

# INSTITUTE AND FACULTY OF ACTUARIES

## EXAMINATION

13 September 2023 (am)

### Subject CP2 – Modelling Practice Core Practices

#### Paper Two

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

# Exam requirements

Read the background document, which describes the scenarios that have been modelled and documented for this project and the work that remains outstanding.

Read the audit trail that has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

**You are not required to add to or amend the audit trail.**

**You should assume that your colleague's calculations have been checked and are correct.**

## 1 Spreadsheet model

- (i) Expand the spreadsheet model to determine the nightly room rates such that the average of the fifth largest daily losses over a month ('Risk Metric 1') in the post-renovation scenario will be the same as the pre-renovation scenario. [3]
- (ii) Expand the spreadsheet model to determine the nightly room rates such that the average of the average five largest daily losses over a month ('Risk Metric 2') in the post-renovation scenario will be the same as the pre-renovation scenario. [2]

**You should ensure that the additional work you undertake on the spreadsheet contains appropriate self-checks.**

## 2 Charts

Construct, for each of the following, a single suitable chart to illustrate:

- (i) the relationship between the average profit in the renovation analysis and the number of premium rooms converted. [3]
- (ii) the day 1 profit simulation of the post-renovation scenario, ranked in ascending order. [3]
- (iii) the utilisation rate over the course of a month for the pre-renovation and the post-renovation scenario split by standard and premium rooms. [2]
- (iv) the monthly profits for the pre-renovation, post-renovation, post-renovation Risk Metric 1 and post-renovation Risk Metric 2, split by standard, premium and upgraded rooms. [3]

### 3 Analysis

Analyse, by constructing either a chart, a table of appropriate results or otherwise, Risk Metric 1 and Risk Metric 2 split by the pre-renovation, post-renovation and the two post-renovation updated scenarios. [3]  
**[Sub-total 19]**

#### Summary document

Prepare a summary document capturing the main features and results of the work done by you and your colleague. You can assume that the summary is being prepared for your manager, a senior actuary, who will present the work to Dr Hilbert.

Your summary should include the following:

- Purpose of the project, data, method and assumptions used by you and your colleague
- Results, including charts
- Commentary on the results
- Key conclusions
- Suggested next steps.

Commentary on the results should cover but not be limited to:

- analytical comments on each stage of the results, including explaining patterns in the results and any unusual features.
- an explanation of the differences between the results under the various strategies modelled.

Next steps need to be specific to the project, with some mention of why each is valid.

The summary should cover the full scope of the project, including the current approach that was modelled in the spreadsheet provided.

**You are not required to add to or amend the audit trail.**

#### 4 Marks available for the summary:

- |       |  |                       |
|-------|--|-----------------------|
| (i)   | Methodology (including purpose, data, method and assumptions). | [26]                  |
| (ii)  | Results, including charts.                                     | [6]                   |
| (iii) | Commentary on results and conclusions.                         | [21]                  |
| (iv)  | Next steps.  | [18]                  |
| (v)   | Drafting.  | [10]                  |
|       |  | <b>[Sub-total 81]</b> |
|       |  | <b>[Total 100]</b>    |

# Background

You are a student actuary working for a management consultancy. Your client, Dr Hilbert, the owner of Hilbert Hotels, is asking your company to propose ways that could help them increase profits.

Hilbert Hotels currently has 100 rooms, all of which are standard rooms.

A colleague in your team has been analysing Hilbert Hotels' current level of profits. For the purpose of this analysis, the profits for a 'typical month' were considered.

Your colleague engaged an external consultancy, which provided anticipated demand data for standard rooms for each day in a typical month. It is assumed there are 30 days in this month.

The anticipated demand dataset consists of the expected number of standard rooms demanded on a daily basis for a month across 500 simulations. Where the expected demand for rooms is more than the number of rooms available, the hotel will be fully booked and will not be able to accommodate the excess demand.

Monthly profits have been calculated using the projected data supplied by the external consultancy and back tested using the hotel's historic booking data. Both Dr Hilbert and your colleague are happy that the dataset is fit for purpose.

Dr Hilbert would also like to understand the potential losses associated with the projected monthly profits of Hilbert Hotels better. Your colleague has proposed two risk measures to assess these potential losses. The two risk measures are:

- the average of the fifth largest daily loss over the month ('Risk Metric 1').
- the average of the average five largest daily losses over the month ('Risk Metric 2').

Your colleague has calculated the utilisation rate (defined to be the average number of rooms occupied across the simulations for a given day divided by the number of standard rooms in the hotel (i.e. 100)) for each day of the 'typical month'.

Your colleague suggests that the utilisation rate, and hence profits, could be increased if some of the standard rooms in the hotels are converted to premium rooms. Premium rooms will be twice as big as a standard room and will command a higher price for an overnight stay. The number of units in the hotel is fixed at 100. A standard room occupies one unit, and a premium room occupies two units.

Your colleague has done some market research and determined that after changes are made to the rooms, the price for a standard room will remain at \$150 per night, whereas a premium room will be at 2.5 times the standard room rate, i.e. \$375 per night. The anticipated demand for premium rooms for each day in a typical month has been provided by the same external consultancy.

The hotel has a fixed cost for cleaning rooms each night that a room has been used. The cost of cleaning a standard room is \$20 per night's stay, and \$35 per night's stay for premium rooms. Only rooms that have been occupied require cleaning.

The hotel also has overhead costs of \$30 per day for a standard room and \$70 per day for a premium room. Overhead costs are calculated based on the numbers of rooms in the hotel irrespective of whether the rooms are occupied or not.

Hotel guests can be upgraded from a standard room to a premium room at no additional cost to the guest if all the standard rooms are occupied and there are still premium rooms available after all the premium room demand has been met.

For the purpose of this analysis, the cost of converting two standard rooms to one premium is assumed to be \$100 per night (for every pair of standard rooms converted). This renovation cost is split equally across standard and premium for profit and loss calculation purposes.

## Renovation

Using the day 1 projected demand data from the external consultancy, your colleague has projected the average profit for a range of options:

<i>Option</i>	<i>Number of</i>	
	<i>Standard rooms</i>	<i>Premium rooms</i>
1	100	0
2	90	5
... and so on, trading 10 standard rooms for five premium rooms at each option, to reach a final option of...		
11	0	50

Based on this analysis (and without regards to either Risk Metric 1 or Risk Metric 2), your colleague has determined that having 20 premium rooms (and 60 standard rooms) will maximise the expected average profit.

## Profit (after renovation)

The projected monthly profit calculation is then repeated assuming that the hotel is converted to having 20 premium rooms and 60 standard rooms.

## Further development

Prior to going on holiday, your colleague presented the results internally to your manager. The results show that while average monthly profits increase, both Risk Metric 1 and Risk Metric 2 have also increased.

As Dr Hilbert is risk adverse, your manager would like you to expand the analysis to determine what rate per night needs to be charged for standard rooms and premium rooms (while keeping the ratio of the premium room rate to standard room rate of 2.5 times unchanged) for the following two additional scenarios, such that:

1. Risk Metric 1 will be the same as that in the pre-renovation scenario.
2. Risk Metric 2 will be the same as that in the pre-renovation scenario.

The above two scenarios are independent of each other.

## Summary report

Your manager has asked you to prepare a summary covering all elements of the work (both the original work of your colleague and the additional modelling you are undertaking).

Within this summary, your manager wishes you to include charts as outlined in the examination instructions above and to include some analysis of the change in the risk metrics on the pre-renovation, post-renovation and updated post-renovation scenarios.

Your summary should include the following:

- Purpose of the project, data, method and assumptions used by you and your colleague
- Results, including charts
- Commentary on the results and key conclusions
- Suggested next steps.

**You are not expected to include the additional modelling you undertake in the audit trail, but the methodology and results for all the modelling should be included in the summary.**

# Audit trail

The following audit trail should be read alongside the model provided.

## Objective

Our objective is to help Hilbert Hotels increase profits. We can achieve this by converting some existing standard rooms into premium rooms, which command a higher price. We enhance this further by finding the optimal standard and premium room split.

## Data

An external consultancy provided us with the projected demand for standard rooms and premium rooms across 500 daily simulations over a month (assumed to be 30 days long).

Our market research has determined costs associated with running and renovating the hotel.

Dr Hilbert, the hotel owner, has informed us that there are currently 100 rooms in the hotel.

## Data checks

The following checks have been completed:

- Summary statistics (minimum, maximum and average) of the projected demand data for standard and premium rooms for each day are calculated.
- We check that all projected demand simulations are non-negative and all the simulations provided contain a number.
- Global minimum, maximum and average are calculated across all simulations in the month ( $500 \times 30$ ) for standard and premium rooms respectively.

These summary statistics do not look unreasonable. No changes were made to the data in the 'Data' tab.

## Assumptions

The following assumptions are made in the model:

- The simulations provided are correct and fit for purpose.
- Renovation cost is spread equally between standard and premium rooms.
- There are no additional overheads for room upgrades.
- There are only 30 days in a month.
- Day 1 simulations used for the renovation analysis are representative of monthly results.

## Calculation

This sheet calculates the average projected daily profits and average projected monthly profit for the standard rooms, using the simulation data provided.

Where  $x$  (ranges from 1 to 30 in columns B to AE) is the number of days in the month, and  $y$  (ranges from 1 to 500 in rows 12 to 511) is the simulation number.

$C(x,y)$  is the simulation data provided.  $C(x,y)$  represents the standard room projected occupancy on the  $x$ th day of the month and the  $y$ th simulation.

Given that demand sometimes exceeds supply (as the hotel only has 100 standard rooms), we first need to work out  $D(x,y)$  – the capped number of rooms occupied in cells B12:AE511 – which is given by the below formula:

$$D(x,y) = \min(100, C(x,y))$$

The utilisation rate, which is the average of  $D(x,y)$  across all simulations  $y$  from 1 to 500 on each day ( $x$ ) divided by the number of standard rooms in the hotel (100), is calculated in cells B2:AE2.

The profit for each  $D(x,y)$  is calculated in cells AH12:BK511 as:

$$\text{Profit} = D(x,y) \times (\text{standard room price} - \text{standard room servicing cost}) \\ - \text{number of standard rooms in the hotel} \times \text{overhead cost}$$

For each day ( $x$ ), we calculate the average profit in cells AH2:BK2. The standard deviation of the profit for each day is also calculated in cells AH3:BK3 using the STDEV.S function.

The **fifth largest daily loss** is calculated in AH4:BK4 by identifying the fifth worst loss in the 500 simulations using the SMALL function with a parameter of 5.

The **average five largest losses** for each day ( $x$ ) is calculated to be the average of the five largest losses in cells AH5:BK5. This is calculated using:

$$\text{AVERAGE}(\text{SMALL}(D(x,y) \text{ for each day } x, \text{row}(1:5)))$$

[**Hint:** The argument, row(1:5), tells the SMALL function to look up the five smallest/worst values of which an average is then taken. If you want the average of the smallest  $x$  values, change 1:5 to 1: $x$ ). An alternative calculation using AVERAGE(SMALL( $D(x,y)$ )) for the five smallest values have also been calculated as a check.]

We calculate the **average projected daily profit** in cell BN10 using the AVERAGE function based on the data calculated in cells AH2:BK2. The **total average profit for the month** is calculated in cell BN12 using the SUM function.

In cell BN13, **Risk Metric 1** is calculated by taking the average of the fifth largest daily loss across the 30 days. In cell BN14, we calculate **Risk Metric 2** by taking the average of the average five largest losses over the month.

## Reno analysis

This sheet calculates the expected profits after renovation, i.e. when some of the standard rooms are converted to premium rooms.

In row 8, for each relevant table, we have the number of standard rooms starting at 100, decreasing at a step size of 10 rooms until it reaches 0. In row 9, we have the number of premium rooms, starting at 0, increasing at a step size of five rooms until it reaches 50 (where there will be no standard rooms). This satisfies the constraint that a standard room occupies one unit and a premium room occupies two. There are 11 combinations in total.

For each of the simulations from row 11 onwards, we calculate the following:

1. **Profit for standard rooms** (columns B to L):

- If the standard room demand is greater than the number of standard rooms available, return the number of standard rooms available, else return the standard room demand.
- The above is then multiplied by the difference of the nightly rate and the cost of servicing a standard room.
- We then take off the overheads cost multiplied by the number of standard rooms available from the resulting product.

2. **Profit for premium rooms** (columns O to Y) using the same logic as for standard rooms.

3. **Profit for upgrades minus the renovation costs** (columns AB to AL):

- The number of upgrades available is  
$$\text{Min}\{\text{Max}(0, \text{Standard room demand less the number of standard rooms available}),$$
$$\text{Max}(0, \text{the number of premium rooms available less premium room demand})\}.$$
- The above is then multiplied by the difference of the nightly price of a standard room and the cost of servicing a premium room to get to the profit for upgrades.
- Renovation cost is the number of premium rooms converted multiplied by the renovation cost (\$100).

The overall nightly profit (columns AO to AY) is then:

$$\begin{aligned} & \text{Profits from standard rooms} + \text{profits from premium rooms} \\ & + (\text{profits from upgrades} - \text{renovation cost}) \end{aligned}$$

The **summary statistics** (minimum, average, maximum, fifth largest loss and average five worst largest losses) are calculated in rows 2 to 6.

It is found that option 5, converting 40 standard rooms to 20 premium rooms, yields the highest average profit.

## Updated\_calc

This sheet does the same calculation as the 'Calculation' worksheet above except that (1) it allows for premium rooms and upgrade profits calculation and (2) it uses 60 standard rooms and 20 premium rooms as the basis for projection.

Columns B:AE determine the number of standard rooms occupied. This is the same as the programming in the 'Calculation' tab but the number of rooms occupied has been capped at 60.

Columns AH:BK determine the number of premium rooms occupied. For each day of each simulation, the number of premium rooms occupied is calculated as the premium room occupancy projections from the 'Data' worksheet, capped at 20.

Columns BN:CQ calculate the number of upgrades offered. For each day of each simulation, this is calculated as the minimum of:

- (i) the difference between the number of available premium rooms (20) and the number of premium rooms occupied.
- (ii) the difference between the number of standard rooms occupied pre renovation and the number of standard rooms occupied post renovation, subject to a floor of 0.

The **utilisation rate** calculation is performed in row 2 for standard rooms, premium rooms and upgrades separately.

The standard room profit, premium room profit and upgrade profit are calculated using the same methodology described above. These are done in columns CT:DW for standard rooms, columns DZ:FC for premium rooms and columns FF:GI for upgrades. The total profit is calculated in columns GL:HO using the following formula:

Standard rooms profit + premium rooms profit + upgrade profits – renovation cost.

Summary statistics are calculated in rows 2 to 6.

The monthly total profits for standard rooms, premium rooms, upgrades and the renovation costs are summed up in cells HS12:HW12, with **Risk Metric 1** in cell HW13 and **Risk Metric 2** in cell HW14. These calculations are the same as those completed in the 'Calculation' worksheet.

### Sense check:

The summary statistics (average, average five largest losses, fifth largest loss) for the day 1 simulations (column GL) are the same as that in option 5 in the 'Reno Analysis' tab.

**END OF PAPER**