

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

11 April 2022 (am)

Subject CM1 – Actuarial Mathematics Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team at T. 0044 (0) 1865 268 873.

1 State four data items relating to the holding of equity shares in a single company that could be used to model the future value of the equity shareholding. [2]

2 An investor is considering making an investment and is deciding between two possible alternatives:

- A 6-month investment which can be purchased at a simple rate of discount of 4.15% p.a.
- A bank deposit, for 6 months, offering an effective rate of interest of 4.35% p.a.

Determine which of these two alternatives offers the higher rate of return. [4]

3 An insurance company has liabilities of £4 million due in 7 years' time and £13 million due in 11 years' time. The company has assets consisting of two zero-coupon bonds, one paying £6.9617 million in 4 years' time and the other paying £11.4007 million in 18 years' time. The current interest rate is 6% p.a. effective.

Demonstrate that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [6]

- 4**
- (a) Write down the formula for the variance of the present value of an n -year temporary annuity of £1 p.a. payable at the end of each year, issued to a life aged x exact.
- (b) Calculate, showing all working, the variance of the present value of a 20-year temporary annuity of £5,000 p.a., payable at the end of each year, issued to a life aged 44 exact.

Basis:
Mortality AM92 Ultimate
Interest 4% p.a. effective

[7]

5 At a particular insurance company, actuarial students study for a maximum of 3 years.

Students are subject to the following decrements:

- Mortality
- Not progressing with their studies but staying with the company (withdrawal)
- Leaving the company to join another employer (transfer).

The forces of mortality, withdrawal and transfer are assumed to be independent and to be constant over individual years of study.

In addition, at the end of each year of study, a proportion of students will complete their studies and will be deemed to have qualified.

The following forces of decrement will apply for each year of study:

<i>Year of study</i>	<i>Force of mortality</i>	<i>Force of withdrawal</i>	<i>Force of transfer</i>
1	0	0.2	0.25
2	0.025	0.1	0.35
3	0.030	0	0.45

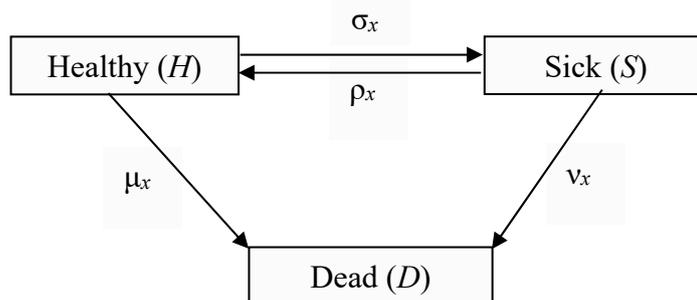
The proportion of students who qualify each year is as follows:

<i>End of year of study</i>	<i>Proportion of students who qualify (%)</i>
1	3
2	12
3	100

Calculate, showing all working, the probability that a student who starts the programme will qualify while still being at the company.

[7]

- 6 An insurance company issues a 30-year combined death and sickness policy to a healthy life aged 25 exact. The company uses the following multiple state model in respect of the policy.



The expected present value of the benefits provided by the policy are set out in expressions (a), (b) and (c) below:

$$(a) \quad 50,000 \int_0^{30} e^{-\delta t} \left({}_tP_{25}^{HH} \times \mu_{25+t} + {}_tP_{25}^{HS} \times \nu_{25+t} \right) dt$$

$$(b) \quad 20,000 \int_0^{29.5} e^{-\delta t} \times {}_tP_{25}^{HH} \times \sigma_{25+t} \int_0^{29.5-t} {}_{0.5+s}P_{25+t}^{\overline{SS}} \times \nu_{25.5+t+s} ds dt$$

$$(c) \quad 5,000 \int_0^{30} e^{-\delta t} {}_tP_{25}^{HS} dt$$

Describe, in words, the **benefits** provided by the policy. You may assume the time periods are measured in years. [6]

- 7 A company issues a fixed-interest security paying coupons at a rate of 6% p.a. payable half-yearly in arrears.

The security is to be redeemed at 103% on any coupon payment date from 10 to 15 years after issue, with the exact date of redemption at the discretion of the company.

An investor, liable to income tax at 20% and capital gains tax at 25%, purchases the security on the date of issue at a price which gives a minimum net yield to redemption of 5% p.a. effective.

Calculate, showing all working, the price per £100 nominal paid by the investor. [6]

- 8 (i) Show, using expressions in the form of integrals, that $\overline{A}_{xy:\overline{n}|}^2 = \overline{A}_{x:\overline{n}|}^1 - \overline{A}_{xy:\overline{n}|}^1$. [2]
- (ii) Calculate $100,000 \overline{A}_{50:50:\overline{1}|}^2$.

Basis:
Mortality AM92 Ultimate
Interest 7% p.a. effective

Assume that the lives are independent with respect to mortality. [5]

[Total 7]

- 9 The annual effective forward rate applicable over the period t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years.

You are given the following information:

$$f_{0,1} = 4.2\%, \quad f_{1,1} = 4.8\%, \quad f_{2,1} = 5.3\%, \quad f_{2,2} = 5.9\% .$$

- (i) Calculate $f_{3,1}$, showing all working and giving your answer as a percentage to four decimal places. [2]
- (ii) Calculate, showing all working and giving your answers as a percentage to four decimal places, all possible spot rates of interest that the above information allows you to calculate. [4]
- (iii) Calculate, using linear interpolation and showing your working, the annual effective gross redemption yield of a 4-year bond, redeemable at 105%, with a 2.5% coupon payable annually in arrears. [6]

[Total 12]

- 10** A life insurance company issues a 3-year without-profit endowment assurance. The same sum assured is payable on survival to the end of the policy term or at the end of year of death if earlier. Premiums are payable annually in advance throughout the term of the policy or until earlier death.

Surrenders are only allowed at the end of the policy year. The surrender value is calculated as the sum of the premiums paid to date and is payable at the time of surrender.

A colleague has started to complete the profit test for this policy. Unfortunately, they have mislaid the profit testing basis. The only details they can remember are:

- the annual effective interest rate earned on cashflows is constant over the 3-year term.
- renewal expenses payable at the beginning of the second and third policy years are the same.
- renewal commission is a fixed percentage of premium and is payable at the beginning of the second and third policy years.
- the risk discount rate is constant over the 3-year term.

The incomplete multiple decrement table is given by:

Age	q_x^{death}	q_x^{surr}	$(aq)_x^{death}$	$(aq)_x^{surr}$	$(ap)_x$	${}_{t-1}(ap)_{62}$
62	0.003550	0.1	0.003550	0.099645	0.896805	1.000
63	0.004251	0.05	0.004251	(a)	(b)	(c)
64	0.005073	0	0.005073	0	0.994927	(d)

- (i) Calculate, showing all working, the values of the missing entries (a), (b), (c) and (d) in the table. [2]

The incomplete calculation of the present value of profit is given by:

Policy year	Premium	Initial expense	Renewal expense	Initial commission	Renewal commission	Interest
1	5,000	200	0	1,250	0	(e)
2	5,000		20		125	242.75
3	5,000		(f)		(g)	(h)

Policy year	Expected cost of death benefit	Expected cost of surrender payment	Expected cost of maturity payment	Profit vector	Profit signature	Discount factor	Present value of profit
1	(i)	498.23		(l)	(n)	(p)	(s)
2	85.02	(j)		4,514.86	4,048.95	(q)	3,407.92
3	101.46	0	(k)	(m)	(o)	(r)	(t)

- (ii) Calculate, showing all working, the values for the missing entries (e) to (t). [10]
[Total 12]

11 A life aged 30 exact purchases a 35-year term assurance policy. Level monthly premiums are payable in advance throughout the duration of the contract, ceasing on death, and the sum assured of \$250,000 is payable immediately on death.

(i) Calculate, showing all working, the monthly premium.

Basis:

Mortality AM92 select

Interest 4% p.a. effective

Expenses Initial: \$250 plus 60% of the first monthly premium

Renewal: 3% of the second and subsequent monthly premiums

[7]

The insurance company actually charges a premium of \$50 per month.

The company calculates gross premium **retrospective** reserves, assuming the same basis as in part (i) above but using a rate of interest of 6% p.a. effective.

(ii) Calculate, showing all working, the reserve held for the policyholder at age 55 exact, immediately before the premium then due. [6]

The insurance company is proposing to calculate reserves using the same basis as in part (ii) but adopting a gross premium **prospective** reserving approach.

(iii) Explain, without performing any further calculations, whether the proposal will result in a higher or lower reserve than that calculated in part (ii). [4]

[Total 17]

- 12** A life insurance company offers two alternative 10-year endowment assurance products, each with an initial sum assured of £50,000, to lives aged 55 exact. For both products, level premiums are payable annually in advance throughout the term of the policy or until earlier death.

The benefits are payable at the end of the year of death, or on survival to the end of the term.

Product A

A with-profit endowment assurance with an expected simple reversionary bonus of 2.1% p.a. added at the end of each year. Each increase is applied at the end of the year if the policyholder is still alive.

Product B

An endowment assurance where the initial benefit increases at a guaranteed compound rate of $k\%$ p.a. Each increase is applied at the end of the year if the policyholder is still alive.

The company sets k so that the maturity benefit will be the same under both products.

- (i) Show that k is approximately equal to 1.924%. [2]
- (ii) Calculate the annual premium for **each** product. Show all working. [12]

Basis:

Mortality AM92 Ultimate
Interest 6% p.a. effective

[Total 14]

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