INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

26 April 2023 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

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1 A singer has the following utility function:

$$U(w) = \ln(w) \quad (w > 0).$$

(i) Show that this utility function is iso-elastic. [3]

The singer measures their wealth as the present value of their future earnings. They are concerned about possible future damage to their voice and consider the following scenarios:

Scenario	*Probability (%)*	*Present value of earnings ($)*
No damage	94.9	1,500,000
Slight damage	5.0	1,300,000
Severe damage	0.1	10,000

The singer is considering buying insurance to cover their loss of earnings if their voice is damaged. The insurance will pay a lump sum equal to the reduction in the present value of earnings compared to the 'no damage' scenario.

(ii) Calculate the singer's expected loss when compared to no damage. [1]

(iii) Calculate the maximum premium the singer would be willing to pay for this insurance based on their utility function. [3]

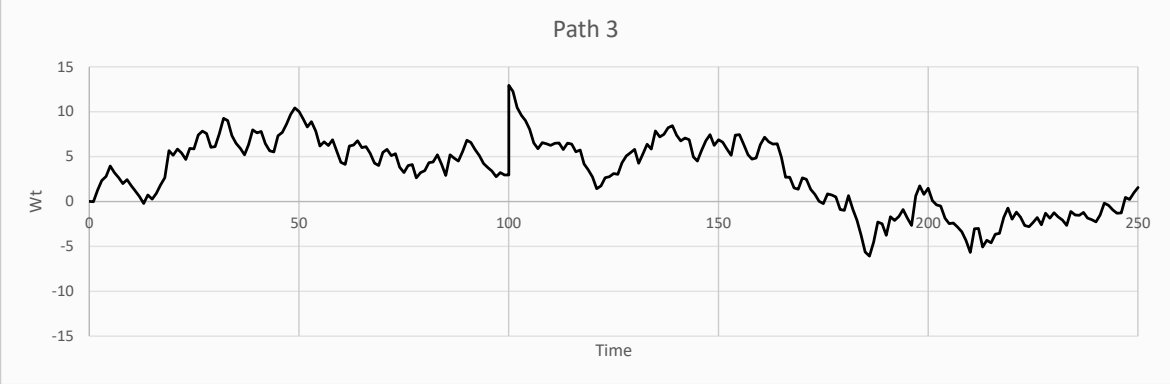
(iv) Discuss briefly what your answers to parts (ii) and (iii) suggest about the singer's appetite to risk. [3]

(v) Suggest two reasons why an insurer may not want to cover this risk. [5]

[Total 15]

2 (i) State three potential uses for stochastic security price models. For each use, explain why the model should be stochastic. [3]

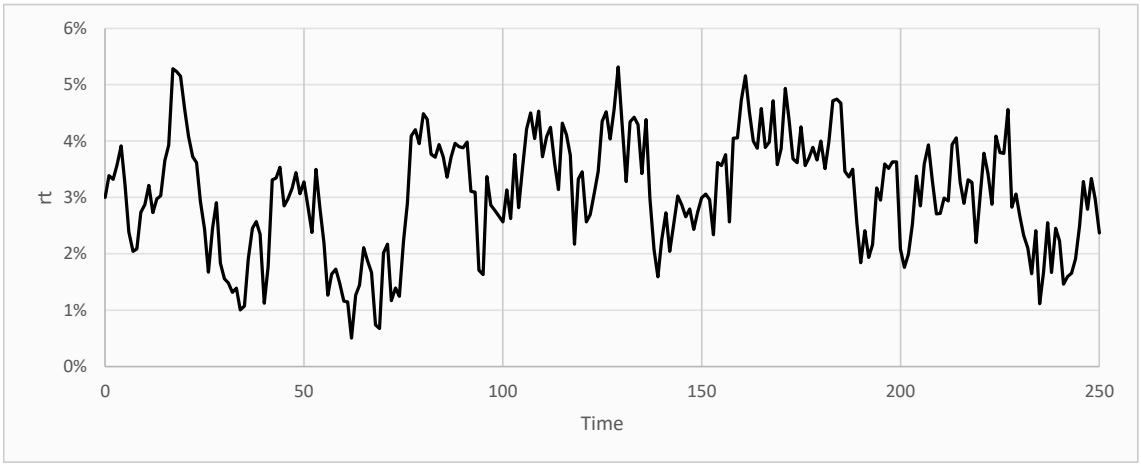
The charts below show sample paths for three different stochastic processes.



(ii) For each chart explain if, and why, the path is likely to be a Wiener process. [3]

The chart below shows a single sample path of a Vasicek model represented by the following stochastic differential equation:

$$dr_t = \kappa(\theta - r_t)dt + \sigma d\widehat{W}_t$$



(iii) Describe the likely impact on the chart of the following parameter changes:

(a) An increase in σ

(b) An increase in θ

(c) A reduction in κ.

[4]
[Total 10]

3 A company specialises in long-term insurance contracts. It holds reserves in respect of these contracts and it wants to build a model of its future investment returns.

(i) Identify the similarities and differences between a deterministic and stochastic model of investment returns. [3]

(ii) Explain why the company may prefer to use a stochastic model. [3]

The company has decided to build a stochastic model. It assumes that the rate of return in each calendar year is Normally distributed and independent of the rates of return in previous years. The company plans to review this model once every 2 years.

(iii) Set out **two** possible disadvantages of using this stochastic model. [2]

(iv) Suggest how the company may overcome each disadvantage identified in part (iii). [2]

[Total 10]

4 A biologist is modelling the population of a group of tortoises on an island over the next 5 years. There are currently 100 tortoises and the biologist has decided it is appropriate to model the population after n years as:

$$S_n = 100(1 + i_1)(1 + i_2)\ldots.(1 + i_n)$$

where:

- i_k is the growth rate of the tortoise population in year k.
- i_k are independent and identically distributed.
- $\log(1 + i_k) \sim N(\mu, \sigma^2)$ for each k.

(i) Write down the distribution of $\log(S_5)$. [1]

The biologist has calculated that:

- there is a 99% chance that $S_5 \geq 75$.
- there is a 99% chance that $S_5 \leq 140$.

(ii) Determine the values of μ and σ. [5]

A university research team has asked the biologist to determine how likely it is that there will be at least 100 tortoises after 10 years. The biologist assumes that the model can be extended to S_{10} with $i_6, i_7, \ldots i_{10}$ having the same properties as $i_1, i_2, \ldots i_5$.

(iii) Calculate the probability that $S_{10} \geq 100$. [3]

(iv) Identify the risks of the biologist using this model to produce the answer in part (iii). [2]

[Total 11]

5 In a Black–Scholes market, the stock price is given by:

$$S_t = S_0 \exp(0.2B_t + 0.2t)$$

where B_t is a standard Brownian motion under the real-world probability measure.

A derivative security written on the stock in the same market at time t has price:

$$D_t = 2\exp\left(0.6\left(\widetilde{B}_t - ct\right) + 0.39t\right)$$

where $\widetilde{B}_t$ is a standard Brownian motion under the equivalent martingale measure.

(i) Determine the value of c such that $B_t + ct$ is a standard Brownian motion under the equivalent martingale measure. [7]

(ii) Calculate the risk-free rate of interest. [1]

[Total 8]

6 An insurer has been offered two possible investments.

Investment A: a derivative that will produce a payoff $\$X_A$ as set out below:

X_A	*Probability*
3.0	$^1/_3$
2.6	$^1/_3$
1.0	$^1/_3$

Investment B: a diversified portfolio that will produce a payoff of $\$X_B$, where

$$X_B = 0.2 + N$$

and N is a Normal random variable with $\mu = 2$ and $\sigma = 2$.

The insurer uses a quadratic utility function.

(i) Calculate, for each investment:

(a) the expected payoff.

(b) the variance of the payoff.

[3]

(ii) State, with reasons, which investment the insurer would choose. [2]

(iii) Calculate, for each investment, the shortfall probability of the payoff falling below:

(a) \$0.5

(b) \$2.0.

[3]

(iv) Explain whether either investment would be more attractive for this investor in all circumstances. [2]

[Total 10]

7 An investor is considering investing in an asset with a payoff of X, where X has a probability density function f, given by:

$$f(s) = 0 \text{ for } s < 1$$

$$f(s) = 4/s^5 \text{ for } s \geq 1$$

(i) Calculate the semi-variance of return. [5]

(ii) Calculate the value at risk in the:

(a) worst 5% of outcomes.

(b) worst 10% of outcomes.

[4]

(iii) Discuss the differences between your answers to part (ii). [3]

[Total 12]

8 The run-off triangle below shows the incremental claims incurred on a portfolio of car insurance policies.

	0	*1*	*2*	*3*
2018	362	272	506	350
2019	444	116	165	
2020	487	195		
2021	518			

(i) Calculate the outstanding claims using the basic chain ladder method. [6]

(ii) Discuss whether the chain ladder method is likely to be suitable for this pattern of claims. [4]

(iii) Discuss, with reference to the pattern of claims, whether any other methods would be better than the basic chain ladder method for this portfolio. [2]

[Total 12]

9 Consider a market that satisfies the assumptions of the capital asset pricing model.

The market contains only three assets, with the following attributes:

	Market capitalisation ($m)	*Expected return (p.a.) (%)*
Asset 1	6	5.0
Asset 2	4	7.0
Asset 3	10	8.5

The variance/covariance matrix is as follows:

	Asset 1	*Asset 2*	*Asset 3*
Asset 1	0.0005	0.0006	0.0005
Asset 2	0.0006	0.0015	0.0026
Asset 3	0.0005	0.0026	0.0042

(i) Show, using the figures provided, that the market portfolio has a beta of 1. [4]

A portfolio, P, on the efficient frontier is invested 30% in Asset 1 and 70% in Asset 3.

(ii) Calculate the expected return and standard deviation of portfolio P. [2]

(iii) Determine the market price of risk and the risk-free rate. [3]

(iv) Suggest reasons why it may not be appropriate to estimate parameters for an asset pricing model using past data. [3]

[Total 12]

END OF PAPER