

INSTITUTE AND FACULTY OF ACTUARIES



EXAMINATION

16 April 2019 (pm)

Subject CM2A – Financial Mathematics and Loss Reserving Core Principles

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all questions, begin your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1**
- (i) Describe the three forms of the Efficient Markets Hypothesis. [4]
 - (ii) Describe the evidence against market efficiency in relation to:
 - (a) over-reaction of market prices to events
 - (b) under-reaction of market prices to events.

[6]
[Total 10]

- 2**
- (i) Describe the first order stochastic dominance theorem. [2]
 - (ii) Describe the second order stochastic dominance theorem. [2]

Consider two portfolios 1 and 2, which have normally distributed returns with parameters (μ_1, σ_1) and (μ_2, σ_2) respectively. Investors in this market meet the assumptions of the dominance theorems.

- (iii) Explain which portfolio, if any, dominates the other in the following situations, indicating whether any dominance is first or second order and why:
 - (a) $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$
 - (b) $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$
 - (c) $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$

[6]
[Total 10]

3 Consider a process S_t which follows a lognormal process with parameters μ and σ^2 .

(i) State the mean and variance of S_t . [2]

The price of the security at time 0 is $S_0 = \$1,000$. The expected price at time 3 is \$2,042 and the standard deviation is \$1,290.

(ii) Determine the parameter values for the corresponding lognormal model. [3]

(iii) Calculate the probability that the security price at time 5, S_5 , will fall between \$2,000 and \$2,500. [3]

(iv) Define the following risk measures algebraically:

(a) Value at Risk at the level $p\%$

(b) Expected shortfall below a benchmark level L .

[2]

(v) Suggest why it can be difficult to apply Value at Risk to real-world situations at higher percentage levels (for example at the 99% level). [3]

[Total 13]

4 (i) List the assumptions underlying the Black-Scholes model of option prices. [3]

Consider a market where the assumptions behind the Black-Scholes model hold.

A non-dividend paying share is currently priced at €30. A put option is available on the share with a strike price of €32 and one year to expiry. The share has a volatility of 15% and the risk-free rate is 5% per annum.

(ii) Calculate the fair price of the put option. [4]

A call option is also available on the same share, with the same strike price and time to maturity.

(iii) Calculate the fair price of the call option. [1]

(iv) Discuss how the option prices above would change if the share were dividend-paying. [2]

[Total 10]

- 5**
- (i) Define an efficient portfolio in the context of portfolio theory. [1]
 - (ii) State the two assumptions required in portfolio theory for the existence of an efficient portfolio. [2]
 - (iii) Explain what it means if a portfolio is not on the efficient frontier. [1]

Consider a market with just two securities, S_A and S_B , with expected return E_A and E_B , variance V_A and V_B and covariance C_{AB} .

- (iv) Derive a formula for the amount, x_A , that should be invested in S_A to minimise the portfolio variance. [4]
- [Total 8]

- 6** Consider a share with price S_t at time t . The continuously-compounded risk-free rate is r per annum.

- (i) Show that the fair price of a forward contract on S_t maturing at time T is $K = S_0 e^{rT}$. [4]

A share S_0 is currently worth £12. The continuously-compounded risk-free rate is 5% per annum.

- (ii) Calculate the fair price of a forward contract written on the share at time $t = 0$ with expiry at time $t = 5$. [1]

An investor takes a long position in the forward contract at time 0. At time 1 the share price has fallen to £10.

- (iii) Calculate the value to the investor of the forward contract at time $t = 1$. [3]

At time $t = 2$ the share unexpectedly pays a one-off dividend.

- (iv) Explain, with reasons, how the forward price might change as a result of the one-off dividend. [2]
- [Total 10]

- 7 (i) Describe six factors that affect the price of a European put option on a dividend-paying share, including how they impact the option price. [3]

Consider a European call option currently priced at \$15.60, based on an underlying share priced at \$34.55. The option has a delta of 0.5, gamma of \$0.10 and a theta of $-\$0.05\text{day}^{-1}$. Two days later the underlying share price has risen to \$35.25.

- (ii) Calculate the new approximate value of the call option. [3]
- (iii) Explain why a low value of gamma is desirable when using options in a delta-hedged portfolio. [3]

[Total 9]

- 8 A portfolio of derivatives is valued today at £1,000,000. The one-day return on the portfolio between today and tomorrow is normally distributed with parameters $\mu = 1\%$ and $\sigma = 20\%$.

- (i) Calculate, in respect of the value of the portfolio tomorrow:
- (a) the expected value of the portfolio
 - (b) the variance of the portfolio value
 - (c) the downside semi-variance of the portfolio value relative to the expected value
 - (d) the shortfall probability relative to £1,000,000.
- [5]

- (ii) Determine the 99% Value at Risk (VaR) over the one-day period, relative to the expected portfolio value. [3]

- (iii) Determine the minimum number of days that would need to pass such that the probability of a 99% one-day VaR event is at least 50%. You should assume that the returns each day are independent of each other. [3]

[Total 11]

- 9 An insurance company offers a special type of policy to actuarial students to cover the cost of celebrations when they qualify. The policy pays a fixed sum of £500 when a student passes his or her final exam. Students pay a premium of £50 at each exam session irrespective of how many exams they are sitting and the last premium is paid in the exam session where the student qualifies.

There are two exam sessions each year, in April and October. Premiums are paid in the month of the exam. Exam results are published immediately after the exam and the £500 is paid if the student has qualified.

The insurer has issued three policies on 1 January 2019 to students who are close to qualifying. The insurer assumes that each student has a 25% chance of qualifying in each future exam session. The insurer's initial assets are £750 and this does not earn any interest.

- (i) Calculate the insurer's assets after the April 2019 premiums are received. [1]
- (ii) Calculate the probability of ruin when the exam results are released. [3]
- (iii) Assuming that no students qualified in April 2019:
 - (a) Calculate the insurer's assets after the October 2019 premiums are received.
 - (b) Calculate the probability of ruin when the October 2019 exam results are released. [2]
- (iv) Assuming that one student qualified in April 2019:
 - (a) Calculate the insurer's assets after the October 2019 premiums are received.
 - (b) Calculate the probability of ruin when the October 2019 exam results are released. [2]
- (v) Calculate the overall probability of ruin before the end of the year 2019. [3]
- (vi) Explain the impact on the ultimate probability of ruin if the insurer issues more policies on the same terms. [2]

[Total 13]

- 10** Consider a model in which the annual rates of return are independent and identically distributed. Let the rate of return each year have mean j and variance s^2 .

Let S_n represent the accumulated amount at time $t = n$ of a single premium of \$1 paid at time $t = 1$.

(i) Derive algebraically an expression for the mean value of S_n . [3]

(ii) Derive algebraically an expression for the variance of S_n . [3]

[Total 6]

END OF PAPER