer could also be produced from first principles and many candidates did so. This approach was perfectly acceptable, and candidates were not penalised.

Q6

$$EPV = v^{10} \times \frac{l_{60}}{l_{50}} \times 5000 \times \int_0^{\infty} 1.025^t \times v^t \times {}_t p_{60} dt \quad [2\frac{1}{2}]$$

$$= 1.045^{-10} \times \frac{91,732}{96,247} \times 5000 \times \int_0^{\infty} \left(\frac{1.025}{1.045}\right)^t \times e^{-0.041t} dt \quad [1]$$

Evaluating the integral:

$$\int_0^{\infty} \left(\frac{1.025}{1.045}\right)^t \times e^{-0.041t} dt$$

$$= \int_0^{\infty} e^{\left(\ln\left(\frac{1.025}{1.045}\right) - 0.041\right)t} dt$$

$$= \int_0^{\infty} e^{-0.0603242728t} dt$$

$$= \frac{1}{-0.0603242728} \times (0 - 1) = 16.57707509 \quad [2]$$

$$EPV = 1.045^{-10} \times \frac{91,732}{96,247} \times 5000 \times 16.57707509 = \pounds 50,868.47$$

[1/2]

[Total 6]

This question was not very well answered.

Common errors included:

- ignoring the growth of the annuity payment;
- using incorrect integral boundaries;
- inability to correctly perform the integration and evaluate the answer.

Q7

(i)

Let g be the payment increase rate = 4.3902% pa

The duration of the liabilities is given by:

$$\begin{aligned} \tau &= \frac{\sum_{k=1}^{25} C_{t_k} t_k v^{t_k}}{\sum_{k=1}^{25} C_{t_k} v^{t_k}} = \frac{1.2 \times (v + 2(1+g)v^2 + 3(1+g)^2 v^3 + \dots + 25(1+g)^{24} v^{25})}{1.2 \times (v + (1+g)v^2 + (1+g)^2 v^3 + \dots + (1+g)^{24} v^{25})} \\ &= \frac{v(1 + 2(1+g)v + 3(1+g)^2 v^2 + \dots + 25(1+g)^{24} v^{24})}{v(1 + (1+g)v + (1+g)^2 v^2 + \dots + (1+g)^{24} v^{24})} \end{aligned}$$

[3]

$$= \frac{(I\ddot{a})_{\overline{25}|}}{\ddot{a}_{\overline{25}|}} = \frac{(Ia)_{\overline{25}|}}{a_{\overline{25}|}} \quad \text{calculated at } i^* \% \text{ pa.} \quad [1]$$

Where $\frac{1}{1+i^*} = \frac{1+g}{1+i} = \frac{7\% - 4.3902\%}{1+4.3902\%} = 2.5\% \text{ p.a.}$ [1]

From actuarial tables, or otherwise, we have:

$$\tau = \frac{216.0088}{18.4244} = 11.72 \text{ years} \quad [1]$$

(ii)

If the increase rate is greater than 4.3902% p.a. then: -

a. The size of future cashflows would increase. [1]

b. This would give a greater weighting to the present value of payments made in the future, [1]

c. and so, the mean term of the cashflows weighted by present value (i.e., the duration) would increase. [1]

[Total 9]

Part (i) Common errors included:

- Not showing all working and only writing the final annuity formula.
- Making the same error in both the numerator and denominator would mean that the error would often cancel out and result in a numerical answer that appears to be correct but obtained through incorrect logic. Such candidates were penalised for the error.

Part (ii) was not very well answered by many candidates.

Q8

(i)

$$0.97 \times P \times \ddot{a}_{\overline{45:15}|6\%} = 155,000 \times 1.02 \times A_{\overline{45:15}|6\%} - 5,000 \times 1.02 \times (IA)_{\overline{45:15}|6\%} + 300 + 0.22 \times P \quad [3\frac{1}{2}]$$

$$\ddot{a}_{\overline{45:15}|6\%} = 10.149$$

$$A_{\overline{45:15}|6\%} = 0.42556 \quad [1\frac{1}{2}]$$

$$(IA)_{\overline{45:15}|6\%} = (IA)_{45} - v^{15} {}_{15}P_{45} \times [(IA)_{60} + 15A_{60} - 15]$$

$$(IA)_{\overline{45:15}|6\%} = 4.37062 - 0.41727 \times \frac{9287.2164}{9801.3123} \times (5.46572 + 15 \times 0.32692 - 15) = 6.20141 \quad [2]$$

Therefore,

$$0.97 \times P \times 10.149 = 155,000 \times 1.02 \times 0.42556 - 5,000 \times 1.02 \times 6.20144 + 300 + 0.22 \times P$$

$$9.62453P = 67,281.04 - 31,627.34 + 300$$

$$P = \frac{35953.70}{9.62453} = \text{£}3,735.63 \quad [1\frac{1}{2}]$$

(ii)

$${}_5V = 130,000 \times 1.02 \times A_{50:\overline{10}|6\%} - 5,000 \times 1.02 \times (IA)_{50:\overline{10}|6\%} - 0.97 \times P \times \ddot{a}_{50:\overline{10}|6\%} \quad [2]$$

$$\ddot{a}_{50:\overline{10}|6\%} = 7.694$$

$$A_{50:\overline{10}|6\%} = 0.56449 \quad [1\frac{1}{2}]$$

$$(IA)_{50:\overline{10}|6\%} = (IA)_{50} - v^{10} {}_{10}P_{50} \times [(IA)_{60} + 10A_{60} - 10] \quad [1\frac{1}{2}]$$

$$(IA)_{50:\overline{10}|6\%} = 4.84555 - 0.55839 \times \frac{9287.2164}{9712.0728} \times (5.46572 + 10 \times 0.32692 - 10) = 5.52106 \quad [2]$$

$${}_5V = 130,000 \times 1.02 \times 0.56449 - 5,000 \times 1.02 \times 5.52106 - 0.97 \times 3735.63 \times 7.694$$

$${}_5V = 46693.97 - 27879.68 = \text{£}18,814.29 \quad [1]$$

(iii)

If the rate of interest had been 6% pa, then both reserving bases would have been the same, and they would have been the same as the premium basis.

The prospective and retrospective reserves would therefore have always been equal.

However, as the rate of interest used for the retrospective reserve is lower than 6% then the premiums less benefits and expenses will accumulate at a lower rate.

The resultant surrender value will therefore be lower than the amount calculated in (ii).

[2]

[Total 16]

Part (i) Generally well answered.

Common errors included

- Using an incorrect age (often using $x = 60$, instead of $x = 45$)
- Ignoring the survival benefit of 80,000.
- Basing the claim expense on part of the EPV of claims (e.g. only adding it to the death benefit and not the survival benefit).

Part (ii) A common error was using an incorrect sum assured.

Part (iii) was not very well answered. The link between prospective reserve (based on discounting values) and retrospective reserve (based on an accumulation of values) seemed to be poorly understood.

Q9

(i)

It is a principle of prudent financial management that once sold and funded at outset a product should be self-supporting.

[1]

Some products can have more than one financing phase (i.e. a negative cashflow in any year after year 1). [½]

The life office will set up provisions in the non-unit fund if the overall cash flow in any year other than year 1 would otherwise be negative. [½]

Reserves are set up early in the contract so that money can be released from the reserves as required to eliminate the negative cash flow. [½]

The company may pay out money to their shareholder when cashflow are positive which could lead to insufficient liquidity if faced with negative cashflows in the future. [½]

[Marks available 3, maximum 2]

(ii)(a)

$$NPV = \frac{-40.75}{1.08} - \frac{p_{[45]} \times 20.14}{(1.08)^2} - \frac{{}_2p_{[45]} \times 15.76}{(1.08)^3} + \frac{{}_3p_{[45]} \times 111.54}{(1.08)^4}$$
 [2]

Age x	q_x
[45]	0.001201
[45]+1	0.001557
47	0.001802

$$= -37.73148148 - 17.24606641 - 12.47631474 + 81.61193763$$
 [1]

$$= 14.158075$$
 [1]

(b)

(-40.75, -20.14, -15.76, 111.54)

$${}_2V = \frac{15.76}{1.04} = 15.15385$$
 [½]

Revised cashflow at t = 2: $-20.14 - 15.15385 \times p_{[45]+1}$ [1]

$$= -20.14 - 15.15385 \times 0.998443 = -35.2703$$
 [½]

$${}_1V = \frac{35.2703}{1.04} = 33.9137$$
 [1]

(c)

Revised cashflow at t = 1: $-40.75 - 33.91370348 \times p_{[45]} = -74.62297312$ [1]

(-74.623, 0, 0, 111.54)

$$NPV = \frac{-74.623}{1.08} + \frac{{}_3p_{[45]} \times 111.54}{(1.08)^4} = 12.51659215$$
 [1]

(d)

The net present value after zeroization is smaller than before. Setting up non-unit reserves has the effect of deferring the emergence of profit, (or equivalently, bringing forward the early year losses) as earlier cashflows are used to set-up these reserves - profits are now discounted for longer. Also, the rate at which profits are discounted (risk discount rate) is greater than the accumulation rate (interest earned on the reserve) [2]

(iii)

It will have no impact on the NPV calculated in (ii)(a) as no reserves were set-up. The NPV calculated in (ii)(c) will increase because the interest earned on funds has increased (or equivalently the reserves needed to be set aside will be smaller). The NPV in (ii)(c) will still be lower than the NPV value in (ii)(a).

[2]

[Total 15]

Part(i) was not very well answered, despite this being straight forward theory. A common error was to provide answers that were applicable to a conventional policy (whereas the question specified a unit-linked policy).

In Part (ii) common errors in (a) included

- Ignoring mortality rates
- Working with incorrect select mortality rate
- Using incorrect notation for select mortality
- Not showing all workings (even if excel was used to calculate the answers, all formulae must be shown for full credit).

A common error in (b) was using an incorrect mortality rate relative to the age or using a cumulative survival probability instead of a survival probability that covered one year of age.

(d) was not very well answered, most of the points could be found in the core reading.

Part (iii) was also not very well answered. Only well prepared candidates understood the interplay between the reserving interest rate and the risk discount rate and the impact on the net present value of profit.

Q10

(i)

Case $t < 8$

$$\begin{aligned} v(t) &= e^{-\int_0^t (0.03+0.005s)ds} \\ &= e^{-\left[0.03s+0.0025s^2\right]_0^t} \\ &= e^{-\left[0.03t+0.0025t^2\right]} \end{aligned}$$

[2]

Case $t \geq 8$

$$\begin{aligned} v(t) &= e^{-\left\{\int_0^8 (0.03+0.005s)ds + \int_8^t 0.07 ds\right\}} \\ &= v(8) \times e^{-[0.07(t-8)]} \\ &= e^{-0.40} e^{-[0.07t-0.56]} \end{aligned}$$

$$= e^{[0.16-0.07t]} \quad [3]$$

(ii)(a)

$A(t) = \frac{1}{v(t)}$ is an increasing function of t

We can see that

And $A(8) = e^{[0.07 \times 8 - 0.16]} = e^{0.40} = 1.49 < 2$

Therefore, it will take more than 8 years for a unit investment to double. [2]

(b) So, $t > 8$. We must find t such that:

$$A(t) = e^{[0.07t - 0.16]} = 2$$

Hence, $0.07t - 0.16 = \ln 2$

$$t = \frac{\ln 2 + 0.16}{0.07} = 12.188 \text{ years} \quad [2]$$

(iii)

$$A(10) = e^{0.54} = 1.71601 \quad [1]$$

Let effective inflation rate per annum = f

$$1 + f = (1.005)^4 = 1.020150501$$

$$\Rightarrow f = 2.0150501\% \text{ pa} \quad [1]$$

Let effective real rate of return per annum be i^*

$$(1 + i^*)^{10} = \frac{1.71601}{(1.020150501)^{10}} \Rightarrow i^* = 3.4636355\% \text{ pa}$$

Then [1]

Therefore, real rate of return per annum convertible quarterly is:

$$4 \times [(1.034636355)^{\frac{1}{4}} - 1] = 3.4195\% \text{ pa} \quad [1]$$

[Total 13]

Part (i) was generally well answered.

In Part (ii) all subparts were well answered, although only well prepared candidates included sufficient workings to gain full marks.

Part (iii) was reasonably well answered. A common error was to attempt to combine a nominal interest rate with an inflation rate applicable over inconsistent time periods.

Q11

(i)

First 10 years

$$i^{(12)} = 9\% \Rightarrow i = 9.3806898\% \quad [1/2]$$

$$5,300 \times a_{\overline{10}|i\%}^{(12)} - 300 \times a_{\overline{1}|i\%}^{(12)} \times (I\ddot{a})_{\overline{10}|i\%} \quad [3]$$

$$a_{\overline{10}|i\%}^{(12)} = 6.578474389$$

$$a_{\overline{1}|i\%}^{(12)} = 0.952909389 \quad [1]$$

$$(I\ddot{a})_{\overline{10}|i\%} = \frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{d} = \frac{6.903567603 - 10(0.407937304982494)}{0.085761845}$$

$$= 32.930664577 \quad [1\frac{1}{2}]$$

$$34,865.914262142 - 9,413.981842655$$

$$= 25,451.93$$

Second 10 years

$$j^{(12)} = 10.5\% \Rightarrow j = 11.020345\% \quad [1\frac{1}{2}]$$

$$2,300 \times a_{\overline{10}|j\%}^{(12)} \times v_{i\%}^{10} = 5,794.492557 \quad [1\frac{1}{2}]$$

$$a_{\overline{10}|j\%}^{(12)} = 6.175813193 \quad [1\frac{1}{2}]$$

Loan amount is given by: -

$$24,451.932419487 + 5,794.492557 = 31,246.424976895$$

$$= \text{£}31,246.42 \quad [1\frac{1}{2}]$$

(ii)

Loan outstanding after the 5th monthly payment has been made:

$$31,246.424976895 \times \left(1 + \frac{0.09}{12}\right)^5 - \left(\frac{5,000}{12}\right) \times s_{\overline{5}|0.75\%}$$

OR

$$31,246.424976895 \times \left(1 + \frac{0.09}{12}\right)^5 - 5,000 \times s_{\overline{5}|i\%}^{(12)}$$

$$= 32,435.874343504 - 2,114.818588558 = 30,321.055754946$$

$$= \text{£}30,321.06 \quad [2\frac{1}{2}]$$

$$s_{\overline{5}|i\%}^{(12)} = 0.422963718 \quad [1]$$

Interest component of the 6th instalment:

$$30,321.055754946 \times \frac{0.09}{12} = 227.4079182 = \text{£}227.41 \quad [1]$$

Capital component of 6th instalment:

$$\left(\frac{5,000}{12} - 227.4079182 \right) = 189.2587485 = \text{£}189.26$$

[1]

[Total 15]

Part (i) was reasonably well answered. Many candidates had difficulty in dealing with monthly payments where decreases occur annually. A common error was using notation with ambiguous meanings, which should be avoided. It shows a lack of understanding on the meaning of the notation.

Part (ii) Two methods are available for calculating the capital outstanding for a loan - the prospective method and the retrospective method. Depending on the pattern of repayments for the loan, one of these methods will often be easier to calculate than the other. In this question the retrospective method was the easier method because the repayment pattern was such that the first 5 repayments were constant. Many candidates created extra work for themselves by using the other method to calculate their answer.

[Paper Total 100]

END OF EXAMINERS' REPORT



Institute and Faculty of Actuaries

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