

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

27 April 2023 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper B

Time allowed: One hour and fifty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 An insurer is modelling its probability of ruin to determine whether to purchase reinsurance. Its starting wealth is \$5 million and it receives premium income of \$0.1 million per year. You have been given 100 simulations of claim amounts in \$million over the next 5 years in the 'Q1 Data' tab of the spreadsheet.

You should assume that once the surplus falls below zero, it is set to zero and remains there indefinitely (i.e. the insurer becomes insolvent).

(i) Calculate the probability that the insurer is insolvent at time $t = 5$. [5]

The insurer considers two reinsurance options:

- Option A: reinsurance covering 20% of every claim. This will cost the insurer \$0.02 million per period.
- Option B: reinsurance where total losses above \$1 million in each year will be covered by the reinsurer. This will cost the insurer \$0.50 million at time $t = 0$.

(ii) Calculate the probability that the insurer becomes insolvent after 5 years, assuming it takes up Option A. [4]

(iii) Calculate the probability that the insurer becomes insolvent after 5 years, assuming it takes up Option B. [4]

(iv) Discuss why each reinsurance option may be preferred by the insurer. [6]

An actuary has suggested purchasing **both** types of reinsurance so that:

- the insurer pays both reinsurance premiums set out above.
- the insurer's losses are first reduced by 20% using Option A.
- any residual losses are capped at \$1 million with the Option B reinsurer covering the difference.

(v) Calculate the probability that the insurer becomes insolvent after 5 years assuming it applies this strategy. [4]

[Total 23]

- 2 An individual is looking at ways to set aside funds to cover the cost of car repairs over the next 10 years.

The distribution of the yearly repair cost of the car has been determined to be $\$100 \times G$, where $G \sim \text{Gamma}(0.5, 1)$. Each year is assumed to be independent.

The individual decides to set aside \$500 into a savings account at time $t = 0$. The fixed rate of interest is 4% p.a. and the same rate applies whether the account is in credit or in debit. The repair bill is paid from the account at the end of each year.

You have been provided with five simulations of the underlying gamma(0.5,1) distribution in the 'Q2 Data' tab of the spreadsheet.

- (i) Calculate the expected value of the savings account at time $t = 10$. [6]

The individual hopes that the savings account will cover the cost of all repairs over the 10 years.

- (ii) Calculate the probability that the savings account at the end of the 10-year period is positive. [2]

The individual decides to assess their level of wealth w using a utility function $U(w)$ as follows:

$$U(w) = \begin{cases} \ln(w) & \text{if } w > 0 \\ 0 & \text{otherwise} \end{cases}$$

- (iii) Calculate the expected utility of the savings account at time $t = 10$. [3]

A company has decided to offer an insurance product that will pay for all repairs to the car for the next 10 years in return for a single premium at the start of the first year. The individual expects the premium to be in the range of \$0 to \$500. The individual will pay the premium from the savings account and leave the remaining funds in that account.

- (iv) Calculate, in increments of \$1, for the range of premiums above, the expected utility of the savings account at time $t = 10$. [4]

- (v) Determine, to the nearest \$1, the maximum premium the individual would be willing to pay. [2]

- (vi) Explain why the customer is willing to pay a premium that exceeds the expected cost of all future claims. [4]

[Total 21]

- 3 An insurance company has invested \$14 million in a share index and has since become concerned about potential losses. The insurer is therefore considering hedging solutions to eliminate its market exposure, and has identified two hedging approaches: Strategy A and Strategy B.

The objective of Strategy A would be to reduce the portfolio delta to zero.

The following table includes details about a 3-month put option that the insurer is considering buying. The options will be funded by selling some of the shares currently held.

<i>Detail</i>	<i>Data for put option</i>
Option type	European style
Current share index level (\$)	7,000
Strike price (\$)	6,800
Implied volatility (% p.a.)	30
Continuously compounded interest rate (% p.a.)	5

- (i) For Strategy A:
- calculate the value of the put option.
 - calculate the delta of the put option.
 - calculate how many shares and how many put options the insurer should hold to achieve a portfolio value of \$14 million and a portfolio delta of zero.

[8]

The objective of Strategy B would be to reduce both the portfolio delta and gamma to zero.

The insurer has identified a 3-month call option that it is considering buying in addition to the put option described above. The following table includes details about the call option.

<i>Detail</i>	<i>Data for call option</i>
Option type	European style
Current share index level (\$)	7,000
Strike price (\$)	8,000
Implied volatility (% p.a.)	30
Continuously compounded interest rate (% p.a.)	5

The gamma of the put option is 0.043 and the gamma of the call option is 0.03.

- (ii) For Strategy B:
- (a) calculate the value of the call option.
 - (b) calculate the number of stocks, put options and call options that should be held in the insurer's portfolio to achieve a portfolio value of \$14 million, a portfolio delta of zero and a portfolio gamma of zero.
- [10]

The insurer wants to see how effective each strategy may be if the value of the share index falls by 10%.

- (iii) Calculate, assuming that the index falls by 10%:
- (a) the value of the put option.
 - (b) the value of the call option.
 - (c) the value of the portfolio in Strategy A.
 - (d) the value of the portfolio in Strategy B.
- [11]
- (iv) Explain the reasons for the difference in the portfolio profit between Strategy A and Strategy B.
- [5]
- [Total 34]

- 4 A bank has issued 500 independent loans. The borrower of each loan has four possible credit states:

<i>Credit state</i>
A (highest)
B
C
D (default)

The probability of a one-step downgrade in a single year is 10%. The probability of a one-step upgrade in a single year is 5%. State D is an absorbing state. All borrowers start in state A.

A borrower can also move from states A or B straight to state D in a single year with the following probabilities:

- 2% from state A
- 5% from state B.

- (i) Construct the 1-year transition rate matrix. [4]
- (ii) Calculate the expected number of borrowers in each credit state at the end of year 3. [7]

Each loan is for \$1,000 at time $t = 0$ and attracts annually compounded interest of 5%, which is paid at maturity. At time $t = 3$, the bank expects to receive:

- repayment of the full loan plus accrued interest from any borrower in states A or B.
- 75% of the loan plus 75% of the accrued interest from a borrower in state C.
- nothing from a borrower in state D.

- (iii) Calculate the total repayments the bank expects to receive at the end of year 3. [5]
- (iv) Calculate the implied annual return on the portfolio of loans from your answer to part (iii). [2]

The bank is concerned that the implied annual return is too low.

- (v) Suggest how the bank may increase its return. [4]
- [Total 22]

END OF PAPER