INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

23 September 2022 (am)

Subject SP6 – Financial Derivatives Specialist Principles

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 An investor seeking limited upside participation in a stock, S (with stock price at time 0, $S_0 = 100$), enters into a 1-year call-spread strategy by simultaneously buying a call option with strike $K_1 = 110$ and selling a call option with strike $K_2 = 120$. Currently, the risk-free interest rate is zero and the stock does not pay any dividends. The implied volatility varies over the strikes and term structure, with implied volatility of 20% at K_1 and 15% at K_2 for 1-year options.

(i) Calculate, showing all workings, the call-spread price at time 0. [4]

(ii) State the effect on the price of the call-spread of increasing one variable while keeping all others constant for S_0, T, σ, r and q. [2]

where:

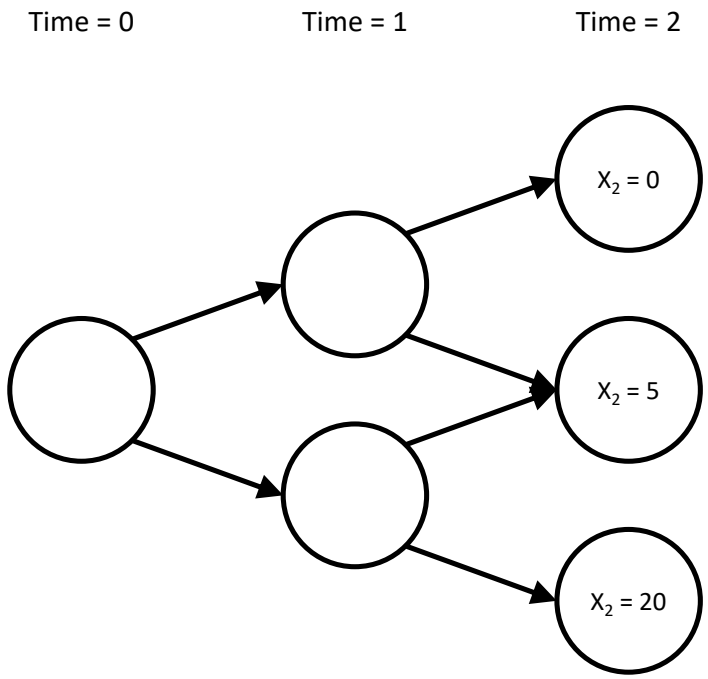
- T is time to expiry of the call options.
- σ is implied volatility.
- r is the risk-free rate, continuously compounded.
- q is the dividend yield, continuously compounded.

(iii) Derive the upper and the lower bound of the call-spread option price. [3]

(iv) Comment on the impact of the volatility surface (K_1 and K_2 being different) to the cost and effectiveness of the investor's strategy of seeking limited upside using a call-spread strategy. [3]

[Total 12]

2 Consider a two-step binomial tree with the probability of an up-move $p = 0.5$ under real-world measure, P, and $q = 0.4$ under risk-neutral measure, Q, and a derivative with a claim X_2 that is known by time 2.



(i) Recommend one example of a derivative that could have this payoff profile. [1]

(ii) Comment on whether measures P and Q are equivalent. [2]

(iii) Calculate, showing all workings, $E_Q(X)$ at time 0 using P and the Radon–Nikodym derivatives of dQ/dP. [4]

(iv) Calculate, showing all workings, the conditional Radon–Nikodym derivatives at time 1 after an up-move. [2]

(v) Comment on the economic meaning of $E_P(X) < E_Q(X)$. [3]

[Total 12]

3 The following volatility surface is used for pricing equity index options.

	K/S_0		
	0.9 (%)	1 (%)	1.1 (%)
1 year	25.5	22.6	21.0
2 years	18.1	16.3	14.8
3 years	14.5	12.0	11.0

(i) Suggest three possible explanations for the shape of this volatility surface. [3]

(ii) Calculate, showing all workings, the implied volatility a trader would use for a 1.6-year option with $K/S_0 = 1.04$. [2]

(iii) Calculate, showing all workings, the value of a 2-year at-the-money European call option assuming $S_0 = 100$, $q = 0\%$ and $r = 0\%$. [2]

(iv) Comment on the impact of the volatility surface if a central government announces unexpected new policies that guarantee a minimum level of the index. [3]

[Total 10]

4 The Pension Regulator in Country A is reviewing its reporting and risk management requirements for its numerous Defined Benefit (DB) pension schemes.

As part of this review, it is considering a new proposal that would require all DB pension schemes to report a regulatory valuation of their assets and liabilities, calculated on a standardised basis.

One aspect of the proposal that has generated a lot of debate is how the discount rates used to value the long-term liabilities would be set. The Pension Regulator's preferred option is to create a regulatory risk-free curve based on Country A's Overnight Indexed Swap (OIS) rates, which would be used to discount the liability cashflows.

The pension schemes' liabilities are typically projected to be longer-dated than the longest-dated liquid swap contracts regularly traded in Country A. Therefore, the Regulator proposes to extend the regulatory risk-free curve using forward rates.

The annualised zero-coupon OIS rates of the liquid swap rates on 31 December 2021 are provided in the following table.

1 year	*2 years*	*5 years*	*10 years*	*15 years*	*20 years*
–0.50%	–0.45%	0.55%	1.90%	3.05%	3.25%

(i) Calculate, showing all workings, the 1-/10-year and 1-/15-year forward rates, assuming forward rates are constant between the liquid tenors. [2]

The 20-year tenor has been assessed to be the Longest Liquid Tenor (LLT) in Country A. To extend the regulatory curve beyond 20 years, two extrapolation options are being considered:

(a) Assume constant forward rates beyond the LLT.

(b) Create a 'final forward rate' that is the weighted average of the 1-/10-year and 1-/15-year forward rates. The weights would be updated annually and would be determined based on the total traded volume of the respective contracts.

The initial proposed split is as follows:

1/10 years	60%
1/15 years	40%

(ii) Calculate, showing all workings, the 30-year annualised spot rate under (a) and (b) assuming the weights given in the table above. [2]

A Risk Manager is considering how the proposals may impact their pension scheme's approach to hedging interest rate risk.

(iii) Outline why pension schemes use derivatives to hedge interest rate risk. [2]

(iv) Discuss how the proposed regulatory approach to valuing the pension scheme's long-dated liabilities may impact the pension scheme's approach to interest rate hedging. [4]

[Total 10]

5 The European Market Infrastructure Regulation (EMIR) introduced rules to reduce the counterparty credit risk of derivatives contracts.

(i) State the main requirements for Over-The-Counter (OTC) derivatives introduced by EMIR. [2]

A Central Clearing Party (CCP) uses its proprietary model to determine the margin requirements of OTC interest rate derivatives. The model calculates the basic margin requirement using 5-day Value at Risk (VaR) with a 99% confidence interval.

Adjustments are then made to the basic margin requirement to determine the total margin requirement, depending on the CCP member's profile.

(ii) Compare the historical simulation method of calculating VaR with the Monte Carlo simulation method in the context of a CCP margin requirement model. [5]

(iii) Recommend, with reasons, adjustments that the CCP may wish to make to the basic margin requirement to give a more representative picture of the risk that is being collateralised. [3]

[Total 10]

6 A bank uses a trinomial tree based on the Hull–White one-factor model to value its portfolio of fixed-income derivatives.

As a first step, the bank assumes that R, which is the interest rate of term Δt, follows the same process as the short rate. It then constructs a preliminary tree for a variable R^* that begins at zero and follows the simplified process.

$$dR^* = -aR^*dt + \sigma\, dz$$

where z = standard Brownian motion.

(i) Describe the three different branching methods that can be used to construct the trinomial tree and when each of these would be used. [3]

The model parameters chosen by the bank are such that only the standard branching pattern is needed. It assigns Pu, Pm and Pd to the probabilities of the high, middle and low branches. The following equations must be satisfied to calculate the risk-neutral probabilities at each node of the tree:

$$Pu\Delta R - Pd\,\Delta R = -aj\,\Delta R\,\Delta t$$

$$Pu\Delta R^2 - Pd\,\Delta R^2 = \sigma^2\,\Delta t + a^2j^2\,\Delta R^2\,\Delta t^2$$

$$Pu + Pm + Pd = 1$$

where

- $t = i\Delta t$
- $R^* = j\Delta R$
- $\Delta R = \sigma\sqrt{2\Delta t}$

(ii) Explain why these equations need to be satisfied. [2]

A Quantitative Analyst has generated the following risk-neutral probabilities and values of R^* for the trinomial tree, where each node is given by (i,j).

	Node								
	(0,0)	(1,1)	(1,0)	(1,–1)	(2,2)	(2,1)	(2,0)	(2,–1)	(2,–2)
R^*	0.0000	0.1414	0.0000	–0.1414	0.2828	0.1414			
Pu	0.1667	0.1605	0.1667	0.1730	0.1545	0.1605			
Pm	0.6667	0.6665	0.6667	0.6665	0.6660	0.6665			
Pd	0.1667	0.1730	0.1667	0.1605	0.1795	0.1730			

(iii) Determine the missing values for columns (2,0), (2,–1) and (2,–2). [3]

(iv) Explain, in your own words, how the analyst can fit the R^* tree to the prevailing term structure. [4]

[Total 12]

7 (i) Explain, in your own words, why a bank cannot completely diversify credit risk in a credit portfolio. [2]

Companies A, B and C are companies in the bank's credit portfolio.

(ii) Explain, with a separate example for each of the following, how if Company A defaults, then:

(a) it may be possible that Company B is more likely to default.

(b) it may be possible that Company C is less likely to default.

[4]

A bank has a credit portfolio with a current value of $105 million. The bank is using a one-factor Gaussian copula model to estimate credit value at risk for regulatory compliance.

(iii) Explain the benefits to the bank of using this one-factor Gaussian copula model in relation to their credit portfolio. [2]

The bank assumes that the 1-year probability of default is 0.5% and is the same for each asset in the credit portfolio. It also assumes that the copula correlation between each pair of assets is 0.15 and is the same for each asset in the portfolio and the average recovery rate for each asset under default is 65%.

(iv) Calculate, showing all workings, an estimate of the 1-year 99.9% credit value at risk of the bank's credit portfolio using this one-factor Gaussian copula model. [3]

(v) Comment on the importance of credit VaR to a bank in managing credit risk. [3]

[Total 14]

8 (i) Describe, in your own words, the characteristics of an interest rate swap, including how the cashflows will work. Please do not comment on the uses of interest rate swaps. [3]

(ii) List the two main risks an investor is exposed to in an interest rate swap. [1]

A researcher at a university is investigating the uses of interest rate swaps. In one part of their research, they have concluded that there is empirical evidence that non-financial companies use interest rate swaps for both hedging and speculation.

(iii) Discuss why a non-financial company could use interest rate swaps for speculation. [3]

The researcher has also concluded that fixed-income traders make regular use of interest rate swaps for speculation.

(iv) Suggest possible ways that fixed-income traders could use interest rate swaps for speculation. [3]

[Total 10]

9 An investment firm is planning to develop an online market for futures that will depend on the financial performance of a movie. The payoff of one contract at expiry will be equal to Gross Worldwide Revenue (in \$)/1,000,000 – K, where K is the strike price.

(i) Outline the details that would be required to be defined in the futures contract by the investment firm. [4]

(ii) Suggest possible reasons why each of the following investor types may want to invest in these futures:

- Movie studios
- People interested in movies
- Other traders.

[3]

(iii) Comment on the reasons why the investment firm may not proceed with these futures. [3]

[Total 10]

END OF PAPER