

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **EXAMINATION**

14 September 2023

### **Subject CM2 – Financial Engineering and Loss Reserving Core Principles**

#### **Paper B**

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

**1** You have been provided with 100 simulations of a random variable representing annual investment returns for the next 5 years. The investment returns are independent and identically distributed, and the distribution is normal with  $\mu = 5\%$  and  $\sigma = 2\%$ .

- (i) Calculate the mean and standard deviation of the investment returns separately for each of the 5 years across the 100 simulations. [4]
- (ii) Comment on how your answer to part (i) compares to the statistical distribution of the returns. [5]

An investor invests a single amount of \$1,000 at time zero.

- (iii) Calculate the value of the investment at times 0, 1, 2, 3, 4 and 5 for each of the 100 simulations. [3]
- (iv) Calculate the following for the investment value at times 0, 1, 2, 3, 4 and 5:
- minimum
  - 25th percentile
  - median
  - 75th percentile
  - maximum.
- [5]
- (v) Plot your results from part (iv) on a suitable chart. [4]
- (vi) Calculate the mean and standard deviation of the value of the investment at time  $t = 5$  using the simulations. [2]

Because the investment returns are independent and identically distributed, we can use the following result to determine the expected mean and standard deviation of the accumulated investment based on the underlying distribution:

$$E[S_n^k] = \prod_{t=1}^n E[(1 + i_t)^k]$$

where  $S_n$  is the accumulated value at time  $n$  of a single investment of 1 at time zero and  $i_t$  is the investment return over the  $t$ -th year.

- (vii) Calculate the expected mean and standard deviation at  $t = 5$  of the value of the investment of \$1,000 at time  $t = 0$  using the above equation. [8]
- (viii) Comment on the difference between your answers to parts (vi) and (vii). [4]
- [Total 35]

- 2** An insurance company's cumulative incurred claims for the last 3 accident years are given in the 'Q2 Data' worksheet.

It can be assumed that claims are fully run off after 2 years and that all incurred claims have been paid. The premiums received for each year are also given in the 'Q2 Data' worksheet.

You do not need to make any allowance for inflation.

- (i) Calculate the reserve at the end of 2022 using the basic chain ladder method. [7]
- (ii) Calculate the reserve at the end of 2022 using the Bornhuetter–Ferguson method. [7]
- (iii) Comment on the differences between the reserves calculated using the two methods. [4]
- [Total 18]

- 3** An investor holds a portfolio containing one European call option and one European put option. Both options are on the same underlying share, have the same strike price of \$16 and mature in exactly 3 years. The underlying share is currently priced at \$17, and the continuously compounded risk-free rate is 3% p.a.

- (i) Plot the payoff of this portfolio at maturity for a share price range of \$0 to \$30 on a suitable chart. [6]

The investor wants to calculate an appropriate price for the portfolio using a binomial tree with 1-year time steps and assuming  $u = 1.06$  and  $d = 1/u$ .

- (ii) Calculate, using a binomial tree, the risk-neutral price of the portfolio at time  $t = 0$ . [8]

After 2 years, the share price is \$14. The risk-free rate has not changed.

- (iii) Show that a portfolio at time  $t = 2$  consisting of \$15.53 cash and short one share is a replicating portfolio for the investor. [7]

As the share price has fallen more than the investor expected, they consider changing their assumption to  $u = 1.02$ .

- (iv) Give an example of how arbitrage could be achieved given this assumption. [6]
- [Total 27]

- 4 A hedge fund analyst has been studying the chart of an investment index. They believe they have developed a model to predict price movements and would like to quantify the excess performance they are expecting to generate. Excess performance is defined as the difference between the actual return and the expected return of the investment index. The model takes the form:

$$Y_i = \alpha + \beta X_i + \xi_i$$

where

- $\alpha$  and  $\beta$  are constants.
- $Y_i$  is the absolute return on the investment index.
- $X_i$  is a dependent variable.
- $\xi_i$  are independent, identically distributed random variable error terms representing the excess performance of the index; the distribution of  $\xi_i$  does not change over time.

The hedge fund analyst is currently considering the model:  $Y_i = 2 + 4X_i + \xi_i$

Values of  $X_i$  and  $Y_i$  for 1,000 historic scenarios can be found in the 'Q4 Data' worksheet.

- (i) Calculate the mean and standard deviation of the historic excess performance. [6]
- (ii) Show, with the aid of a suitable chart, that the excess performance can be modelled as a standard Normal random variable. [10]
- (iii) Explain why the hedge fund model does not provide sufficient evidence to reject the weak form of the Efficient Markets Hypothesis. [4]
- [Total 20]

**END OF PAPER**