

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2019 (am)

Subject CM2B – Financial Engineering and Loss Reserving Core Principles

Time allowed: One hour and forty-five minutes

INSTRUCTIONS TO THE CANDIDATE

1. *You are given this question paper and four Excel files.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions. Each question is to be answered in a separate document.*

If you encounter any issues during the examination, please contact the Examinations Team at
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1 You have been given claims data for an insurance product with a development period of five years. The figures given are cumulative claim payments in £m and represent the total amount paid by the end of each development year. They have been produced at the end of accident year 2018.

- (i) Calculate, to three decimal places, the development factors for years 1 to 5 using the chain ladder method. You do not need to make any allowance for inflation. [6]
- (ii) Calculate projected cumulative claim payments for all future development years, using the development factors from part (i). You do not need to make any allowance for inflation. [4]
- (iii) Calculate the reserve required at the end of 2018 in respect of future claim payments, using your answer from part (ii). You do not need to make any allowance for inflation or discounting. [4]

You have now been given annual claim payments inflation rates for the 12 months up to the middle of the year for each of the years 2014 to 2018. You can assume all payments are made in the middle of each calendar year.

- (iv) Calculate the projected inflation-adjusted incremental claim payment for accident year 2017 and development year 5. You should use the same method as you used in parts (i) and (ii). You do not need to allow for any inflation beyond 2018. [16]
- [Total 30]

- 2** Two companies, A and B, have each applied to a bank to borrow £10m with a term of ten years and an interest rate of 10% per annum. The loan will be repaid as a lump sum with compound interest at the end of the term. Both companies are already funded by debt and equity.

You can assume that taking out more debt will not affect the companies' share prices. The risk-free force of interest is 6% p.a. The bank is willing to lend only £10m in total.

Data for each company is in the spreadsheet provided.

- (i) Calculate, using the Merton model, the implied volatility of each company's assets before any additional borrowing. [6]
- (ii) Calculate the probability that Company A would be able to repay the loan if it received the full £10m loan. [3]
- (iii) Calculate the probability that Company B would be able to repay the loan if it received the full £10m loan. [3]

The bank is considering lending £5m to each company.

- (iv) Calculate the probability that both companies will be able to repay their respective loans. [4]
- (v) Explain, using further calculations where appropriate, why the bank might want to split its lending equally between companies A and B despite the lower probability of all the money being repaid. [8]

[Total 24]

- 3 An investor holds a portfolio of shares and European derivatives as set out below.

	<i>Units held</i>	<i>Current price</i>	<i>Strike price</i>	<i>Term</i>	<i>Volatility</i>
Share	1,000	\$10	n/a	n/a	15%
Call option	200	(a)	\$15	2 years	n/a
Put option	1,000	(b)	\$8	2 years	n/a

You have been given a distribution of the possible share prices at time $t = 2$ in the spreadsheet. The risk-free rate is 4% per annum.

- (i) Calculate, using the information in the table above, the price of:
- (a) the call option
 - (b) the put option.
- [2]
- (ii) Calculate the value of the investor's portfolio at time $t = 0$. [1]
- (iii) Calculate, for each possible share price provided, the value of the investor's portfolio at time $t = 2$. [4]

The investor needs at least \$15,000 at time $t = 2$ to repay a loan.

- (iv) Calculate the probability that the investor will be able to repay the loan. [2]
- (v) Calculate the expected shortfall relative to the loan repayment. [2]

The investor wishes to maximise the probability of being able to repay the loan. She wishes to retain the same holding in shares and is therefore considering changing only her holding of options. Short selling is allowed.

- (vi) Derive a portfolio that will maximise the probability that the investor can repay the loan at time $t = 2$. [4]

The investor is now considering changing her holding in shares as well in order to maximise the probability that she will be able to repay the loan.

- (vii) Using only the three assets above:
- (a) Suggest how the investor could maximise the chances of being able to repay the loan.
 - (b) Discuss the risks involved with your suggestion in part (vii)(a).

[6]
[Total 21]

- 4 An individual joins a pension scheme at age 50 exact and intends to retire at age 60 exactly. He will pay £5,000 into the scheme annually at the start of each year. Contributions will be invested in an asset with annual returns which follow a lognormal distribution.

You have been provided with five simulations of the annual return on the scheme's assets for each of the next ten years. Each simulation is considered to be equally likely.

- (i) Calculate the expected value of the fund at age 60. [6]

The individual hopes to retire at age 60 with a fund value of £70,000.

- (ii) Calculate the probability that the fund value at age 60 will exceed £70,000. [2]

The investor decides to assess his level of wealth w using a utility function $U(w)$ as follows:

$$U(w) = \begin{cases} 0.001w & \text{for } w \leq £70,000 \\ 70 & \text{for } w > £70,000 \end{cases}$$

- (iii) Calculate the expected utility of the fund at age 60. [2]

Instead of contributing £5,000 into the scheme on an annual basis, the individual plans to invest a lump sum of £50,000 into the fund at age 50. To do this, he will borrow £50,000 using a loan which attracts interest at 4% per annum.

The individual plans to make regular loan repayments of £5,000 annually in arrears for the next ten years in order to repay the loan. Each instalment will cover any interest accumulated to date with the balance used to reduce the outstanding loan amount. At the end of the ten years, at age 60, the outstanding loan amount will be repaid from the pension fund.

- (iv) Calculate the probability that the fund value at age 60, after repaying any outstanding loan, will exceed £70,000. [6]

- (v) Calculate the expected utility of the fund at age 60, after repaying the outstanding loan. [2]

- (vi) Comment on:

- (a) The optimal approach suggested by your answers to (ii) and (iv).
(b) The optimal approach suggested by your answers to (iii) and (v).
(c) Why these approaches might suggest different courses of action.

[7]

[Total 25]

END OF PAPER